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A Colored Petri Net-based Approach to the Design of Controllers[1]

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Abstract

In this paper, we extend the supervisory control theory and a supervisor synthesis problem to a class of colored Petri nets. More specifically, we investigate the forbidden state control problem with full observation in which the discrete-event system is modeled as a colored Petri net with a symmetry specification. This problem is decidable if the colored Petri net has finite color sets and bounded places. A new algorithm for deriving a controller is presented in detail with a proof of its correctness. Unlike conventional algorithms that explore the entire reachable set of states, our algorithm avoids an exhaustive search of the state space by exploiting a symmetry specification. It performs particularly well when applied to large but structured processes with similar components. Furthermore, this approach allows to represent a controller in a compact form.

1 Introduction

A discrete-event system (DES) is a dynamic system whose the internal state changes instantaneously in response to the occurrence of an event. Typically, they are used to represent the behavior of reactive, communication, or manufacturing systems. The control theory for discrete-event systems pioneered by Ramadge and Wonham [16] is a framework for modeling supervised discrete-event systems and applying synthesis algorithms to solve control problems. This theory has been primarily studied in the context of automaton-based models. Automaton-based modeling is, however, cumbersome, particularly in representing large systems consisting of numerous similar interacting components. Even though the computational complexity can be polynomial in the number of system states, it grows exponentially with the number of components. This phenomenon, called the *state explosion problem* [2], can be overcomed by partitioning the components into a small number of equivalence classes so that all components in a given class are essentially similar. This paper makes a contribution in this direction and presents an algorithm that reduces the complexity of the supervisory control problem under full observation for instances consisting of large but structured processes with similar behaviors.

Recently, Petri nets have begun to receive attention as models for investigating the control of discrete-event systems. The Petri net-based models are more powerful than the automaton-based models in that the set of Petri net languages is a superset of regular languages. Furthermore, they allow more compact representations of multiple interacting components than equivalent automaton models. Indeed, a single Petri net structure can describe the behavior of several concurrent components modeled as tokens marking the structure.

One may identify mainly two different approaches. In the first approach, introduced by Krogh [12] and Ichikawa [9], a DES is described by using a *Controlled Petri net* (CtlP-net). A CtlP-net is an extension of a standard Petri net in which the enabling of transitions can be influenced by external binary control inputs. A tutorial survey of works related to this approach can be found in [6]. Recently, Boel et al. [1] addressed the forbidden state problem for a special class of CtlP-nets. In contrast, the second approach does not include the concept of control input. A DES is specified by using an ordinary Petri net and its dynamic is viewed with a linear algebraic perspective [3, 5, 13].

Among the Petri net-based models, colored Petri nets (CP-nets) [10] are powerful enough to describe complex systems in a manageable way, particularly when they contain many interacting components that are similar but not identical. It is well-known that the class of CP-nets with finite color sets is equivalent to the class of place/transition nets (PT-nets). The CP-nets describe the same systems as the PT-nets but often in a more compact form and allow the use of equivalence classes to represent similar components. Specifications can be

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more readable and eventually more tractable. If we do not impose any restrictions on the definition of CP-nets, then they are equivalent to Turing machines.

The work presented in this paper addresses the forbidden state control problem for a class of CP-nets in which color sets are finite and behaviors satisfy boundedness properties. Given a control specification expressed as a set of forbidden markings, a procedure computes the unique maximal set of admissible markings and a controller by means of an occurrence graph with symmetries. The symmetries guarantee that equivalent markings have similar behavior [11]. Our aim is not to include, as control logic, additional places and transitions to the CP-net being controlled. Such an approach is generally used when a simulation or a performance analysis is performed for a given control policy [12]. In this work, the process being controlled is separated from the control logic. This approach is suitable to controller synthesis in which the process to be controlled already exists and the aim is to compute, from a control specification, a separated controller which, when embodied with the process in a closed-loop system, satisfies the control specification. Thus, this approach permits formulations and solutions of different control synthesis problems for the same process.

The layout of the paper is as follows. The next section introduces the notation and definitions used in CP-nets. Section 3 extends the supervisory control theory to processes modeled by CP-nets with a consistent symmetry specification. Section 4 formulates the control synthesis problem for avoiding a set of forbidden markings. Section 5 describes in detail a new synthesis algorithm and gives a proof of its correctness. Section 6 illustrates its application with a short example. Finally, concluding remarks are provided in Section 7.

2 Notation and Preliminaries

We use a CP-net to model a discrete-event system (*DES*). A colored Petri net is an ordered tuple: $CPN = (\Sigma, P, T, A, N, C, E, M_0)$, where Σ is a finite set of non-empty types, called *color sets*; P is a finite set of *places*; T is a finite set of *transitions*; A is a finite set of *arcs* connecting places and transitions; N is a *node function* that maps each arc to a pair of nodes of different kinds (i.e., one is a place, while the other is a transition); C is a *color function* that associates a color set with each place; E is an *arc expression function* that maps each arc to a multi-set over the color set that is attached to the corresponding place; and M_0 is the initial marking. We consider only CP-nets with decidable properties as valid models for DESs; that is, the color sets are finite and the contents of places are bounded. Furthermore, for the sake of simplicity, we assume that guards (occasionally used in the definition of CP-nets) always evaluate to true.

The following notation is introduced to consider the behavior of a CP-net that is based on the concepts of *enabling* and *occurrence*. The reader is referred to Jensen's book [10] for a detailed exposition. For all $t \in T$, let $Var(t)$ be the set of *variables* appearing on arcs that have t as source or destination. For all $(p,t) \in P \times T$, let $E(p,t) = \sum_{a \in A(p,t)} E(a)$,[1] where $A(p,t)$ gives the set of arcs from p to t. $E(t,p)$ is defined in a similar fashion.

A *binding* b of a transition t is a substitution, noted $b = < v_1 = c_1, \ldots, v_n = c_n >$, that assigns a color c_i to the variable $v_i \in Var(t)$ $(i = 1, \ldots, n)$. A *binding element* is a pair (t,b) where $t \in T$ and $b \in B(t)$, the set of all bindings for t. Let BE be the set of all binding elements.

The state of a CP-net is given by its current *marking* M that maps each place p to a multi-set over $C(p)$. A marking gives the current distribution of tokens on the places. A *step* Y is a function that maps each $t \in T$ to a multi-set over $B(t)$ such that $Y(t)$ is finite for all $t \in T$ and non-empty for at least one $t \in T$. A step Y is *enabled* in a marking M if and only if

$$\sum_{b \in Y(t)} E(p,t) < b > \leq M(p) \text{ for all } p \in P$$

where $E(p,t) < b >$ yields the multi-set of tokens removed from p when t occurs with the binding b. The sets of all markings and steps are denoted by $\mathcal{M}$ and $\mathcal{Y}$, respectively. When a step Y is enabled in a marking M, it may *occur*, changing it to another marking M', defined by:

$$M'(p) = (M(p) - \sum_{b \in Y(t)} E(p,t) < b >) + \sum_{b \in Y(t)} E(t,p) < b > \forall p \in P \quad (1)$$

Moreover, we say that M' is *directly reachable* from M by the *occurrence* of the step Y, which is also denoted by $M[Y> M'$. The concept of *occurring step* is extended to a *finite occurrence sequence* of markings and steps as follows: $M_1[Y_1> M_2[Y_2> \ldots M_n[Y_n> M_{n+1}$, where $n \in \mathbb{N}$ and $M_i[Y_i> M_{i+1}$ for all $i \in \{1, \ldots, n\}$. The integer n is the *number of steps* in the sequence. The abbreviated form $M_1[Y_1 Y_2 \ldots Y_n> M_{n+1}$ is also used. A marking M' is *reachable* from a marking M if and only if there exists a finite occurrence sequence

[1] The expression $\sum_{s \in S} m(s)s$ represents a multi-set. The non-negative integer $m(s)$ is the number of occurrences of the element s in the multi-set m.

having M as start marking and M' as end marking. A marking M is reachable if and only if it is reachable from M_0. The set of markings that are reachable from M is denoted as $[M>$ and the marking directly reachable from M by the occurrence of Y is represented by $M[Y>$. It is always true that $M \in [M>$ for all markings $M \in \mathcal{M}$. In this paper, we assume that an occurring step contains only one binding element, although a number of binding elements can be concurrently enabled. Therefore, $M[Y> M'$ is written as $M[(t,b) > M'$ and $M_1[Y_1Y_2 \ldots Y_n > M_{n+1}$ as $M_1[(t_1,b_1)(t_2,b_2)\ldots(t_n,b_n)> M_{n+1}$.

Symmetry Specification

It is often the case that large systems have numerous interacting components with similar behaviors. Theses components are so alike that we may abstract the differences between them. This can be captured in a CP-net by defining a set of symmetries called a *symmetry specification* Φ over markings or binding elements [11]. Each symmetry $\phi \in \Phi$ is a function (more precisely a bijection renaming tokens) that maps a given marking (or a given binding element) to another marking (or binding element) with similar properties. In order to capture the fact that two symmetrical markings have similar properties, the symmetry specification Φ must be consistent with the behavior of the CP-net. A symmetry specification Φ is *consistent* if and only if the following properties are satisfied for all symmetries $\phi \in \Phi$, all markings $M_1, M_2 \in [M_0>$, and all binding elements $(t,b) \in BE$:

$$\phi(M_0) = M_0 \quad (2)$$

$$M_1[(t,b)> M_2 \Leftrightarrow \phi(M_1)[(t,\phi(b))> \phi(M_2) \quad (3)$$

The consistency property has been verified for particular symmetry specifications such as permutation, rotation, and identity [11]. A consistent symmetry specification Φ induces the equivalence relations $\approx_M$ and $\approx_{BE}$. We use $M_\approx$ and $BE_\approx$ to denote the set of all equivalence classes for $\approx_M$ and $\approx_{BE}$, respectively. The notation $[X]$, where $X \subseteq \mathcal{M}$, represents all the markings equivalent to a marking from X: $[X] = \{M \in \mathcal{M} \mid (\exists x \in X)\ M \approx_M x\}$. The notation [{M}], where $M \in \mathcal{M}$, is simplified to $[M]$. Finally, the notation $[Y]$, where $Y \subseteq M_\approx$, represents all the markings that belong to one of the equivalence classes in Y: $[Y] = \{M \in \mathcal{M} \mid (\exists y \in Y)\ M \in y\}$.

3 CP-net Supervisory Design under Full Observation

The basic problem in supervisory control is to construct a controller that can turn off various events of an uncontrolled discrete-event system (DES), called a *process* (Pr), according to some requirements. The process Pr is defined as a triple (CPN, Φ, K), where CPN is a CP-net with a consistent symmetry specification Φ and $K \in [P \to \mathbb{N} - \{0\}]$ is a *capacity function* bounding the contents of every place. Therefore, we require that $M_0(p) \leq K(p)$ and $M'(p) \leq K(p)$ for all $p \in P$ in the enabling rule (1).

Let Γ be the set of all functions γ, called *control patterns*, that assign a subset of $B(t)$ to every transition t of T. If $b \in \gamma(t)$, then the controller prevents the transition t from occurring with the binding b. Let T_c and T_u be fixed disjoint subsets of T denoting the sets of *controllable* and *uncontrollable* transitions, respectively.

A *controlled discrete-event system* (CDES) is an ordered tuple $Pr_c = (CPN_c, \Phi, K, \Gamma)$. In a CPN_c, $M[_c(t,b)> M'$ denotes that a marking M' is directly reachable from M by the occurrence of the binding element (t,b) under the control of $\gamma \in \Gamma$. This is defined as:

$$M[(t,b)> M' \quad \text{if } t \in T_u$$
$$M[(t,b)> M', b \notin \gamma(t) \quad \text{if } t \in T_c \text{ and } \gamma \in \Gamma$$

A *controller* is a pair $S = (G, \varphi)$, where G is a subgraph of an occurrence graph with symmetries (*OS-graph*) and φ the feedback function. An *OS-graph* is a 4-tuple (V, A, N, v_0), where

- V is the finite set of nodes $\{[M] \in M_\approx \mid [M] \cap [M_0> \neq \emptyset\}$;
- A is the finite set of arcs $\{([M_1],[(t,b)],[M_2]) \in V \times BE_\approx \times V \mid \exists (M_1', (t,b'), M_2') \in [M_1] \times [(t,b)] \times [M_2]$ such that $M_1'[(t,b')> M_2'\}$;
- N is the node function from A into $V \times V$. If $a = ([M_1],[(t,b)],[M_2])$, then $N(a) = ([M_1],[M_2])$;
- $v_0 \in V$ is the initial node ($v_0 = [M_0]$).

The *feedback function* $\varphi \in [V \to \Gamma]$ satisfies the following conditions:[2]

$$\varphi(v)(t) = \emptyset \quad \text{if } t \in T_u, v \in V$$
$$\varphi(v)(t) \in \wp(B(t)) \quad \text{if } t \in T_c, v \in V$$

The graph G can be interpreted as the transition graph of an automaton modeling the behavior of the controller as in the original framework of Ramadge and Wonham. It is driven by a sequence of steps occurring in CPN. That is, after the firing of a transition, G moves to a node v which represents the marking reached by CPN. The role of the feedback function

[2] $\wp(B(t))$ denotes the power set of $B(t)$.

φ is to provide, after each execution step of CPN and G, the control pattern γ that represents the binding elements inhibited for the next step.

The CDES and controller are then embodied in a closed-loop system to constitute a *supervised discrete-event system* (SDES) $S/Pr_c = (S, Pr_c)$. A state of a $SDES$ is a pair (v, M) where $v \in V$ and M is a marking of CPN_c such that $M \in v$. The behavior of S/Pr_c is illustrated in Fig. 1. Let M_i be the current marking of the process and $v_i = [M_i']$ be a node of G such that there exists $\phi \in \Phi$ such that $M_i' = \phi(M_i)$. Let (t_i, b_i) the next step such that $M_i[(t_i, b_i)> M_{i+1}$ and $b_i \notin \gamma(t_i)$ with $\gamma = \varphi(v_i)$. First, the controller S moves to the next node $v_{i+1} = [M_{i+1}']$ by executing the transition from node v_i on the arc labeled (t_i, b_i'). The binding element (t_i, b_i') is the representative member of binding elements equivalent to the process step (t_i, b_i), that is, $(t_i, b_i') = (t_i, \phi(b_i))$. The controller S includes control patterns for representative marking of each equivalence class of reachable markings. Let $\gamma_{i+1}' = \varphi(v_{i+1})$ be the control pattern of node v_{i+1}. To compute the control pattern corresponding to the next marking M_{i+1} of the process, the controller uses the inverse of symmetry function ϕ to map the control pattern γ_{i+1}' to the control pattern $\gamma_{i+1} = \phi^{-1}(\gamma_{i+1}')$.[3] Formally, if (v_i, M_i) is the current state of S/Pr_c, then the next state is (v_{i+1}, M_{i+1}) if and only if there exists a binding element (t_i, b_i) such that $M_i[(t_i, b_i)> M_{i+1}$, $b_i \notin \gamma(t_i)$ with $\gamma = \varphi(v_i)$, and $(v_i, [(t_i, b_i')], v_{i+1}) \in A$ with $(t_i, b_i) \approx_{BE} (t_i, b_i')$. The control pattern of M_{i+1} is $\phi^{-1}(\varphi(v_{i+1}))$. The controller S must always be *complete* in the sense that $(v_i, [(t_i, b_i')], v_{i+1}) \in A$ whenever S/Pr_c is in state (v_i, M_i) with $M_i[_c(t_i, b_i)> M_{i+1}$ and $b_i' \notin \varphi(v_i)(t_i)$.

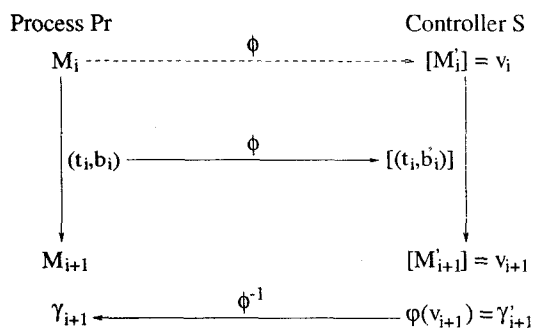

Figure 1: An execution step of the closed-loop system

4 The Forbidden State Control Problem

Several types of control specifications can be applied to control synthesis methods, including: avoiding a set of forbidden states [12]; enforcing event language specifications [16]; and enforcing liveness [8]. In this paper, we consider the forbidden state control problem in which the control specification is expressed as a set of forbidden markings $\mathcal{M}_b$. Based on the consistency property of Φ, if a marking is forbidden, all its equivalent markings are also forbidden. Therefore, only one representative per each equivalence class of forbidden markings is included in $\mathcal{M}_b$. This problem is expressed as follows. Given a set of forbidden markings $\mathcal{M}_b$, an uncontrolled discrete-event system (CPN, Φ, K), and an admissible (the definition of inadmissible marking is given in the next subsection) initial marking $M_0 \notin [\mathcal{M}_b]$, derive a *maximally permissive* controller S, that is: (1) the closed-loop system S/Pr_c is safe (Pr_c cannot reach a forbidden marking under the control of S); (2) a reachable marking of Pr, which is a nonreachable marking of Pr_c under the control of S, is either forbidden or can uncontrollably lead to a forbidden marking. Before describing the synthesis algorithm, lets us introduce an admissibility assessment predicate and the notion of latest controllable binding elements.

Inadmissible marking

Given a set of forbidden markings $\mathcal{M}_b$, there is, in general, a larger set of markings which must be avoided, due to uncontrollable transition sequences. The markings from which the process can uncontrollably reach forbidden markings are characterized by the following recursive predicate:

$$\begin{aligned} Inadmissible(M) \Longleftrightarrow\ & (\exists n \geq 0)(\exists t_1, \ldots, t_n \in T_u) \\ & (\exists b_1 \in B(t_1)) \ldots (\exists b_1 \in B(t_n)) \\ & M[(t_1, b_1) \ldots (t_n, b_n)> \in [\mathcal{M}_b] \end{aligned} \quad (4)$$

A node with associated inadmissible markings is inadmissible. When the initial marking is safe and admissible, and steps contain only one binding element, the maximally permissive solution to the forbidden state control problem exists and prevents the process from reaching any inadmissible marking [7].

Latest Controllable Binding Elements

Let y denote a node in the *OS-graph*. The *latest controllable binding elements* of y is a set (denoted as $LCBE$) of all triples of the form (x, t, b) such that:[4]

1. x is a node in the *OS-graph*;
2. (t, b) is a binding element, where t is a controllable transition;
3. $(\exists n \geq 0)\ M_x[(t, b)(t_1, b_1) \ldots (t_n, b_n)> M_y$ with $t_i \in T_u$, for $i = 1, \ldots, n$.

A triple (x, t, b) contained in the $LCBE$ of y is interpreted as follows. The occurrence of the step (t, b)

[3] Since Φ is a group, ϕ^{-1} always exists.

[4] The expressions M_x and $lcbe_x$ denote a representative marking M and $LCBE$ of the node x, respectively.

from M_x is controllable whereas the sequence of steps $(t_1, b_1) \ldots (t_n, b_n)$ from $M_x[(t, b)>$ to M_y are uncontrollable. Therefore, to make unreachable M_y and its equivalents in the process, it is necessary to disable the binding b for t when the process is in marking M_x.

5 The Synthesis Algorithm

The basic idea behind our algorithm is to reduce the number of markings that must be examined by gathering the components that "behave in the same way" into the same equivalence class. Therefore, the sets of all markings and steps are partitioned into disjoint nonempty equivalence classes. The algorithm, given in Fig. 2, is based on the notion of *latest controllable binding elements*, *equivalent marking*, and *inadmissible marking*. It accepts as input a process Pr, a consistent symmetry specification represented by the equivalence relations $\approx_M$ and $\approx_{BE}$, a set of forbidden markings $\mathcal{M}_b$, and a set of controllable transitions T_c. A maximally permissive compact controller $S = (G, \varphi)$ is computed from representative members, one per class of equivalent markings and class of equivalent steps.

```
1  function Synthesize_Controller(Pr,≈M,≈BE,Mb,Tc)
2  V ← {}; A ← {}; processed_nodes ← {};
                   unprocessed_nodes ← {New_Node(M0,{})}
3  repeat
4    select x in unprocessed_nodes;
5    if not (Mx ≈M My) for some y in processed_nodes then
6      for all (t,b) enabled in Mx do
7        M ← New_Marking(Mx,(t,b));
8        if M ≈M M' for some M' in Mb then
9          if t ∈ Tc then
10           φ(x)(t) ← φ(x)(t) ∪ {b}
11         else
12           Inadmissible(x); break
13       else
14         if not (M ≈M My) for some y being a son of x then
15           if t ∈ Tc then lcbez ← {(x,t,b)} else lcbez ← lcbex;
16           z ← New_Node(M,lcbez);
17           unprocessed_nodes ← unprocessed_nodes ∪ {z};
18           a ← New_Arc(x,(t,b),z); A ← A ∪ a
19     if x.status = admissible then V ← V ∪ {x}
20   else
21     if y.status = inadmissible then Inadmissible(x)
                                  else lcbey ← lcbey ∪ lcbex
22   processed ← processed ∪ {x};
23   unprocessed_nodes ← unprocessed_nodes \ {x}
24 until unprocessed_nodes = {};
```

Figure 2: Algorithm for synthesizing a compact controller

The algorithm uses many functions briefly described hereafter. The function **New_Node** creates a new node from a marking and an $LCBE$. The function **New_Marking** yields the marking reached after the occurrence of a binding element from a given marking. The function **New_Arc** creates a new arc from a source node, a binding element, and a destination node. The function **From_Nodes** takes as argument a node x and returns the set of nodes in V from which x is directly reachable on an uncontrollable binding element.

Each node has a status indicating whether or not its associated marking is *admissible*. A new node has its status set to *admissible*. The function **Inadmissible**, given in Fig. 3, fixes the status of a node to *inadmissible* (line 2), determines those that become inadmissible among its predecessors (lines 3 and 4), and updates the feedback function by inserting, for each (x', t, b) in the $LCBE$ of x, the binding b in the set of forbidden bindings for t of x' (lines 6 and 7). When a node becomes inadmissible, all its son nodes in V are removed (including their bound arcs) by using the procedure **Remove_Sons** (line 2). If the $LCBE$ of x is empty, then there are no solutions (line 8).

```
1 procedure Inadmissible(x) :
2     x.status ← inadmissible; Remove_Sons(x); V ← V \ {x}
3     for each x' in From_Nodes(x)
                  such that x'.status = admissible do
4         Inadmissible(x')
5     if lcbex ≠ {} then
6         for each (x',t,b) ∈ lcbex do
7             φ(x')(t) ← φ(x')(t) ∪ {b}
8     else "no solutions"
9 end
```

Figure 3: Procedure for determining inadmissible nodes

The algorithm works as follows. Initially, the sets of nodes V and arcs A of the graph are both empty. The set of processed nodes is also empty. The node $v_0 = (M_0, \{\})$ is created and inserted into the set of unprocessed nodes (line 2). While there are unprocessed nodes, a node x is selected (line 4) and processed. The processing of a node starts with a test for an equivalence between the marking M_x and the marking M_y of an already processed node y (line 5). Only the first-picked node in each equivalence class is developed further. If such a node y exists, then the algorithm checks if it is an inadmissible node. If so, then the **Inadmissible** function is called on node x to disable the latest controllable binding elements on the path leading to x (line 21). Otherwise, the contents of $lcbe_x$ is inserted in $lcbe_y$ (line 21). If such a node y does not exist, every binding element (t, b) enabled in M_x is analyzed (line 6). The marking M reached after the occurrence of the binding element (t, b) is computed (line 7) and checked for an equivalence with some other marking M' included in the set of forbidden markings (line 8). If so, there are two cases: either t is controllable or not. If transition t is controllable, the binding b is inserted in the set of forbidden bindings of x (line 10). If t is uncontrollable, then the function **Inadmissible** is called on node x (line 12). If M is not equivalent to some forbidden marking in $\mathcal{M}_b$, then the algorithm checks if marking M is equivalent to a marking M_y where y is a son of x (line 14). If all the above conditions are not satisfied, it means that the marking M is not equivalent to a marking of a son of x and not forbidden. In this case, a new node z is created with the following attributes: the marking M and $lcbe_z$ which is

defined as $\{(x,t,b)\}$ if the transition t is controllable; otherwise, it is the $lcbe_x$ of its parent node x (lines 15 and 16). Furthermore, the set of unprocessed nodes is updated (line 17) and a new arc is created from node x to node z and added to A (line 18). At the end of the analysis of all binding elements (t,b) enabled in M_x, if the status of x is *admissible*, then x is included in V. Finally, the node x is included in the set of processed nodes and removed from the set of unprocessed nodes (lines 22 and 23).

The following theorem shows that the set of nodes V generated by our algorithm represents the unique maximal set of markings that solves the forbidden state control problem with respect to a set of forbidden markings $\mathcal{M}_b$. The proof of the theorem is based on two lemmas. Let us first introduce some properties of the Jensen's OS-graph [11] that are used in the proof. Hereafter, a node is identified to its representative marking.

Proposition 1 *Let Φ be a consistent symmetry specification and (W,A,N,v_0) an OS-graph. The following properties hold for all $M_1,\ldots,M_{n+1} \in [M_0>$ and all $\phi \in \Phi$:*

1. $[M_0> = [W]$
2. $M_1[(t_1,b_1)> \ldots M_n[(t_n,b_n)> M_{n+1} \iff$
   $\phi(M_1)[(t_1,\phi(b_1))> \ldots \phi(M_n)[(t_n,\phi(b_n))> \phi(M_{n+1})$
3. $M \in [M_0> \iff \phi(M) \in [M_0>$

Lemma 1 *Let Φ be a consistent symmetry specification. For all $M \in [M_0>$ and all $\phi \in \Phi$, $Inadmissible(M) = Inadmissible(\phi(M))$.*

Proof: Because of the definition of the predicate *Inadmissible*, there exists a sequence of uncontrollable binding elements $(t_1,b_1)\ldots(t_n,b_n)$ with $t_i \in T_u$ and a marking $M' \in \mathcal{M}$ such that $M[(t_1,b_1)\ldots(t_n,b_n)> M'$ and $M' \in [\mathcal{M}_b]$. From Proposition 1, $\phi(M)[(t_1,\phi(b_1))\ldots(t_n,\phi(b_n))> \phi(M')$. By the definition of $\mathcal{M}_b$, $\phi(M') \in [\mathcal{M}_b]$. Therefore, $Inadmissible(\phi(M)) = true$. The proof of the converse is similar, using the function ϕ^{-1}. □

Lemma 2 *Let predicate Inadmissible be restricted to markings labeling nodes in the set processed_nodes, $v \in V$ iff $Inadmissible(M_v) = false$.*

Proof: We use induction on the number of times k the **repeat-until** loop is executed.

Basis: $(k = 0)$. Trivially true since *processed_nodes* (the set of markings reachable form M_0) is empty.

Induction hypothesis: $v \in V$ iff $Inadmissible(M_v) = false$.

Induction: $(k > 0)$ We show that, if the hypothesis holds at the start of the loop, then it holds at the end. Let $x \in$ *processed_nodes* at the end of the loop. There are two cases: whether $x \in$ *processed_nodes* at the start of the loop or not.

Case 1: When $x \notin$ *processed_nodes* at the start of the loop, x is selected from the set *unprocessed_nodes* by the current loop, yielding two subcases.
Subcase 1.1: The conditions of the **if** statement at line 5, the **for** loop at line 6, and the **if** statement at line 8 are satisfied. The condition of the **if** statement at line 9 is not satisfied (the **else** part starting at line 11 is entered). The procedure `Inadmissible` is called with the actual parameter x (line 12). The node x is not included in V (line 19) since its status has been set to *inadmissible* in procedure `Inadmissible` (line 2, Fig. 3). These conditions are true *iff* there exists a binding element (t,b) such that $M_x[(t,b)> M$, $t \in T_u$, and $M \in [M']$ for some $M' \in [\mathcal{M}_b]$. Then $Inadmissible(M_x) = true$ because of Eq. 4.

Subcase 1.2: The conditions of the **if** statement at line 5 is not satisfied (the **else** part starting at line 20 is entered) and the **if** statement at line 21 is satisfied. The procedure `Inadmissible` is called with the actual parameter x (line 21) and its status is set to *inadmissible* (line 2, Fig. 3). These conditions are true *iff* there exists an inadmissible node $y \in$ *processed_nodes* with a marking that is equivalent to M_x ($M_x = \phi(M_y)$). Therefore, $Inadmissible(M_x) = true$ because of Lemma 1.

Case 2: $x \in$ *processed_nodes* at the start of the loop. At the start of the loop, either $x \in V$ or not. It can easily be checked that if $x \notin V$ at the start of the loop, then the algorithm never inserts a node with a marking equivalent to M_x in V (because of line 5). Let us consider the case where $x \in V$ at the start of the loop. The node x is removed from V at the end of loop *iff* a node x' from *unprocessed_nodes* has been selected and the procedure `Inadmissible` has been called with actual parameter x' (lines 12 or 21). Then, $x'.status$ received the value *inadmissible* and the recursive calls to `Inadmissible` follow backward all the paths incoming to x', while transitions are uncontrollable. Furthermore, every encountered inadmissible node in paths leading to x' are removed, including x (line 2, Fig. 3). The above is, however, possible *iff* x' has been selected, $Inadmissible(M_{x'}) = true$, in accordance with Case 1, and there is a sequence of uncontrollable steps $Y_1 \ldots Y_n$ with $M_x[Y_1 \ldots Y_n> M_{x'}$. These conditions are true *iff* $Inadmissible(M_x) = true$ at the end of the loop because of Eq. 4. □

Theorem 1 *The procedure* `Synthetize_Controller` *always terminates and, if $M_0 \notin [\mathcal{M}_b]$ and $Inadmissible(M_0) = false$, then it returns a controller*

$S = (G, \varphi)$ *that is the unique maximal solution to the forbidden state control problem with respect to the set of forbidden markings* $\mathcal{M}_b$.

Proof:
Note that the procedure **Synthetize_Controller** always terminates since the color sets are finite and the CP-net Pr satisfies boundedness properties.

Now, let us assume that $M_0 \notin [\mathcal{M}_b]$ and $Inadmissible(M_0) = false$. Let $S = (G, \varphi)$ be the controller computed by **Synthetize_Controller**. We can easily show the following by using Lemma 2 and the definition of the predicate *Inadmissible* which expresses the converse of controllability. For every $v \in V$, $M_v \in [M_0>$ and $Inadmissible(M_v) = false$. Furthermore, for all $t \in T_u$ and $b \in B(t)$, if $M_v[(t,b)> \in [M_0>$ then $Inadmissible(M_v[(t,b)>) = false$. From Lemma 1, this property holds for all markings equivalent to M_v. Therefore, the controller is safe.

From Proposition 1, we know that a marking is reachable *iff* it belongs to $[W]$, where W is the set of nodes of the *OS-graph* generated from Pr. Remark that $V \subseteq processed\_nodes \subseteq W$. Let $w \in W$ such that $Inadmissible(M_w) = false$, M_w is reachable under the control of S, and $w \notin V$. If $w \in processed\_nodes$, then, from Lemma 2, $Inadmissible(M_w) = true$, a contradiction. If $w \notin processed\_nodes$, then for all sequences of steps $(t_1, b_1) \ldots (t_n, b_n)$ such that $M_0[(t_1, b_1)> M_1 \ldots M_{n-1}[(t_n, b_n)> M_w$, there exist $i \in \{1, \ldots, n-1\}$ and a node v with $v \in processed\_nodes$, $Inadmissible(M_v) = true$ (that is $v \notin V$), and $M_i \in [M_v]$. Therefore, $[W]$ are nonreachable under the control of S, a contradiction. Hence, the controller is maximal. □

6 A short example

As an example, Fig. 4 shows a CP-net that models a flexible assembly cell [4]. It contains two color sets $C = \{c_0, c_1, c_2\}$ and $R = \{r_0, r_1, r_2\}$, representing three conveyors and three robots, respectively. A conveyor and two neighboring robots are needed to carry out an assembly task. Each conveyor requests the left robot first (defined by the function $Left(c_i) = r_i$, where $i = 0, 1, 2$), and after acquiring it, requests the right one (defined by the function $Right(c_i) = r_{i \oplus 1}$, where $i \oplus 1$ means $i + 1$ mod 3 and $i = 0, 1, 2$). Then the assembly operation starts. When the task is completed, the conveyor releases both robots. The states of a conveyor or a robot are represented by places (circles), that is, *Unused_Conveyor*, *Free_Robot*, *Waiting*, and *Working*. A conveyor passage from one state to another is modeled by a transition (a rectangle). There are three transitions (*Request_Left_Robot*, *Request_Right_Robot*, and *Release_Robots*). Places can be marked by tokens. In our example, every place may contain tokens which values are of type C, except the place *Free_Robot* which has the color set R. A token in a given place represents the fact that the corresponding conveyor or robot is in the corresponding state.

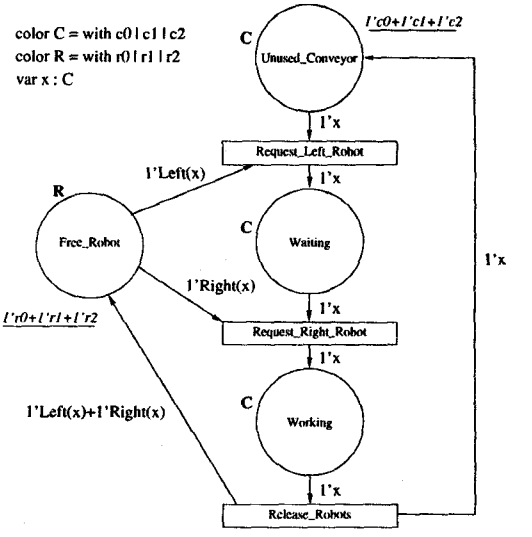

Figure 4: A CP-net describing a flexible assembly cell

Initially, the conveyor are unused and robots are free. The initial marking is represented by the expressions $\Gamma c_0 + \Gamma c_1 + \Gamma c_2$ and $\Gamma r_0 + \Gamma r_1 + \Gamma r_2$ next to places *Unused_Conveyor* and *Free_Robot*, respectively. The integer before the back quote indicates the number of occurrences of the value after the back quote. The execution of a transition always extracts one token from its incoming place and inserts one or two tokens in its outgoing place. The variable x ranges over the color set C. In this model, no constraint is given concerning the state of the three conveyors relative to each other. In this example, we use a particular consistent symmetry specification called a rotation [11] and noted Φ_r. In this symmetry specification, equivalent markings are obtained by changing the identity of all conveyors and robots with the same rotation which adds k (in a cyclic way) to the index of conveyors and robots. Formally, we have functions $\phi_c^k \in [C \longrightarrow C]$ defined by $\phi_c^k(c_i) = c_{i \oplus k} = i + k$ mod 3 $(k = 0, 1, 2)$ and $\phi_r^k \in [R \longrightarrow R]$ defined by $\phi_r^k(r_i) = r_{i \oplus k}$ $(k = 0, 1, 2)$. Thus, $\Phi_r = \{\phi_c^0, \phi_c^1, \phi_c^2, \phi_r^0, \phi_r^1, \phi_r^2\}$.

Let us consider the forbidden markings for this example. We want to prevent the assembly cell from ever reaching deadlock markings. A marking is dead when no step is enabled. The marking in which the three conveyors have acquired their left (right) robots and are waiting for their right (left) one are the only forbidden markings. We assume that only transition *Request_Left_Robot* is controllable. The graph and feedback function φ are given in Fig. 5 and Table 1,

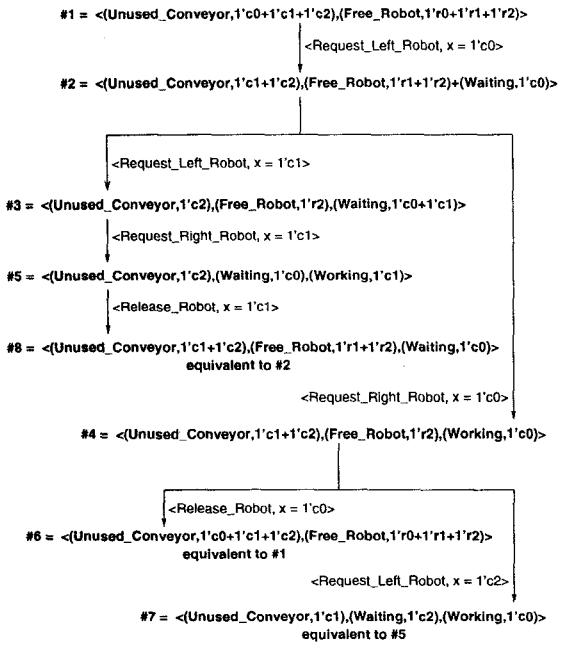

Figure 5: The subgraph for the assembly cell system

respectively. Since the $LCBE$ of each node is useless in the closed-loop system, only a representative marking of each node is kept in the solution.

Marking	Request_Left_Robot
$< (Unused\_Conveyor, \hat{1}c_2), (Free\_Robot, \hat{1}r_2), (Waiting, \hat{1}c_0 + \hat{1}c_1) >$	$< x = c_2 >$

Table 1: The feedback function φ

7 Conclusion

In this paper, we have presented a colored Petri-net approach to the control of discrete-event systems. One of the benefits of using CP-nets instead of equivalent PT-nets is the more compact and readable representation of the system. The algorithm developed for synthesizing the controller avoids an exhaustive search of the state space by the use of equivalence relations. Our experiments on different systems with many similar components shown that our algorithm is generally more efficient than methods requiring the construction of the entire reachable set of states [14, 15]. The theoretical limitation of our algorithm is, however, due to the restrictions introduced in Section 3. Future efforts may be devoted to model the controller by using a colored Petri net equipped with the required symmetry specifications.

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