Randomized Protocols for Node Discovery in Ad-hoc Multichannel Broadcast Networks

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Abstract

The first step in forming an ad-hoc network is the discovery of other nodes. This procedure, called *node discovery*, involves two or more nodes in an ad-hoc, multichannel broadcast system that want to establish a common communication channel. Each node can either talk or listen in anyone of f frequencies, here denoted by $1, 2, \ldots, f$. The decision on whether or not a node will talk or listen as well as on which channel depends on a given probability distribution representing channel allocation. Node discovery is accomplished when all nodes agree on a common communication channel. We consider protocols for node discovery in a two node, multi-frequency system. We also propose and study protocols for node discovery in ad-hoc, multi-node, multi-frequency system.

1 Introduction

The increasing use of wireless devices is inevitably bringing to the forefront new hortizons and paradigms in applications of wireless networking. Ad-hoc networks are an "infrastructureless" technology that lies at the forefront of new wireless solutions in home networking, personal area networks, and connectivity of sensor devices, to mention a few. One of the main tasks of today's research in ad-hoc networking is to eliminate the shortcomings of mobility and wireless computing [10]. Ad-hoc networks form wireless, self-organizing systems by co-operating nodes within communication (either short- or long-) range of each other, thus constituting a decentralized but dynamically changing topology consisting of moving but inter-communicating nodes.

Forming an ad-hoc network is a complicated task involving the application of a sequence of fundamental procedures that enforce self-organization and establish node connectivity. *Node discovery* is one of these fundamental procedures. A given node tries to discover which other nodes are within its range as follows: (a) the node bradcasts a message, and (b) waits to receive response from other nodes. After receiving a response from another node the former node knows that its presence is known to the latter node. However, in a decentralized environment nodes are initially uncoordinated and *collisions* occur due to the simultaneous access of a shared channel.

Several solutions have been attempted in the past. In a packet radio network, collisions are caused by either *direct* or *secondary* interference [4]. The former occurs when two nodes transmit to each other at the same time, while the latter occurs when nodes unaware of each other's presence attempt to transmit at the same time. Collision avoidance in a shared broadcast channel has been widely studied [4]. Existing collision avoidance protocols are based on the exchange of control messages among the nodes in order to dynamically establish a transmission schedule with the highest possible throughput. However, all these protocols assume nodes are already aware of the presence of other nodes. In other words, they assume that node discovery has already taken place.

To solve this problem when a single broadcast channel is shared among all nodes, randomized backoff protocols [2] are employed. However, ad-hoc networks differ from traditional broadcast systems, like Ethernet [2], because collision detection may not be possible. Another technique is frequency hopping: it minimizes the probability of collisions by having nodes transmitting in multiple channels (e.g., in Bluetooth [6]).

In the context of Bluetooth Salonidis *et al.* [12, 13] propose and analyze a symmetric protocol for *2-node link formation.* which is based on random schedules. The resulting *alternating states* protocol has been used as the basic building block for a scatternet formation protocol [13]. Law *et al.* [8] propose a probabilistic protocol for node discovery: a node decides with probability p to start discovering other nodes or, with probability $1-p$, to listen until it is discovered by another node. A node gives up either if it does not discover another node or does not hear from any other node in a period of time. The protocols aim at establishing only one-to-one connections. The number of connections established in each round of the protocol is the smaller of the number of nodes in discovering mode and the number of nodes waiting to be discovered. In addition, in Alonso *et al.* [1] we proposed and analyzed probabilistic protocols for node discovery in ad-hoc, *single broadcast channel* networks.

This paper makes several contributions. We extend the model of Alonso *et al.* [1] in order to handle two-node and multiple-node, ad-hoc, *multiple channel (i.e., frequency)*, broadcast networks. We propose and analyze several randomized protocols for node discovery: (a) in the case of two node systems we propose and analyze random and non-random protocols, while (b) in the case of multinode systems we analyze the random protocol and the random unicast protocols. We also distinguish static and dynamic protocols (in the first case success occurs within the same frequency, while in the second success may occur in different frequencies). Our optimization criteria are based on the (expected waiting) time for two nodes to discover each other in the presence of other nodes also engaged in node discovery.

An outline of the paper is as follows. Section 2, gives the model used throughout the paper. Sections 3 and 4 consider random and non-random protocols, respectively, for two node systems and Section 5 studies random protocols in a multinode ad-hoc network and Section 6 the random unicast protocols.

2 Node Discovery in Multiple-Node Ad-hoc Networks

2.1 Communication model

First we discuss and extend the model of Alonso *et al* [1]. We assume a system with K nodes communicating by broadcasting messages. At any given time a node can be at a given frequence $i = 1, 2, \ldots, f$ either T (Talking) or L (Listening). The state of a node is denoted by the pair (S, i) where $S = T$ or $S = L$ and $i = 1, 2, \ldots, f$. The nodes are synchronized: they change states at the same time and remain in a given state for a period of time that is identical for all nodes. An event E describes the state of the K nodes of the system:

$$E = \begin{pmatrix} S_1 & i_1 \\ S_2 & i_2 \\ \vdots & \vdots \\ S_K & i_K \end{pmatrix}, \tag{1}$$

where (S_k, i_k) is the state of the k-th node. For each event E we denote by k^E the state of the kth node in event E. The conditions for a node k to receive a message from another node l are defined as follows. A node k receives a message from the other node l if, at any given time,

1. nodes k, l are at the same frequency, say i,
2. node k is listening, while node l is talking at frequency i, and
3. all other nodes in the system are either in another frequency or else they are *listening* in frequency i.

A run of a protocol is a sequence of events. A run terminates if two nodes have discovered each other. This means that the last two events E, E' satisfy the following property:

1. in event E, node k receives a message from node l, and
2. in event E', node l receives a message from node k.

The decision on whether or not a node will talk or listen as well as on which channel depends on a given deterministic/randomized algorithm and channel allocation depends on a given probability distribution. We use the notation $E \to E'$ to indicate that event E' succeeds event E in a given run of the protocol.

2.2 Probability distribution of talking/listening in a frequency

A node is represented by a random variable X assuming the values (S, i), where S is either T or L and $i = 1, 2, \ldots, f$. Associated with this random variable is a probability distribution of the frequencies. If F_i is the probability that a node is in frequency i, p is the probability that a node is talking, and q is the probability that a node is listening then we have

$$p_i = \Pr[X = (T, i)] = pF_i, q_i = \Pr[X = (L, i)] = qF_i$$

It is therefore clear that

$$p + q = 1, \quad \sum_{i=1}^{f} F_i = 1. \quad \sum_{i=1}^{f} p_i + \sum_{i=1}^{f} q_i = 1.$$

We assume that the nodes have identical probability distributions of frequency allocation, i.e., the frequency allocations are i.i.d. (independent, identically distributed) random variables.

2.3 Dynamic and static frequency allocation

Given the model above, we analyze two types of node discovery protocols: *dynamic* and *static* frequency allocation. In either case, the protocol succeeds if two nodes discover each other. The two types of protocols differ in the way they allocate frequencies. In the first type, with *static* frequency allocation, the first node talks and the second listens in a given frequency and, in the next step, the second node talks and the first listens in this same frequency. In the second type, with *dynamic* frequency allocation, the first node talks and the second listens in a given frequency and, in the next step, the second node talks and the first listens in this same or in a different frequency.

3 Random Protocols in Two-Node Systems

The behavior of the nodes is represented by two i.i.d. r.v.s X, X'. An event is a pair $\begin{pmatrix} S & i \\ S' & i' \end{pmatrix}$ where S, S' are either T or L and i, i' are frequencies. Consider the events

$$\mathcal{A} := \{A_1, A_2, \ldots, A_f, B_1, B_2, \ldots, B_f\}$$

defined as follows:

$$A_i := \begin{pmatrix} T & i \\ L & i \end{pmatrix}, B_i := \begin{pmatrix} L & i \\ T & i \end{pmatrix}$$

As a consequence of the assumptions of our model we have that A_i and B_i occur with probability $p_i q_i$, respectively.

By Random Protocol (**RP**), we understand a protocol in which each node decides at random whether to talk or listen. At each time instance, the two nodes generate a event which consists of an instantantiation of the the two random variables X and X'. We consider and analyze two **RP** algorithms: with static frequency allocation and with dynamic frequency allocation. Our analysis of these two algorithms uses the technique described in [3, 9].

3.1 Static frequency allocation

Theorem 1 (RP with Static Frequency Allocation) *The expected waiting time for protocol* **RP** *with static frequency allocation to succeed is given by*

$$1/\sum_{i=1}^{f} \frac{2}{\left(\frac{1}{p_i q_i} + \frac{1}{p_i^2 q_i^2}\right)} \tag{2}$$

Proof (Theorem 1). The protocol succeeds if, for some $i = 1, 2, \ldots, f$, it terminates with either

$$\begin{pmatrix} T & i \\ L & i \end{pmatrix} \to \begin{pmatrix} L & i \\ T & i \end{pmatrix} \text{ or } \begin{pmatrix} L & i \\ T & i \end{pmatrix} \to \begin{pmatrix} T & i \\ L & i \end{pmatrix} \tag{3}$$

A success pattern is a pair such that the two nodes discover each other. Consider the following $2f$ "success" patterns $A_1B_1, A_2B_2, \ldots, A_fB_f, B_1A_1, B_2A_2, \ldots, B_fA_f$ in this order. Using the previous notation, the algorithm succeeds when, for some $i = 1, 2, \ldots, f$ either of the patterns A_iB_i or B_iA_i occurs.

To analyze the protocol we use the method of patterns as described in [3, 9]. A_iB_i can overlap only with itself or with the first letter of B_iA_i. In the first case, the overlap is of length 2 and in the second of length 1. Similarly, B_iA_i can overlap only with itself or with the first letter of A_iB_i.

As a measure of the amount of overlap between patterns we have the quantities

$$e_{i,j} = \begin{cases} \frac{1}{p_i^2 q_i^2} & \text{if } i = j \leq f \\ \frac{1}{p_{i-f}^2 q_{i-f}^2} & \text{if } i = j > f \\ \frac{1}{p_i q_i} & \text{if } |i - j| = f \\ 0 & \text{otherwise} \end{cases}$$

i.e., we have the $2f$ patterns described above in the order

$$A_1B_1, A_2B_2, \ldots, A_fB_f, B_1A_1, B_2A_2, \ldots, B_fA_f.$$

The probability that the letter A_i occurs is p_iq_i and similarly for B_i. When a pattern A_iB_i (respectively, B_iA_i) overlaps with itself, the probability of its occurrence is $p_i^2q_i^2$. When the last letter of the pattern A_iB_i overlaps with the first letter of B_iA_i, the probability of the occurrence of this letter is p_iq_i.

Using the notation N for the waiting time the resulting system of linear equations is as follows.

$$\left[\begin{array}{cccc|cccc} \frac{1}{p_1^2q_1^2} & 0 & \cdots & 0 & \frac{1}{p_1q_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{p_2^2q_2^2} & \cdots & 0 & 0 & \frac{1}{p_2q_2} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \frac{1}{p_f^2q_f^2} & 0 & 0 & \cdots & \frac{1}{p_fq_f} \\ \hline \frac{1}{p_1q_1} & 0 & \cdots & 0 & \frac{1}{p_1^2q_1^2} & 0 & \cdots & 0 \\ 0 & \frac{1}{p_2q_2} & \cdots & 0 & 0 & \frac{1}{p_2^2q_2^2} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \frac{1}{p_fq_f} & 0 & 0 & \cdots & \frac{1}{p_f^2q_f^2} \end{array}\right] \cdot \begin{bmatrix} \pi_1 \\ \pi_2 \vdots \\ \pi_f \\ \pi_{f+1} \\ \pi_{f+2} \\ \vdots \\ \pi_{2f} \end{bmatrix} = \begin{bmatrix} E[N] \\ E[N] \\ \vdots \\ E[N] \\ E[N] \\ E[N] \\ \vdots \\ E[N] \end{bmatrix} \tag{4}$$

Here, π_i (respectively, π_{i+f}) is the probability that pattern A_iB_i (respectively, B_iA_i) occurs before any other pattern. In addition, we have the condition

$$\sum_{i=1}^{2f} \pi_i = 1. \tag{5}$$

Now we solve the System consisting of the linear equations in (4) and (5). If we add the i-th and $i+f$-th equations in System (4), we obtain

$$(\pi_i + \pi_{i+f}) \left(\frac{1}{p_i q_i} + \frac{1}{p_i^2 q_i^2} \right) = 2E[N],$$

which implies that

$$(\pi_i + \pi_{i+f}) = \frac{2E[N]}{\left(\frac{1}{p_i q_i} + \frac{1}{p_i^2 q_i^2} \right)}. \tag{6}$$

If we add Equations (6), for $i = 1, 2, \ldots, f$, we obtain

$$1 = \sum_{i=1}^{f} \frac{2E[N]}{\left(\frac{1}{p_i q_i} + \frac{1}{p_i^2 q_i^2} \right)}.$$

It follows that the expected number of steps is given by Equation (2). This completes the proof of Theorem 1. □

3.2 Dynamic frequency allocation

Theorem 2 (RP with Dynamic Frequency Allocation) *The expected waiting time for the protocol* **RP** *with dynamic frequency allocation to succeed is given by*

$$\frac{1 + \sum_{j=1}^{f} p_j q_j}{2 \left(\sum_{j=1}^{f} p_j q_j \right)^2}. \tag{7}$$

Proof (Theorem 2). The protocol succeeds if for some $i, j = 1, 2, \ldots, f$, either of the events depicted in 3 occurs. In this instance, we have $2f^2$ "success" patterns, namely,

$$\begin{array}{cccc}
A_1B_1 & A_1B_2 & \cdots & A_1B_f \\
A_2B_1 & A_2B_2 & \cdots & A_2B_f \\
\vdots & \vdots & \cdots & \vdots \\
A_fB_1 & A_fB_2 & \cdots & A_fB_f \\
\hline
B_1A_1 & B_1A_2 & \cdots & B_1A_f \\
B_2A_1 & B_2A_2 & \cdots & B_2A_f \\
\vdots & \vdots & \cdots & \vdots \\
B_fA_1 & B_fA_2 & \cdots & B_fA_f
\end{array}$$

Observe that, for each $i, j = 1, 2 \ldots, f$, either the pattern A_iB_j (respectively, B_iA_j) may overlap with itself, or the last letter of A_iB_j (respectively, B_iA_j) may overlap with the first letter of B_jA_k (respectively, A_jB_k), for $k = 1, 2, \ldots, f$. In the first case, the probability of occurrence is $\frac{1}{p_i q_i p_j q_j}$ while, in the second, it is $\frac{1}{p_j q_j}$.

This gives rise to a $(2f^2) \times (2f^2)$ matrix which is defined as follows $\left[\begin{array}{c|c} D & U \\ \hline U & D \end{array}\right]$. The entries of the matrices are as follows.

- D is a diagonal $f^2 \times f^2$ matrix such that the $((i,j),(i,j))$-entry is $\frac{1}{p_i^2 q_j^2}$.
- U is an $f^2 \times f^2$ matrix which is a column of f matrices $U = [V, V, \ldots, V]^T$, where V is an $f \times f^2$ matrix defined as follows.

$$V = \begin{bmatrix}
\frac{1}{p_1q_1} & \frac{1}{p_2q_2} & \cdots & \frac{1}{p_fq_f} & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & \frac{1}{p_1q_1} & \frac{1}{p_2q_2} & \cdots & \frac{1}{p_fq_f} & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & \frac{1}{p_1q_1} & \frac{1}{p_2q_2} & \cdots & \frac{1}{p_fq_f}
\end{bmatrix}$$

We have a system of linear equations consisting of $2f^2+1$ unknowns and $2f^2+1$ equations. Let $\pi(A_iB_j)$ (respectively, $\pi(B_iA_j)$) be the probability that pattern A_iB_j (respectively, B_iA_j) occurs before any other pattern. These variables and $E[N_2]$ are the unknowns of the system.

The first equation of the system is

$$\sum_{s=1}^{f}\sum_{r=1}^{f}\pi(A_sB_r)+\sum_{s=1}^{f}\sum_{r=1}^{f}\pi(B_sA_r)=1, \tag{8}$$

There are $2f^2$ additional equations which we describe in the sequel. Using the notation N for the waiting time, the top-half f^2 equations of the system consist of the following linear equations

$$\begin{aligned}
E[N] &= \frac{1}{p_iq_ip_1q_1}\pi(B_iA_1)+\frac{1}{p_1q_1}\sum_{r=1}^{f}\pi(B_1A_r)\\
E[N] &= \frac{1}{p_iq_ip_2q_2}\pi(B_iA_2)+\frac{1}{p_2q_2}\sum_{r=1}^{f}\pi(B_2A_r)\\
&\vdots\\
E[N] &= \frac{1}{p_iq_ip_fq_f}\pi(B_iA_f)+\frac{1}{p_fq_f}\sum_{r=1}^{f}\pi(B_fA_r)
\end{aligned}$$

for $i=1,2,\ldots,f$. It follows that

$$\begin{aligned}
p_1q_1E[N] &= \frac{1}{p_iq_i}\pi(B_iA_1)+\sum_{r=1}^{f}\pi(B_1A_r)\\
p_2q_2E[N] &= \frac{1}{p_iq_i}\pi(B_iA_2)+\sum_{r=1}^{f}\pi(B_2A_r)\\
&\vdots\\
p_fq_fE[N] &= \frac{1}{p_iq_i}\pi(B_iA_f)+\sum_{r=1}^{f}\pi(B_fA_r)
\end{aligned}$$

Adding these last equations, we obtain

$$\left(\sum_{j=1}^{f}p_jq_j\right)E[N]=\frac{1}{p_iq_i}\sum_{j=1}^{f}\pi(B_iA_j)+\sum_{s=1}^{f}\sum_{r=1}^{f}\pi(B_sA_r). \tag{9}$$

Similarly, we also have the following f^2 linear equations from the bottom-half of the system

$$\begin{aligned}
E[N] &= \frac{1}{p_iq_ip_1q_1}\pi(A_iB_1)+\frac{1}{p_1q_1}\sum_{r=1}^{f}\pi(A_1B_r)\\
E[N] &= \frac{1}{p_iq_ip_2q_2}\pi(A_iB_2)+\frac{1}{p_2q_2}\sum_{r=1}^{f}\pi(A_2B_r)\\
&\vdots\\
E[N] &= \frac{1}{p_iq_ip_fq_f}\pi(A_iB_f)+\frac{1}{p_fq_f}\sum_{r=1}^{f}\pi(A_fB_r)
\end{aligned}$$

for $i=1,2,\ldots,f$. Using similar transformations we obtain

$$\left(\sum_{j=1}^{f}p_jq_j\right)E[N]=\frac{1}{p_iq_i}\sum_{j=1}^{f}\pi(A_iB_j)+\sum_{s=1}^{f}\sum_{r=1}^{f}\pi(A_sB_r). \tag{10}$$

If we multiply Equations (10) and (9) by $p_i q_i$, add them, and use Equation 8, we obtain

$$2p_i q_i \left(\sum_{j=1}^{f} p_j q_j \right) E[N] = \sum_{j=1}^{f} (\pi(A_i B_j) + \pi(B_i A_j)) + p_i q_i. \tag{11}$$

for $i = 1, 2, \ldots, f$. Adding Equations (11), for $i = 1, 2, \ldots, f$, and using again Equation 8, we obtain

$$2 \left(\sum_{j=1}^{f} p_j q_j \right)^2 E[N] = 1 + \sum_{j=1}^{f} p_j q_j.$$

From this, we easily obtain Equation 7. This completes the proof of Theorem 2. □

4 Non-Random Protocols in Two-Node Systems

Thus far, a node has randomly chosen whether to talk or listen, and has also randomly chosen a frequency. We shall now consider protocols where a node's behaviour is dictated by a simple set of rules. In the *answering* protocol, **AP**, a node that receives a message will answer, i.e., talk, in the next step. In the *listening* protocol, a node that sent a message, i.e., talked, will listen in the next step. The *answering* and *listening* protocols can be implemented with either static or dynamic frequency allocation. The results for the respective protocols are shown below.

4.1 Phases of the non-random prototols

When studying the run of a non-random protocol, it is useful to view a given subrun as having two phases. In the first phase, the protocol acts like a random protocol until the nodes are at the same frequency and one node is listening while the other node is talking. The second phase of a protocol has one step. Phase two is successful, and the algorithm terminates, if the nodes reverse the roles taken in phase one, i.e., both nodes are at the same frequency and the node that talked (listened) in phase one listens (talks) in phase two. If phase two is not successful, another subrun starts, i.e., phase one begins again.

4.2 Wald's identity

It is useful to know the expected number of subruns, as well as the expected length of a subrun, for a given protocol, as Wald's identity then allows us to calculate the expected length of a run of a protocol. Wald's identity can be stated as follows [11]. Let $\{W_i, i \geq 1\}$ be independent and identically distributed random variables with a finite mean (i.e., $E[W] < \infty$) and let N be a stopping time for $W_1, W_2, \ldots$ (i.e., the event $\{N = n\}$ is independent of $W_{n+1}, W_{n+2}, \ldots$, for all $n \geq 1$) such that $E[N] < \infty$. Then

$$E\left[\sum_{i=1}^{N} W_i \right] = E[W]E[N]. \tag{12}$$

To apply Wald's identity to the present problem, let X_i be the length of a subrun of a given protocol and let N be the number of subruns for a given protocol. The X_i are identically distributed random variables with a finite mean, and N is a random variable with non-negative integer values and finite mean. Since the length of a subrun is independent of the number of subruns for a given protocol, X_i is independent of N, Wald's identity implies that the expected length of a run of a given protocol is the product of the expected length of a subrun and the expected number of subruns, i.e.,

$$E[\text{length of run}] = E[\text{length of subrun}] \cdot E[\text{number of subruns}]. \tag{13}$$

In the sequel, we use Wald's identity to analyze the answering and listening protocols.

4.3 AP with dynamic/static frequency allocation

In the *answering* protocol, **AP**, a node that receives a message will answer, i.e., talk, in the next step.

First we analyze the Answering Protocol with dynamic frequency allocation.

Theorem 3 (AP with Dynamic Frequency Allocation) *The expected length of a run of the* **AP** *protocol with dynamic frequency allocation, is given by the expression:*

$$\frac{1 + 2pq\sum_{i=1}^{f} F_i^2}{2pq^2(\sum_{i=1}^{f} F_i^2)^2} \tag{14}$$

Proof (Theorem 3). The expected length of phase 1, and thus the expected length of a subrun, is independent of the type of frequency allocation used. In phase 1, the **AP** protocol acts as a random protocol until either of the events

$$\begin{pmatrix} T & i \\ L & i \end{pmatrix} \text{ or } \begin{pmatrix} L & i \\ T & i \end{pmatrix} \tag{15}$$

occurs for some frequency $i = 1, 2, \ldots, f$. Given that these events occur with probability

$$2pq \sum_{i=1}^{f} F_i^2 \tag{16}$$

and phase 1 is a sequence of independent events, the expected length of phase 1 is

$$\frac{1}{2pq\sum_{i=1}^{f} F_i^2} \tag{17}$$

Phase 2 consists of a single step, so the expected length of a subrun of the **AP** protocol is

$$1 + \frac{1}{2pq\sum_{i=1}^{f} F_i^2} \tag{18}$$

In the answering protocol **AP**, the node that listened at frequency i in the last step of phase 1 "answers", i.e., talks, in phase 2. However, in the dynamic frequency allocation case, the node may choose any of the f frequencies on which to answer. Success in phase 2 occurs if the other node listens on that same frequency, so the probability of success is $\sum_{i=1}^{f} q{F_i}^2$, and thus the expected number of subruns is

$$E[\text{number of subruns}] = \sum_{k=1}^{\infty} k \left(1 - \sum_{i=1}^{f} q{F_i}^2\right)^{k-1} \left(\sum_{i=1}^{f} q{F_i}^2\right) = \frac{1}{\sum_{i=1}^{f} q{F_i}^2}$$

As a result, the expected length of a run of the **AP** protocol with dynamic frequency allocation is

$$E[\text{number of steps}] = \left(1 + \frac{1}{2pq\sum_{i=1}^{f} F_i^2}\right)\left(\frac{1}{\sum_{i=1}^{f} qF_i^2}\right) = \frac{1 + 2pq\sum_{i=1}^{f} F_i^2}{2pq^2(\sum_{i=1}^{f} F_i^2)^2}$$

This completes the proof of Theorem 3. □

We can prove a similar result for static frequency allocation. In the **AP** protocol with static frequency allocation, the node that listened at frequency i in the last step of phase 1 will "answer", i.e., talk, on frequency i in phase 2. Success occurs in phase 2 if the other node listens on frequency i.

Theorem 4 (AP with Static Frequency Allocation) *The expected length of a run of the* **AP** *protocol with static frequency allocation is given by the expression:*

$$\frac{1 + 2pq\sum_{i=1}^{f} F_i^2}{2pq^2(\sum_{i=1}^{f} F_i^2)^2}. \tag{19}$$

Proof (Theorem 4). In phase 1, the **AP** protocol acts as a random protocol until either of the events depicted in 15 occurs for some frequency $i = 1, 2, \ldots, f$. Given that these events occur with the same probability as that given by Formula 16 we obtain the same formula for the expected waiting time. This completes the proof of Theorem 4. □

4.4 LP with dynamic/static frequency allocation

In the *listening* protocol **LP**, a node that sent a message, i.e., talked, will listen in the next step.

Under dynamic frequency allocation, the node that talks in phase one will listen at a randomly chosen frequency, j, in phase two. Success in phase two occurs if the node that listened in phase one subsequently talks at frequency j in phase two.

Theorem 5 (LP with Dynamic Frequency Allocation) *The expected length of a run of the listening protocol with dynamic frequency allocation, is given by the expression:*

$$\frac{1 + 2pq\sum_{i=1}^{f} F_i^2}{2p^2q(\sum_{i=1}^{f} F_i^2)^2} \tag{20}$$

Proof (Theorem 5). The expected length of phase 1, and thus the expected length of a subrun, is independent of the type of frequency allocation used. In phase 1, the **LP** protocol acts as a random protocol until either of the events

$$\begin{pmatrix} T & i \\ L & i \end{pmatrix} \text{ or } \begin{pmatrix} L & i \\ T & i \end{pmatrix} \tag{21}$$

occurs for some frequency $i = 1, 2, \ldots, f$. Given that these events occur with probability

$$2pq\sum_{i=1}^{f} F_i^2$$

and phase 1 is a sequence of independent events, the expected length of phase 1 is

$$\frac{1}{2pq\sum_{i=1}^{f} F_i^2}$$

Phase 2 consists of a single step, so the expected length of a subrun of the AP protocol is

$$1 + \frac{1}{2pq\sum_{i=1}^{f} F_i^2}$$

In the **LP** protocol, the node that talked at frequency i in the last step of phase 1 istens in phase 2. However, in the dynamic frequency allocation case, the node may choose any of the f frequencies on which to answer. Success in phase 2 occurs if the other node talks on that same frequency, so the probability of success is $\sum_{i=1}^{f} p{F_i}^2$, and thus the expected number of subruns is

$$E[\text{number of subruns}] = \sum_{k=1}^{\infty} k \left(1 - \sum_{i=1}^{f} p{F_i}^2\right)^{k-1} \left(\sum_{i=1}^{f} p{F_i}^2\right) = \frac{1}{\sum_{i=1}^{f} p{F_i}^2}$$

As a result, the expected length of a run of the **LP** protocol with dynamic frequency allocation is

$$E[\text{number of steps}] = \left(1 + \frac{1}{2pq\sum_{i=1}^{f} F_i^2}\right)\left(\frac{1}{\sum_{i=1}^{f} pF_i^2}\right) = \frac{1 + 2pq\sum_{i=1}^{f} F_i^2}{2p^2q(\sum_{i=1}^{f} F_i^2)^2}$$

This completes the proof of Theorem 5. □

We can prove a similar result for static frequency allocation. Under static frequency allocation, the node that talks at frequency i in phase one will listen at the same frequency in phase two. Success in phase two occurs if the node that listened at frequency i in phase one subsequently talks at frequency i in phase two.

Theorem 6 (LP with Static Frequency Allocation) *The expected length of a run of the listening protocol,* **LP***, with static frequency allocation is given by the expression:*

$$\frac{1 + 2pq\sum_{i=1}^{f} F_i^2}{2p^2q(\sum_{i=1}^{f} F_i^2)^2}. \tag{22}$$

Proof (Theorem 6). This follows easily from the proof of Theorem 5 in a manner similar to the proof of Theorem 4, and we omit the details □

5 Random Protocols in Multiple Node Systems

In this section we consider the case of K nodes, $K \geq 2$. We analyze two types of node doscovery protocols: static and dynamic frequency allocation. An event describes the state of the K nodes of the system. Consider the events

$$A_i^{ab} := \begin{pmatrix} \vdots & \vdots \\ T & i \\ \vdots & \vdots \\ L & i \\ \vdots & \vdots \end{pmatrix}, \quad \text{respectively, } B_i^{ab} := \begin{pmatrix} \vdots & \vdots \\ L & i \\ \vdots & \vdots \\ T & i \\ \vdots & \vdots \end{pmatrix}$$

for which $a < b$ and

1. the ath node depicted is in state T, i (respectively, L, i),
2. the bth node depicted is in state L, i (respectively, T, i), and
3. for all $c \neq a, b$ either node c is listening at any frequency or else node c is talking at a frequency other than i.

Clearly, there are $2f\binom{K}{2}$ such events A_i^{ab}, B_i^{ab}, where $i = 1, 2, \ldots, f$, $a < b$, and $a, b = 1, 2, \ldots, K$. Consider the event that the K-node system is in the state A_i^{ab}, i.e., $X = A_i^{ab}$. Using the definition of A_i^{ab} above we observe that event $X = A_i^{ab}$ is the intersection of the above three events. Hence, it is easy to see that

$$\Pr[X = A_i^{ab}] = p_i q_i \prod_{c \neq a,b} \left(\sum_{j=1}^{f} q_j + \sum_{j=1, j \neq i}^{f} p_j \right) = p_i q_i (1 - p_i)^{N-2}. \tag{23}$$

Similarly, we obtain that $\Pr[X = B_i^{ab}] = p_i q_i (1 - p_i)^{K-2}$.

5.1 RP with static frequency allocation

In the random protocol with static frequency allocation success occurs if for some frequency $i = 1, 2, \ldots, f$ and two nodes $a < b$, event A_i^{ab} (respectively, B_i^{ab}) is followed by event B_i^{ab} (respectively, A_i^{ab}). It follows that the success patterns are $A_i^{ab} B_i^{ab}$ and $B_i^{ab} A_i^{ab}$.

Theorem 7 *The expected waiting time for protocol* **RP** *with K nodes and static frequency allocation to succeed is given by*

$$1 / \left(2 \binom{K}{2} \sum_{i=1}^{f} \frac{1}{\left(\frac{1}{p_i^2 q_i^2 (1-p_i)^{2K-4}} + \frac{1}{p_i q_i (1-p_i)^{K-2}} \right)} \right) \tag{24}$$

Proof (Theowem 7). For $a < b$, let $\pi(i,a,b)$ (respectively, $\pi(i,b,a)$) be the probability that the success pattern $A_i^{ab}B_i^{ba}$ (respectively, $B_i^{ab}A_i^{ba}$) occurs before any other success pattern. We have the identity

$$\sum_{i=1}^{f}\sum_{a<b}\pi(i,a,b)+\sum_{i=1}^{f}\sum_{a<b}\pi(i,b,a)=1. \tag{25}$$

In addition, using Identity (23), we obtain a system of $2f\binom{K}{2}$ additional linear equations:

$$\begin{aligned} E[N] &= \frac{\pi(i,a,b)}{p_i^2q_i^2(1-p_i)^{2K-4}}+\frac{\pi(i,b,a)}{p_iq_i(1-p_i)^{K-2}} \\ E[N] &= \frac{\pi(i,a,b)}{p_iq_i(1-p_i)^{K-2}}+\frac{\pi(i,b,a)}{p_i^2q_i^2(1-p_i)^{2K-4}}, \end{aligned}$$

where N denotes the waiting time of the protocol. Adding these two equations we obtain

$$2E[N]=\left(\frac{1}{p_i^2q_i^2(1-p_i)^{2K-4}}+\frac{1}{p_iq_i(1-p_i)^{K-2}}\right)(\pi(i,a,b)+\pi(i,b,a)),$$

which implies that

$$(\pi(i,a,b)+\pi(i,b,a))=\frac{2E[N]}{\left(\frac{1}{p_i^2q_i^2(1-p_i)^{2K-4}}+\frac{1}{p_iq_i(1-p_i)^{K-2}}\right)}.$$

Adding these equations and using Identity (25) we obtain Identity (24). This completes the proof of Theorem 7. □

5.2 RP with dynamic frequency allocation

In the random protocol with dynamic frequency allocation success occurs if for some frequencies $i,j=1,2,\ldots,f$ (we do not exclude the case $i=j$) and two nodes $a<b$, event A_i^{ab} (respectively, B_i^{ab}) is followed by event B_j^{ab} (respectively, A_j^{ab}). It follows that the success patterns are

$$A_i^{ab}B_1^{ab},A_i^{ab}B_2^{ab},\ldots,A_i^{ab}B_f^{ab}, \text{ and } B_i^{ab}A_1^{ab},B_i^{ab}A_2^{ab},\ldots,B_i^{ab}A_f^{ab}. \tag{26}$$

Theorem 8 *The expected waiting time for protocol* **RP** *with K nodes and dynamic frequency allocation to succeed is given by*

$$\frac{1+\sum_{j=1}^{f}p_jq_j(1-p_j)^{K-2}}{2\binom{K}{2}\left(\sum_{j=1}^{f}p_jq_j(1-p_j)^{K-2}\right)^2}. \tag{27}$$

Proof (Theowem 8). For $a<b$, let $\pi(A_i,B_j,a,b)$ (respectively, $\pi(B_iA_,j,a,b)$) be the probability that the success pattern $A_i^{ab}B_j^{ba}$ (respectively, $B_i^{ab}A_j^{ba}$) occurs before any other success pattern. We have the identity

$$\sum_{i,j=1}^{f}\sum_{a<b}\pi(A_i,B_j,a,b)+\sum_{i,j=1}^{f}\sum_{a<b}\pi(A_i,B_j,a,b)=1. \tag{28}$$

To obtain the linear system of equations we observe that for each i,j either pattern $A_i^{ab}B_J^{ab}$ (respectively, $B_i^{ab}A_J^{ab}$) can overlap with itself, or the last letter B_j^{ab} (respectively, A_j^{ab}) of $A_i^{ab}B_J^{ab}$ (respectively, $B_i^{ab}A_J^{ab}$) can overlap with the first letter of any of $B_j^{ab}A_k^{ab}$ (respectively, $A_j^{ab}B_k^{ab}$), for $k=1,2,\ldots,f$. It follows that using Identity (23), we obtain a system of $2f^2\binom{K}{2}$ additional linear equations. For each $a < b$ and $i = 1, 2, \ldots, f$ we give the $2f$ linear equations below:

$$
\begin{aligned}
E[N] &= \frac{\pi(B_i, A_1, a, b)}{p_i q_i (1-p_i)^{K-2} p_1 q_1 (1-p_1)^{K-2}} + \frac{1}{p_1 q_1 (1-p_1)^{K-2}} \sum_{r=1}^{f} \pi(A_1, B_r, b, a) \\
&\vdots \\
E[N] &= \frac{\pi(B_i, A_f, a, b)}{p_i q_i (1-p_i)^{K-2} p_f q_f (1-p_f)^{K-2}} + \frac{1}{p_f q_f (1-p_f)^{K-2}} \sum_{r=1}^{f} \pi(A_f, B_r, a, b) \\
E[N] &= \frac{\pi(A_i, B_1, a, b)}{p_i q_i (1-p_i)^{K-2} p_1 q_1 (1-p_1)^{K-2}} + \frac{1}{p_1 q_1 (1-p_1)^{K-2}} \sum_{r=1}^{f} \pi(B_1, A_r, b, a) \\
&\vdots \\
E[N] &= \frac{\pi(A_i, B_f, a, b)}{p_i q_i (1-p_i)^{K-2} p_f q_f (1-p_f)^{K-2}} + \frac{1}{p_f q_f (1-p_f)^{K-2}} \sum_{r=1}^{f} \pi(B_f, A_r, a, b),
\end{aligned}
$$

where N denotes the waiting time of the protocol. Simplifying we obtain

$$
\begin{aligned}
p_1 q_1 (1-p_1)^{K-2} E[N] &= \frac{\pi(B_i, A_1, a, b)}{p_i q_i (1-p_i)^{K-2}} + \sum_{r=1}^{f} \pi(A_1, B_r, b, a) \\
&\vdots \\
p_f q_f (1-p_f)^{K-2} E[N] &= \frac{\pi(B_i, A_f, a, b)}{p_i q_i (1-p_i)^{K-2}} + \sum_{r=1}^{f} \pi(A_f, B_r, a, b) \\
p_1 q_1 (1-p_1)^{K-2} E[N] &= \frac{\pi(A_i, B_1, a, b)}{p_i q_i (1-p_i)^{K-2}} + \sum_{r=1}^{f} \pi(B_1, A_r, b, a) \\
&\vdots \\
p_f q_f (1-p_f)^{K-2} E[N] &= \frac{\pi(A_i, B_f, a, b)}{p_i q_i (1-p_i)^{K-2}} + \sum_{r=1}^{f} \pi(B_f, A_r, a, b).
\end{aligned}
$$

Adding these equations we obtain two identities

$$
\left(\sum_{j=1}^{f} p_j q_j (1-p_j)^{K-2} \right) E[N] = \frac{1}{p_i q_i (1-p_i)^{K-2}} \sum_{j=1}^{f} \pi(B_i, A_j, a, b) + \sum_{s=1}^{f} \sum_{r=1}^{f} \pi(B_s, A_r, a, b). \quad (29)
$$

and its symmetric version

$$
\left(\sum_{j=1}^{f} p_j q_j (1-p_j)^{K-2} \right) E[N] = \frac{1}{p_i q_i (1-p_i)^{K-2}} \sum_{j=1}^{f} \pi(A_i, B_j, a, b) + \sum_{s=1}^{f} \sum_{r=1}^{f} \pi(A_s, B_r, a, b). \quad (30)
$$

Again we multiply Equations (29) and (30) by $p_i q_i (1-p_i)^{K-2}$, add them and use Equation 28 to obtain

$$
2 \binom{K}{2} \left(\sum_{j=1}^{f} p_j q_j (1-p_j)^{K-2} \right)^2 E[N] = 1 + \sum_{j=1}^{f} p_j q_j (1-p_j)^{K-2}. \quad (31)
$$

This completes the proof of Theorem 8. □

6 Random Unicast Protocols in Multiple Node Systems

In this section we again consider the case of $K \geq 2$ nodes. We analyze the Random Unicast protocol (**RUP**). In this protocol, an attempt is made to select a single node and, if successful, this node then attempts to discover another node in the ad hoc network.

The **RUP** protocol has three phases. In the first phase, the protocol uses a discrete uniform distribution on the integers from 1 to K in an attempt to randomly choose exactly one of the K nodes. The protocol requires that each node randomly choose a value from the discrete uniform distribution. Each node then randomly chooses another value from the same distribution. If the two values chosen by a given node are equal, then that node is considered to be *selected* and it proceeds with the remaining phases of **RUP**. Otherwise, the node will act according to the Random protocol, **RP**, described in section 5.

If no node chooses a repeated value in the first phase, then the **RUP** protocol reverts to the Random protocol, **RP**, of section 5. If two or more nodes choose a repeated value, then collision may occur in later phases of the **RUP** protocol. If exactly one node chooses a repeated value in the first phase, then, as mentioned, this node is deemed *selected* and the second phase of the Random Unicast protocol begins.

The remaining phases of the **RUP** protocol can be undertaken either by *listening* or by *answering*. In the former version, the *selected* node talks for one step (the second phase) and then, in the next step (the third phase), listens to hear if contact was made. In the *answering* version, the *selected* node listens until contact is made, and then talks in the next step (the third phase). As a result, the last two phases require two steps in the *listening* version of **RUP**, while the last two phases in the *answering* version take a random number of steps, i.e., a random number of steps for phase two plus one step for phase three. In both versions of the **RUP** protocol, however, once the first phase is completed, the second and third phases are repeated until the *selected* node discovers one of the $K-1$ other nodes.

6.1 RUP with listening

In this version of the **RUP** protocol, the *selected* node talks on a randomly chosen frequency i for one step (the second phase) and then listens on a randomly chosen frequency for the next step (the third phase). Two variations of the **RUP** *with listening* protocol exist, depending on whether static or dynamic frequency allocation is used.

Theorem 9 (Listening RUP with Static Frequency Allocation) *Assuming a unique node was selected in the first phase, the expected length of a run of the last two phases of the* **RUP** with listening *protocol with static frequency allocation is given by the expression:*

$$\frac{2}{\sum_{i=1}^{f}\left((1-F_i(p+q(1-F_i))^{K-1})\left(\sum_{m=1}^{K-1} mpF_i(q+p(1-F_i))^{K-2}\right)\right)}.. \tag{32}$$

Proof Theorem 9. Recall that a node is selected in the first phase. In the second phase the selected node chooses to talk at frequency i with probability F_i. This phase fails if the remaining $K-1$ nodes either talk (at any frequency) or else listen at a frequency different from the frequency chosen by the selected node. As a result, in the second phase success occurs in frequency i with probability: $1 - F_i(p + q(1 - F_i))^{K-1}$. Assuming the second phase succeeded, the third phase succeeds when exactly one node talks among the nodes that listened in frequency i (the frequency used in the second phase); note that there are $1 \leq m \leq K-1$ such nodes. Success in the third phase occurs with probability:

$$\sum_{m=1}^{K-1} mpF_i(q+p(1-F_i))^{K-2}. \tag{33}$$

Hence, the probability of success in the last two phases of a run of the **RUP** *with listening* protocol is:

$$\sum_{i=1}^{f} \left(\left(1 - F_i(p + q(1 - F_i))^{K-1}\right) \left(\sum_{m=1}^{K-1} mpF_i(q + p(1 - F_i))^{K-2} \right) \right). \tag{34}$$

As a result, the expected number of subruns of the last two phases of the **RUP** *with listening* protocol, assuming a unique node was *selected* in first phase, is:

$$\frac{1}{\sum_{i=1}^{f} \left((1 - F_i(p + q(1 - F_i))^{K-1}) \left(\sum_{m=1}^{K-1} mpF_i(q + p(1 - F_i))^{K-2} \right) \right)}.$$

Since the length of a subrun of the last two phases of the **RUP** *with listening* protocol is 2, the expected length of a run of the last two phases of the **RUP** *with listening* protocol, assuming a unique node was *selected* in first phase, is: given by Formula 32. This completes the proof of Theorem 9. □

We can prove a simular result for the **RUP** *with listening* protocol with dynamic frequency allocation.

Theorem 10 (Listening RUP with Dynamic Frequency Allocation) *Assuming a unique node was selected in the first phase, the expected length of a run of the last two phases of the* **RUP** with listening *protocol with dynamic frequency allocation is given by the expression:*

$$\frac{2}{\left(\sum_{i=1}^{f} (1 - F_i(p + q(1 - F_i)^{K-1}) \right) \left(\sum_{j=1}^{f} F_j pF_j(q + p(1 - F_j))^{K-2} \right)}. \tag{35}$$

Proof (Theorem 10). Under dynamic frequency allocation, the *selected* node talks on frequency i in the second phase and then listens on a randomly chosen frequency j in the third phase. Success in the second phase occurs if at least one of the other $K - 1$ nodes listens on frequency i, while success in the third phase occurs if exactly one of the non-selected nodes that listened in the second phase subsequently talks on frequency j. Thua success in the third phase occurs with probability

$$\sum_{j=1}^{f} F_j pF_j(q + p(1 - F_j))^{K-2}. \tag{36}$$

It follows that the probability of success in the last two phases is equal to

$$\left(\sum_{i=1}^{f} (1 - F_i(p + q(1 - F_i)^{K-1}) \right) \left(\sum_{j=1}^{f} F_j pF_j(q + p(1 - F_j))^{K-2} \right). \tag{37}$$

The rest of the proof follows using Formulas 36 and 37 and arguing as in the proof of Theorem 9. □

6.2 RUP with answering

In this version of the **RUP** protocol, the *selected* node listens on a randomly chosen frequency i for each step of the second phase. The second phase ends when the *selected* node hears another node talk. The *selected* node then completes the third phase by talking (answering) on a randomly chosen frequency. Two variations of the **RUP** *with listening* protocol exist, depending on whether static or dynamic frequency allocation is used.

Assume that a *selected* node from the first phase of **RUP** undertakes the remaining stages of the protocol with static frequency allocation. That is, in the second stage the *selected* node will listen on a randomly chosen frequency i and then, in the third stage, it talks, i.e. answers, on the same frequency i.

Theorem 11 (Answering RUP with Static Frequency Allocation) *Assuming a unique node was selected in the first phase, the expected length of a run of the last two phases of the* **RUP** with answering *protocol with static frequency allocation is given by the expression:*

$$\left(\frac{1}{\sum_{i=1}^{f} pF_i^2(p(1-F_i)+q)^{K-2}}+1\right)\left(\frac{1}{\sum_{i=1}^{f} qF_i^2(p(1-F_i)+q)^{K-2}}\right). \tag{38}$$

Proof Theorem 11. The second phase is successful when exactly one of the $K-1$ nonselected nodes from the first phase talks on the frequency i in the second phase, where i is the frequency randomly chosen by the *selected* node. As a result, success in the second phase occurs with probability:

$$\sum_{i=1}^{f} F_i p F_i (p(1-F_i)+q)^{K-2}.$$

This implies that the expected length of the second phase is:

$$\frac{1}{\sum_{i=1}^{f} pF_i^2(p(1-F_i)+q)^{K-2}}.$$

Since the third phase involves one step, the expected length of a subrun of **RUP** with *answering* is:

$$\frac{1}{\sum_{i=1}^{f} pF_i^2(p(1-F_i)+q)^{K-2}}+1. \tag{39}$$

The third phase is successful when the one node (among the $K-1$) that talked on the frequency i in the second phase listens on that same frequency as the selected node in the third phase, while the remaining $K-2$ nodes either listen or talk at a different frequency. Success in the third phase thus occurs with probability:

$$F_i q F_i (p(1-Fi)+q)^{K-2} \tag{40}$$

The probability of success in the last two phases of a run of the **RUP** *with listening* protocol is:

$$\sum_{i=1}^{f} qF_i^2(p(1-F_i)+q)^{K-2} \tag{41}$$

As a result, the expected number of subruns of the **RUP** *with listening* protocol, assuming a unique node was *selected* in first phase, is:

$$\frac{1}{\sum_{i=1}^{f} qF_i^2(p(1-F_i)+q)^{K-2}}. \tag{42}$$

Since the expected length of a subrun of the last two phases of the **RUP** is given by Formula 39 we can apply Wald's Identity and we derive that the expected length of a run of the last two phases of the **RUP** *with answering* protocol, assuming a unique node was *selected* in first phase, is given by Formula 38. This completes the proof of the theorem. □

We can prove a simular result for the **RUP** *with answering* protocol with dynamic frequency allocation. Under dynamic frequency allocation, the *selected* node listens on frequency i in the second phase and then talks on a randomly chosen frequency j in the third phase. Success in the second phase occurs if exactly one of the other $K-1$ nodes talks on frequency i, while success in the third phase occurs if that non-selected node subsequently listens on frequency j.

Theorem 12 (Answering RUP with Dynamic Frequency Allocation) *Assuming a unique node was selected in the first phase, the expected length of a run of the last two phases of the* **RUP** with answering *protocol with dynamic frequency allocation is given by the same Expression 38.*

Proof (Theorem 12). This follows easily from the proof of Theorem 11 and we omit the details. □

7 Conclusions

In this paper we have analyzed node discovery protocols in multinode, multichannel broadcast networks. We studied randomized and non-randomized protocols both with static and dynamic frequency allocation. Under certain conditions, non-randomized static and dynamic node discovery protocols for two node systems are shown to have the same expected waiting time. In the case of multinode systems we analyzed randomized node discovery protocols. Continuing future research will explore additional network models, and adaptive protocols.

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