Appendix: Intermediate Results and Proofs

Proofs for Section 3

Proof of Lemma 1: This result extends a similar result in [3]. We concentrate on the cases not covered there. We have to show $\mathcal{M}(DB, DB') \models p(\bar{x})$: $\mathbf{f_c} \vee \exists y(q(\bar{x}', y){:}\mathbf{t_d} \wedge q(\bar{x}', y){:}\mathbf{t_c}) \vee \exists y(q(\bar{x}', y){:}\mathbf{f_d} \wedge q(\bar{x}', y){:}\mathbf{t_c})$. We have that IC contains the formula $p(\bar{x}) \rightarrow \exists y q(\bar{x}', y)$. As $DB' \models_\Sigma IC$ we must analyze two cases. The first one is $DB' \models_\Sigma \neg p(\bar{a})$. Then $I_P(p(\bar{a})) = \mathbf{f}$ or $I_P(p(\bar{a})) = \mathbf{f_a}$, so $\mathcal{M}(DB, DB') \models p(\bar{a}){:}\mathbf{f_c}$. The second case is $DB' \models_\Sigma q(\bar{a}', b_1), \ldots, q(\bar{a}', b_n)$ for elements $b_1, \ldots, b_n$ in the domain ($n \geq 1$). Hence, $I_P(q(\bar{a}', b_i)) = \mathbf{t}$ or $I_P(q(\bar{a}', b_i)) = \mathbf{t_a}$, for every $1 \leq i \leq n$. Then, $\mathcal{M}(DB, DB') \models \exists y(q(\bar{a}', y){:}\mathbf{t_d} \wedge q(\bar{a}', y){:}\mathbf{t_c})$ or $\mathcal{M}(DB, DB') \models \exists y(q(\bar{a}', y){:}\mathbf{f_d} \wedge q(\bar{a}', y){:}\mathbf{t_c})$. As the analysis was done for an arbitrary value $\bar{a}$, we have that $\mathcal{M}(DB, DB') \models \mathcal{T}(DB, IC)$. □

Proof of Lemma 2: This result extends a similar result in [3]. We concentrate on the cases not covered there. We have to show $DB_\mathcal{M} \models_\Sigma p(\bar{x}) \rightarrow \exists y q(\bar{x}', y)$. Let us suppose first $\mathcal{M} \models p(\bar{a}){:}\mathbf{f_c}$. Then, we either have $\mathcal{M} \models p(\bar{a}){:}\mathbf{f}$ or $\mathcal{M} \models p(\bar{a}){:}\mathbf{f_a}$. Hence, $DB_\mathcal{M} \models_\Sigma \neg p(\bar{a})$, and from there $DB_\mathcal{M} \models_\Sigma p(\bar{a}) \rightarrow \exists y q(\bar{a}', y)$. Let us suppose now $\mathcal{M} \models \exists y(q(\bar{a}', y){:}\mathbf{t_d} \wedge q(\bar{a}', y){:}\mathbf{t_c})$. Therefore, $\mathcal{M} \models q(\bar{a}', b){:}\mathbf{t}$ for some element b in the domain. Hence $DB_\mathcal{M} \models_\Sigma q(\bar{a}', b)$, and from there $DB_\mathcal{M} \models_\Sigma p(\bar{a}) \rightarrow \exists y q(\bar{a}', y)$. Finally, we will assume $\mathcal{M} \models \exists y(q(\bar{a}', y){:}\mathbf{f_d} \wedge q(\bar{a}', y){:}\mathbf{t_c})$. Then, $\mathcal{M} \models q(\bar{a}', b){:}\mathbf{t_a}$ for some element b in the domain. Hence, $DB_\mathcal{M} \models_\Sigma q(\bar{a}', b)$, and from there $DB_\mathcal{M} \models_\Sigma p(\bar{a}) \rightarrow \exists y q(\bar{a}', y)$. As this is valid for any value $\bar{a}$, we have that $DB_\mathcal{M} \models_\Sigma p(\bar{x}) \rightarrow \exists y q(\bar{x}', y)$. □

Proof of Proposition 1: By Lemma 1, we conclude that $\mathcal{M}(DB, DB') \models \mathcal{T}(DB, IC)$. Let us suppose that $\mathcal{M}(DB, DB')$ is not Δ-minimal in the class of models of $\mathcal{T}(DB, IC)$. Then, there exists $\mathcal{M} \models \mathcal{T}(DB, IC)$, such that $\mathcal{M} <_\Delta \mathcal{M}(DB, DB')$. By using this it is possible to prove that $\Delta(DB, DB_\mathcal{M}) \subsetneq \Delta(DB, DB')$.

1. Let us suppose that $p(\bar{a}) \in \Delta(DB, DB_\mathcal{M})$. Then $p(\bar{a}) \in DB$ and $p(\bar{a}) \notin DB_\mathcal{M}$, or $p(\bar{a}) \notin DB$ and $p(\bar{a}) \in DB_\mathcal{M}$. In the first case we can conclude that $p(\bar{a}){:}\mathbf{t_d} \in \mathcal{T}(DB, IC)$ and $\mathcal{M} \models p(\bar{a}){:}\mathbf{f} \vee p(\bar{a}){:}\mathbf{f_a}$. If we suppose that $\mathcal{M} \models p(\bar{a}){:}\mathbf{f}$, then $\mathcal{M} \not\models p(\bar{a}){:}\mathbf{t_d}$, a contradiction. Thus, we have that $\mathcal{M} \models p(\bar{a}){:}\mathbf{f_a}$. But $\mathcal{M} <_\Delta \mathcal{M}(DB, DB')$, and therefore $\mathcal{M}(DB, DB') \models p(\bar{a}){:}\mathbf{f_a}$. Then, we conclude that $p(\bar{a}) \notin DB'$, and therefore in this case it is possible to conclude that $p(\bar{a}) \in \Delta(DB, DB')$. In the second case we can conclude that $p(\bar{a}){:}\mathbf{f_d} \in \mathcal{T}(DB, IC)$ and $\mathcal{M} \models p(\bar{a}){:}\mathbf{t} \vee p(\bar{a}){:}\mathbf{t_a}$. If we suppose that $\mathcal{M} \models p(\bar{a}){:}\mathbf{t}$, then $\mathcal{M} \not\models p(\bar{a}){:}\mathbf{f_d}$, a contradiction. Thus, we have that $\mathcal{M} \models p(\bar{a}){:}\mathbf{t_a}$. But $\mathcal{M} <_\Delta \mathcal{M}(DB, DB')$, and therefore $\mathcal{M}(DB, DB') \models p(\bar{a}){:}\mathbf{t_a}$. Then, we conclude that $p(\bar{a}) \in DB'$, and therefore in this case it is possible to conclude that $p(\bar{a}) \in \Delta(DB, DB')$. Thus, we can conclude that $\Delta(DB, DB_\mathcal{M}) \subseteq \Delta(DB, DB')$.
2. Since $\mathcal{M}(DB, DB') \not\leq_\Delta \mathcal{M}$, there exists $p(\bar{a})$ such that $\mathcal{M}(DB, DB') \models p(\bar{a}){:}\mathbf{t_a} \vee p(\bar{a}){:}\mathbf{f_a}$ and $\mathcal{M} \models p(\bar{a}){:}\mathbf{t} \vee p(\bar{a}){:}\mathbf{f}$. By using the first fact it is

possible to conclude that $p(\bar{a}) \in \Delta(DB, DB')$. If we suppose that $p(\bar{a}) \in DB$, then $p(\bar{a})$:$\mathbf{t_d} \in \mathcal{T}(DB, IC)$, and therefore by considering the second fact it is possible to deduce that $\mathcal{M}$ must satisfy $p(\bar{a})$:$\mathbf{t}$. Thus, we can conclude that in this case $p(\bar{a}) \in DB_{\mathcal{M}}$, and therefore $p(\bar{a}) \notin \Delta(DB, DB_{\mathcal{M}})$. By the other hand, if we suppose that $p(\bar{a}) \notin DB$, then $p(\bar{a})$:$\mathbf{f_d} \in \mathcal{T}(DB, IC)$, and therefore by considering the second fact it is possible to deduce that $\mathcal{M}$ must satisfy $p(\bar{a})$:$\mathbf{f}$. Thus, we can conclude that in this case $p(\bar{a}) \notin DB_{\mathcal{M}}$, and therefore $p(\bar{a}) \notin \Delta(DB, DB_{\mathcal{M}})$. Finally, we conclude that $\Delta(DB, DB') \not\subseteq \Delta(DB, DB_{\mathcal{M}})$.

We know that DB' is a database instance, and therefore $\Delta(DB, DB')$ must be a finite set. Thus, we can conclude that $\Delta(DB, DB_{\mathcal{M}})$ is a finite set, and therefore $DB_{\mathcal{M}}$ is a database instance. With the help of Lemma 2, we deduce that $DB_{\mathcal{M}} \models IC$. But this is a contradiction, since DB' is a repair of DB with respect to IC and $\Delta(DB, DB_{\mathcal{M}}) \subsetneq \Delta(DB, DB')$. □

Proof of Proposition 2: By Lemma 2, we conclude that $DB_{\mathcal{M}} \models_{\Sigma} IC$. Now, we need to prove that $DB_{\mathcal{M}}$ is minimal. Let us suppose this is not true. Then, there is a database instance DB^* such that $DB^* \models_{\Sigma} IC$ and $\Delta(DB, DB^*) \subsetneq \Delta(DB, DB_{\mathcal{M}})$.

1. From Lemma 1, we conclude that $\mathcal{M}(DB, DB^*) \models \mathcal{T}(DB, IC)$.
2. Now, we are going to prove that $\mathcal{M}(DB, DB^*) <_{\Delta} \mathcal{M}$.
   If $\mathcal{M}(DB, DB^*) \models p(\bar{a})$:$\mathbf{t_a}$, then we can conclude that $p(\bar{a}) \notin DB$ and $p(\bar{a}) \in DB^*$, and therefore $p(\bar{a}) \in \Delta(DB, DB^*)$. But $\Delta(DB, DB^*) \subsetneq \Delta(DB, DB_{\mathcal{M}})$, and therefore $p(\bar{a}) \in DB_{\mathcal{M}}$. Thus, we can conclude that $\mathcal{M} \models p(\bar{a})$:$\mathbf{t} \vee p(\bar{a})$:$\mathbf{t_a}$. If we suppose that $\mathcal{M} \models p(\bar{a})$:$\mathbf{t}$, then $\mathcal{M} \not\models p(\bar{a})$:$\mathbf{f_d}$, but we know that $\mathcal{M} \models \mathcal{T}(DB, IC)$ and $p(\bar{a})$:$\mathbf{f_d} \in \mathcal{T}(DB, IC)$, since $p(\bar{a}) \notin DB$, a contradiction. Therefore, $\mathcal{M} \models p(\bar{a})$:$\mathbf{t_a}$.
   If $\mathcal{M}(DB, DB^*) \models p(\bar{a})$:$\mathbf{f_a}$, then we can conclude that $p(\bar{a}) \in DB$ and $p(\bar{a}) \notin DB^*$, and therefore $p(\bar{a}) \in \Delta(DB, DB^*)$. But $\Delta(DB, DB^*) \subsetneq \Delta(DB, DB_{\mathcal{M}})$, and therefore $p(\bar{a}) \notin DB_{\mathcal{M}}$. Thus, we can conclude that $\mathcal{M} \models p(\bar{a})$:$\mathbf{f} \vee p(\bar{a})$:$\mathbf{f_a}$. If we suppose that $\mathcal{M} \models p(\bar{a})$:$\mathbf{f}$, then $\mathcal{M} \not\models p(\bar{a})$:$\mathbf{t_d}$, but we know that $\mathcal{M} \models \mathcal{T}(DB, IC)$ and $p(\bar{a})$:$\mathbf{t_d} \in \mathcal{T}(DB, IC)$, since $p(\bar{a}) \in DB$, a contradiction. Therefore, $\mathcal{M} \models p(\bar{a})$:$\mathbf{f_a}$. Thus, we can deduce that $\mathcal{M}(DB, DB^*) \leq_{\Delta} \mathcal{M}$.
   Finally, we know that there exists $p(\bar{a})$ such that it is not in $\Delta(DB, DB^*)$ and it is in $\Delta(DB, DB_{\mathcal{M}})$. Thus, $p(\bar{a}) \in DB$ and $p(\bar{a}) \in DB^*$, and therefore $\mathcal{M}(DB, DB^*) \models p(\bar{a})$:$\mathbf{t}$, or $p(\bar{a}) \notin DB$ and $p(\bar{a}) \notin DB^*$, and therefore $\mathcal{M}(DB, DB^*) \models p(\bar{a})$:$\mathbf{f}$. Then, we have that $\mathcal{M}(DB, DB^*) \not\models p(\bar{a})$:$\mathbf{t_a}$ and $\mathcal{M}(DB, DB^*) \not\models p(\bar{a})$:$\mathbf{f_a}$. Additionally, since $p(\bar{a}) \in \Delta(DB, DB_{\mathcal{M}})$, we can conclude that $p(\bar{a}) \in DB$ and $p(\bar{a}) \notin DB_{\mathcal{M}}$, or $p(\bar{a}) \notin DB$ and $p(\bar{a}) \in DB_{\mathcal{M}}$. In the first case we can conclude that $\mathcal{M} \models p(\bar{a})$:$\mathbf{f_a}$. In the second case we can conclude that $\mathcal{M} \models p(\bar{a})$:$\mathbf{t_a}$. Thus, we can conclude that $\mathcal{M} \models p(\bar{a})$:$\mathbf{t_a} \vee p(\bar{a})$:$\mathbf{f_a}$. Therefore we can deduce that $\mathcal{M} \not\leq_{\Delta} \mathcal{M}(DB, DB^*)$.

Finally, we deduce that $\mathcal{M}$ is not minimal in the class of the models of $\mathcal{T}(DB, IC)$, with respect to Δ, a contradiction. □

Proofs for Section 4

Lemma 3. *For a minimal model $\mathcal{M}$ of $\mathcal{T}(DB, IC)$ and APC formula $\varphi(\bar{x})$, $\mathcal{M} \models_{APC} (\neg\varphi)^{an}(\bar{t})$ iff $\mathcal{M} \models_{APC} \neg\varphi^{an}(\bar{t})$.*

Proof: By induction on φ.
Initial step: $\varphi(\bar{t}) = p(\bar{t})$. Trivial, by the fact that every model of $\mathcal{T}(DB, IC)$ annotates atoms either with $\mathbf{t}$, $\mathbf{f}$, $\mathbf{t_a}$ or $\mathbf{f_a}$.
Inductive step:

- $\varphi(\bar{t}) = \neg\alpha(\bar{t})$. $\mathcal{M} \models (\neg\neg\alpha)^{an}(\bar{t})$ iff $\mathcal{M} \models (\alpha)^{an}(\bar{t})$ iff $\mathcal{M} \not\models \neg(\alpha)^{an}(\bar{t})$ iff $\mathcal{M} \not\models (\neg\alpha)^{an}(\bar{t})$ (by induction hypothesis) iff $\mathcal{M} \models \neg(\neg\alpha)^{an}(\bar{t})$.
- $\varphi(\bar{t}) = \alpha(\bar{t_1}) \vee \beta(\bar{t_2}) = (\alpha \vee \beta)(\bar{t})$, where $\bar{t_1}$ is the restriction of $\bar{t}$ to α (the same for $\bar{t_2}$ and β). Now, $\mathcal{M} \models (\neg(\alpha \vee \beta))^{an}(\bar{t})$ iff $\mathcal{M} \models (\neg\alpha)^{an}(\bar{t_1})$ and $\mathcal{M} \models (\neg\beta)^{an}(\bar{t_2})$ iff $\mathcal{M} \models \neg(\alpha)^{an}(\bar{t_1})$ and $\mathcal{M} \models \neg(\beta)^{an}(\bar{t_2})$ (by induction hypothesis) iff $\mathcal{M} \models \neg(\alpha \vee \ \beta)^{an}(\bar{t})$. □

Proof of of Proposition 3: We will prove it by induction on φ.
Initial step: $\varphi(\bar{x}) = p(\bar{x})$. $DB \models_c p(\bar{t})$ iff for every repair DB' of DB, $DB' \models_\Sigma p(\bar{t})$ iff for every minimal model $\mathcal{M}$ of $\mathcal{T}(DB, IC)$, $\mathcal{M} \models p(\bar{t})\!:\!\mathbf{t} \vee p(\bar{t})\!:\!\mathbf{t_a}$ iff $\mathcal{T}(DB, IC) \models_\Delta p(\bar{t})\!:\!\mathbf{t} \vee p(\bar{t})\!:\!\mathbf{t_a}$.

Inductive step:

- $\varphi(\bar{x}) = \neg\alpha(\bar{x})$. $DB \models_c \neg\alpha(\bar{t})$ iff for every repair DB' of DB we have that $DB' \not\models_\Sigma \alpha(\bar{t})$ iff for every minimal model $\mathcal{M}$ of $\mathcal{T}(DB, IC)$, $\mathcal{M} \not\models \alpha^{an}(\bar{t})$ (by induction hypothesis) iff for every minimal model $\mathcal{M}$ of $\mathcal{T}(DB, IC)$, $\mathcal{M} \models \neg\alpha^{an}(\bar{t})$ iff $\mathcal{M} \models (\neg\alpha)^{an}(\bar{t})$ (by Lemma 3).
- $\varphi(\bar{x}) = \alpha(\bar{x_1}) \vee \beta(\bar{x_2}) = (\alpha \vee \beta)(\bar{x})$. $DB \models_c (\alpha \vee \beta)(\bar{t})$ iff for every repair DB' of DB it is true that $DB' \models_\Sigma \alpha(\bar{t_1})$ or $DB' \models_\Sigma \beta(\bar{t_2})$, where $\bar{t_i}$ is the restriction of substitution $\bar{t}$ to the variables $\bar{x_i}$, iff for every minimal model $\mathcal{M}$ of $\mathcal{T}(DB, IC)$, $\mathcal{M} \models \alpha^{an}(\bar{t_1})$ or $\mathcal{M} \models \beta^{an}(\bar{t_2})$ (by induction hypothesis) iff $\mathcal{T}(DB, IC) \models_\Delta (\alpha^{an} \vee \beta^{an})(\bar{t})$ iff $\mathcal{T}(DB, IC) \models_\Delta (\alpha \vee \beta)^{an}(\bar{t})$. □

Proofs for Section 5

Lemma 4. *If $\mathcal{M}$ is a coherent stable model of $\Pi^\star(DB, IC)$, i.e. a coherent minimal model of $(\Pi^\star(DB, IC))^{\mathcal{M}}$, then exactly one of the following cases holds:*

- *$p(\bar{a}, \mathbf{t_d})$, $p(\bar{a}, \mathbf{t}^\star)$ and $p(\bar{a}, \mathbf{t}^{\star\star})$ belong to $\mathcal{M}$, and no other $p(\bar{a}, v)$, for v an annotation value, belongs to $\mathcal{M}$.*
- *$p(\bar{a}, \mathbf{t_d})$, $p(\bar{a}, \mathbf{t}^\star)$, $p(\bar{a}, \mathbf{f_a})$, $p(\bar{a}, \mathbf{f}^\star)$ and $p(\bar{a}, \mathbf{f}^{\star\star})$ belong to $\mathcal{M}$, and no other $p(\bar{a}, v)$, for v an annotation value, belongs to $\mathcal{M}$.*
- *$p(\bar{a}, \mathbf{t_a})$, $p(\bar{a}, \mathbf{f}^\star)$, $p(\bar{a}, \mathbf{t}^\star)$ and $p(\bar{a}, \mathbf{t}^{\star\star})$ belong to $\mathcal{M}$, and no other $p(\bar{a}, v)$, for v an annotation value, belongs to $\mathcal{M}$.*
- *$p(\bar{a}, \mathbf{f}^\star)$ and $p(\bar{a}, \mathbf{f}^{\star\star})$ belongs to $\mathcal{M}$, and no other $p(\bar{a}, v)$, for v an annotation value, belongs to $\mathcal{M}$.*

Proof: For an atom $p(\bar{a})$ we have two possibilities:

1. $p(\bar{a}, \mathbf{t_d}) \in \mathcal{M}$. Then, $p(\bar{a}, \mathbf{t}^\star) \in \mathcal{M}$. Two cases are possible now: $p(\bar{a}, \mathbf{f_a}) \in \mathcal{M}$ or $p(\bar{a}, \mathbf{f_a}) \notin \mathcal{M}$. For the first one we also have $p(\bar{a}, \mathbf{f}^{\star\star})$, $p(\bar{a}, \mathbf{f}^\star) \in \mathcal{M}$ and $p(\bar{a}, \mathbf{t_a}) \notin \mathcal{M}$ (because $\mathcal{M}$ is coherent). For the second one, $p(\bar{a}, \mathbf{f}^\star) \notin \mathcal{M}$ (since $\mathcal{M}$ is minimal), $p(\bar{a}, \mathbf{t_a}) \notin \mathcal{M}$ (because $p(\bar{a}, \mathbf{f}^\star) \notin \mathcal{M}$ and $\mathcal{M}$ is minimal) and $p(\bar{a}, \mathbf{t}^{\star\star}) \in \mathcal{M}$. This covers the first two items in the lemma.
2. $p(\bar{a}, \mathbf{t_d}) \notin \mathcal{M}$. Then, $p(\bar{a}, \mathbf{f}^\star) \in \mathcal{M}$. Two cases are possible now: $p(\bar{a}, \mathbf{t_a}) \in \mathcal{M}$ or $p(\bar{a}, \mathbf{t_a}) \notin \mathcal{M}$. For the first one we also have $p(\bar{a}, \mathbf{t}^{\star\star})$, $p(\bar{a}, \mathbf{t}^\star) \in \mathcal{M}$ and $p(\bar{a}, \mathbf{f_a}) \notin \mathcal{M}$ (because $\mathcal{M}$ is coherent).
   For the second one, $p(\bar{a}, \mathbf{t}^\star) \notin \mathcal{M}$ (since $\mathcal{M}$ is minimal), $p(\bar{a}, \mathbf{f_a}) \notin \mathcal{M}$ (because $p(\bar{a}, \mathbf{t}^\star) \notin \mathcal{M}$ and $\mathcal{M}$ is minimal) and $p(\bar{a}, \mathbf{f}^{\star\star}) \in \mathcal{M}$. This covers the last two items in the lemma. □

From two database instances we can define a structure.

Definition 11. *For two database instances DB_1 and DB_2 over the same schema and domain, $\mathcal{M}^\star(DB_1, DB_2)$ is the Herbrand structure $\langle D, I_{\mathcal{P}}, I_{\mathcal{B}} \rangle$, where D is the domain of the database*[6] *and I_P, I_B are the interpretations for the database predicates (extended with annotation arguments) and the built-ins, respectively. I_P is defined as follows:*

- *If $p(\bar{a}) \in DB_1$ and $p(\bar{a}) \in DB_2$, then $p(\bar{a}, \mathbf{t_d})$, $p(\bar{a}, \mathbf{t}^\star)$ and $p(\bar{a}, \mathbf{t}^{\star\star}) \in I_P$.*
- *If $p(\bar{a}) \in DB_1$ and $p(\bar{a}) \notin DB_2$, then $p(\bar{a}, \mathbf{t_d})$, $p(\bar{a}, \mathbf{t}^\star)$, $p(\bar{a}, \mathbf{f_a})$, $p(\bar{a}, \mathbf{f}^\star)$ and $p(\bar{a}, \mathbf{f}^{\star\star}) \in I_P$.*
- *If $p(\bar{a}) \notin DB_1$ and $p(\bar{a}) \notin DB_2$, then $p(\bar{a}, \mathbf{f}^\star)$ and $p(\bar{a}, \mathbf{f}^{\star\star}) \in I_{\mathcal{P}}$.*
- *If $p(\bar{a}) \notin DB_1$ and $p(\bar{a}) \in DB_2$, then $p(\bar{a}, \mathbf{f}^\star)$, $p(\bar{a}, \mathbf{t_a})$, $p(\bar{a}, \mathbf{t}^\star)$ and $p(\bar{a}, \mathbf{t}^{\star\star}) \in I_{\mathcal{P}}$.*

The interpretation $I_{\mathcal{B}}$ is defined as expected: if q is a built-in, then $q(\bar{a}) \in I_{\mathcal{B}}$ iff $q(\bar{a})$ is true in classical logic, and $q(\bar{a}) \notin I_{\mathcal{B}}$ iff $q(\bar{a})$ is false. □

Notice that the database associated to $\mathcal{M}^\star(DB_1, DB_2)$ corresponds exactly to DB_2, i.e. $DB_{\mathcal{M}^\star(DB_1, DB_2)} = DB_2$.

Lemma 5. *If $DB' \models_\Sigma IC$, then there is a coherent model $\mathcal{M}$ of the program $(\Pi^\star(DB, IC))^{\mathcal{M}}$ such that $DB_{\mathcal{M}} = DB'$. Furthermore, the model $\mathcal{M}$ corresponds to $\mathcal{M}^\star(DB, DB')$.*

Proof: As $DB_{\mathcal{M}^\star(DB, DB')} = DB'$, we only need to show that $\mathcal{M}^\star(DB, DB')$ is a model of $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB, DB')}$. Since $DB' \models_\Sigma \bigvee_{i=1}^n \neg p_i(\bar{a_i}) \vee \bigvee_{j=1}^m q_j(\bar{b_j}) \vee \varphi$, we have three possibilities to analyze with respect to the satisfaction of this clause. The first possibility is $DB' \models_\Sigma \neg p_i(\bar{a})$. Then, two cases arise

[6] Strictly speaking, the domain D now also contains the annotations values.

- $p_i(\bar{a}) \in DB$. Then, $p_i(\bar{a}, \mathbf{f}^\star)$, $p_i(\bar{a}, \mathbf{t_d})$, $p_i(\bar{a}, \mathbf{f_a})$, $p_i(\bar{a}, \mathbf{t}^\star)$ and $p_i(\bar{a}, \mathbf{f}^{\star\star})$ belong to $\mathcal{M}^\star(DB,\ DB')$, and the program $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$ contains the following clauses: $p_i(\bar{a}, \mathbf{t_d}) \leftarrow$, $p_i(\bar{a}, \mathbf{t}^\star) \leftarrow p_i(\bar{a}, \mathbf{t_d})$, $p_i(\bar{a}, \mathbf{t}^\star) \leftarrow p_i(\bar{a}, \mathbf{t_a})$, $p_i(\bar{a}, \mathbf{f}^\star) \leftarrow p_i(\bar{a}, \mathbf{f_a})$, $p_i(\bar{a}, \mathbf{t}^{\star\star}) \leftarrow p_i(\bar{a}, \mathbf{t_a})$ and $p_i(\bar{a}, \mathbf{f}^{\star\star}) \leftarrow p_i(\bar{a}, \mathbf{f_a})$. Then, all these formulas are satisfied by $\mathcal{M}^\star(DB, DB')$. The program also contains the clause $\bigvee_{i=1}^n p_i(\bar{a}, \mathbf{f_a}) \vee \bigvee_{j=1}^m q_j(\bar{a}, \mathbf{t_a}) \leftarrow \bigwedge_{i=1}^n p_i(\bar{a}, \mathbf{t}^\star) \wedge \bigwedge_{j=1}^m q_j(\bar{a}, \mathbf{f}^\star) \wedge \bar{\varphi}$, which is satisfied since $p_i(\bar{a}, \mathbf{f_a})$ belongs to $\mathcal{M}^\star(DB, DB')$.
- $p_i(\bar{a}) \notin DB$. Then, $p_i(\bar{a}, \mathbf{f}^\star)$ and $p_i(\bar{a}, \mathbf{f}^{\star\star}) \in \mathcal{M}^\star(DB, DB')$, and $p_i(\bar{a}, \mathbf{f}^\star)$, $p_i(\bar{a}, \mathbf{t}^\star) \leftarrow p_i(\bar{a}, \mathbf{t_d})$, $p_i(\bar{a}, \mathbf{t}^\star) \leftarrow p_i(\bar{a}, \mathbf{t_a})$, $p_i(\bar{a}, \mathbf{f}^\star) \leftarrow p_i(\bar{a}, \mathbf{f_a})$, $p_i(\bar{a}, \mathbf{f}^{\star\star}) \leftarrow$, $p_i(\bar{a}, \mathbf{t}^{\star\star}) \leftarrow p_i(\bar{a}, \mathbf{t_a})$ and $p_i(\bar{a}, \mathbf{f}^{\star\star}) \leftarrow p_i(\bar{a}, \mathbf{f_a})$ are in the program $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$. All these are satisfied by the model considered. Also the clause $\bigvee_{i=1}^n p_i(\bar{a}, \mathbf{f_a}) \vee \bigvee_{j=1}^m q_j(\bar{a}, \mathbf{t_a}) \leftarrow \bigwedge_{i=1}^n p_i(\bar{a}, \mathbf{t}^\star) \wedge \bigwedge_{j=1}^m q_j(\bar{a}, \mathbf{f}^\star) \wedge \bar{\varphi}$ is present, and is trivially satisfied since $p_i(\bar{a}, \mathbf{t}^\star) \notin \mathcal{M}^\star(DB, DB')$.

The second possibility is $DB' \models_\Sigma q_j(\bar{a})$. The following cases arise:

- $q_j(\bar{a}) \in DB$. Then, $\mathcal{M}^\star(DB, DB')$ contains $q_j(\bar{a}, \mathbf{t_d})$, $q_j(\bar{a}, \mathbf{t}^\star)$ and $q_j(\bar{a}, \mathbf{t}^{\star\star})$, and program $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$ contains the formulas $q_j(\bar{a}, \mathbf{t_d}) \leftarrow$, $q_j(\bar{a}, \mathbf{t}^\star) \leftarrow q_j(\bar{a}, \mathbf{t_d})$, $q_j(\bar{a}, \mathbf{t}^\star) \leftarrow q_j(\bar{a}, \mathbf{t_a})$, $q_j(\bar{a}, \mathbf{f}^\star) \leftarrow q_j(\bar{a}, \mathbf{f_a})$, $q_j(\bar{a}, \mathbf{t}^{\star\star}) \leftarrow q_j(\bar{a}, \mathbf{t_d})$, $q_j(\bar{a}, \mathbf{t}^{\star\star}) \leftarrow q_j(\bar{a}, \mathbf{t_a})$ and $q_j(\bar{a}, \mathbf{f}^{\star\star}) \leftarrow q_j(\bar{a}, \mathbf{f_a})$. The structure $\mathcal{M}^\star(DB, DB')$ satisfies all these clauses. The clause $\bigvee_{i=1}^n p_i(\bar{a}, \mathbf{f_a}) \vee \bigvee_{j=1}^m q_j(\bar{a}, \mathbf{t_a}) \leftarrow \bigwedge_{i=1}^n p_i(\bar{a}, \mathbf{t}^\star) \wedge \bigwedge_{j=1}^m q_j(\bar{a}, \mathbf{f}^\star) \wedge \bar{\varphi}$ is also in the program, and is trivially satisfied since it holds that $q_j(\bar{a}, \mathbf{f}^\star)$ does not belong to $\mathcal{M}^\star(DB, DB')$.
- $q_j(\bar{a}) \notin DB$. Then, $q_j(\bar{a}, \mathbf{f}^\star)$, $q_j(\bar{a}, \mathbf{t_a})$, $q_j(\bar{a}, \mathbf{t}^\star)$ and $q_j(\bar{a}, \mathbf{t}^{\star\star})$ are in the structure $\mathcal{M}^\star(DB, DB')$, and the following formulas are in the program $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$: $q_j(\bar{a}, \mathbf{f}^\star) \leftarrow$, $q_j(\bar{a}, \mathbf{t}^\star) \leftarrow q_j(\bar{a}, \mathbf{t_d})$, $q_j(\bar{a}, \mathbf{t}^\star) \leftarrow q_j(\bar{a}, \mathbf{t_a})$, $q_j(\bar{a}, \mathbf{f}^\star) \leftarrow q_j(\bar{a}, \mathbf{f_a})$, $q_j(\bar{a}, \mathbf{t}^{\star\star}) \leftarrow q_j(\bar{a}, \mathbf{t_d})$, $q_j(\bar{a}, \mathbf{t}^{\star\star}) \leftarrow q_j(\bar{a}, \mathbf{t_a})$ and $q_j(\bar{a}, \mathbf{f}^{\star\star}) \leftarrow q_j(\bar{a}, \mathbf{f_a})$. These are satisfied by $\mathcal{M}^\star(DB, DB')$. Also the clause $\bigvee_{i=1}^n p_i(\bar{a}, \mathbf{f_a}) \vee \bigvee_{j=1}^m q_j(\bar{a}, \mathbf{t_a}) \leftarrow \bigwedge_{i=1}^n p_i(\bar{a}, \mathbf{t}^\star) \wedge \bigwedge_{j=1}^m q_j(\bar{a}, \mathbf{f}^\star) \wedge \bar{\varphi}$ is in the program, and is satisfied since $q_j(\bar{a}, \mathbf{t_a})$ belongs to $\mathcal{M}^\star(DB, DB')$.

The third possibility is $DB' \models_\Sigma \varphi$. Then, φ is true. The clause $\bigvee_{i=1}^n p_i(\bar{a}, \mathbf{f_a}) \vee \bigvee_{j=1}^m q_j(\bar{a}, \mathbf{t_a}) \leftarrow \bigwedge_{i=1}^n p_i(\bar{a}, \mathbf{t}^\star) \wedge \bigwedge_{j=1}^m q_j(\bar{a}, \mathbf{f}^\star) \wedge \bar{\varphi}$ is in $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$, and is satisfied since $\mathcal{M}^\star(DB, DB') \not\models \bar{\varphi}$.

As the analysis was done for an arbitrary value $\bar{a}$, it holds that the Herbrand structure $\mathcal{M}^\star(DB, DB')$ is a model of $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$. Moreover, it is also coherent, since $\mathcal{M}^\star(DB, DB')$ was defined in such a way that does not contain both $p(\bar{a}, \mathbf{t_a})$ and $p(\bar{a}, \mathbf{f_a})$. □

The next lemma shows that if $\mathcal{M}$ is a coherent and minimal model of the program $(\Pi^\star(DB, IC))^{\mathcal{M}}$, and represents a finite database instance, then the instance satisfies the constraints.

Lemma 6. *If $\mathcal{M}$ is a coherent stable model of the program $\Pi^\star(DB, IC)$ and $DB_{\mathcal{M}}$ is finite, then $DB_{\mathcal{M}} \models_\Sigma IC$.*

Proof: We want to show $DB_{\mathcal{M}} \models_{\Sigma} \bigvee_{i=1}^{n} \neg p_i(\bar{x_i}) \vee \bigvee_{j=1}^{m} q_j(\bar{y_j}) \vee \varphi$, for every constraint in IC. Since $\mathcal{M}$ is a model of $(\Pi^{\star}(DB,IC))^{\mathcal{M}}$, we have that $\mathcal{M} \models \bigvee_{i=1}^{n} p_i(\bar{x_i},\mathbf{f_a}) \vee \bigvee_{j=1}^{m} q_j(\bar{y_j},\mathbf{t_a}) \leftarrow \bigwedge_{i=1}^{n} p_i(\bar{x_i},\mathbf{t}^{\star}) \wedge \bigwedge_{j=1}^{m} q_j(\bar{y_j},\mathbf{f}^{\star}) \wedge \bar{\varphi}$. Then, at least one of the following cases is satisfied:

- $\mathcal{M} \models p_i(\bar{a},\mathbf{f_a})$. Then, $\mathcal{M} \models p_i(\bar{a},\mathbf{f}^{\star\star})$ and $p(\bar{a}) \notin DB_{\mathcal{M}}$ (by lemma 4). Hence, $DB_{\mathcal{M}} \models_{\Sigma} \neg p_i(\bar{a})$. Since the analysis was done for an arbitrary value $\bar{a}$, $DB_{\mathcal{M}} \models_{\Sigma} \bigvee_{i=1}^{n} \neg p_i(\bar{x_i}) \vee \bigvee_{j=1}^{m} q_j(\bar{y_j}) \vee \varphi$ holds.
- $\mathcal{M} \models q_j(\bar{a},\mathbf{t_a})$. It is symmetrical to the previous one.
- It is not true that $\mathcal{M} \models \bar{\varphi}$. Then $\mathcal{M} \models \varphi$. Hence, φ is true, and $DB_{\mathcal{M}} \models_{\Sigma} \bigvee_{i=1}^{n} \neg p_i(\bar{x_i}) \vee \bigvee_{j=1}^{m} q_j(\bar{y_j}) \vee \varphi$ holds.
- $\mathcal{M} \not\models p_i(\bar{a},\mathbf{t}^{\star})$. Given the model is coherent and minimal, just the last item in Lemma 4 holds. This means $\mathcal{M} \models p_i(\bar{a},\mathbf{f}^{\star\star})$, $p_i(\bar{a}) \notin DB_{\mathcal{M}}$ and $DB_{\mathcal{M}} \models_{\Sigma} \neg p_i(\bar{a})$. Since the analysis was done for an arbitrary value $\bar{a}$, $DB_{\mathcal{M}} \models_{\Sigma} \bigvee_{i=1}^{n} \neg p_i(\bar{x_i}) \vee \bigvee_{j=1}^{m} q_j(\bar{y_j}) \vee \varphi$ holds.
- $\mathcal{M} \not\models q_j(\bar{a},\mathbf{f}^{\star})$. Given the model is coherent and minimal, just the first item in lemma 4 holds. Then, $\mathcal{M} \models q_j(\bar{a},\mathbf{t}^{\star\star})$, $q_j(\bar{a}) \in DB_{\mathcal{M}}$ and $DB_{\mathcal{M}} \models_{\Sigma} q_j(\bar{a})$. Since the analysis was done for an arbitrary value $\bar{a}$, $DB_{\mathcal{M}} \models_{\Sigma} \bigvee_{i=1}^{n} \neg p_i(\bar{x_i}) \vee \bigvee_{j=1}^{m} q_j(\bar{y_j}) \vee \varphi$ holds. □

Lemma 7. *Consider two database instances DB and DB' over the same schema and domain. If $\mathcal{M}$ is a coherent and minimal model of $(\Pi^{\star}(DB,IC))^{\mathcal{M}^{\star}(DB,DB')}$, such that $\mathcal{M} \subsetneqq \mathcal{M}^{\star}(DB,DB')$, then there exists model $\mathcal{M}'$ such that $\mathcal{M}'$ is a coherent and minimal model of $(\Pi^{\star}(DB,IC))^{\mathcal{M}'}$ and $\Delta(DB,DB_{\mathcal{M}'}) \subsetneqq \Delta(DB,DB')$.*

Proof: Since $\mathcal{M}$ is a coherent and minimal model of $(\Pi^{\star}(DB,IC))^{\mathcal{M}^{\star}(DB,DB')}$, we have that $p(\bar{a},\mathbf{t_d}) \in \mathcal{M}$ iff $p(\bar{a}) \in DB$. By the way we defined $\mathcal{M}^{\star}(DB,DB')$ and given $\mathcal{M} \subsetneqq \mathcal{M}^{\star}(DB,DB')$, the only two ways that both models can differ is that, for some $p(\bar{a}) \in DB$, $\{p(\bar{a},\mathbf{f_a}), p(\bar{a},\mathbf{f}^{\star}), p(\bar{a},\mathbf{f}^{\star\star})\} \subseteq \mathcal{M}^{\star}(DB,DB')$ and none of these atoms belong to $\mathcal{M}$, or for some $p(\bar{a}) \notin DB$, $\{p(\bar{a},\mathbf{t_a}), p(\bar{a},\mathbf{t}^{\star}), p(\bar{a},\mathbf{t}^{\star\star})\} \subseteq \mathcal{M}^{\star}(DB,DB')$ and none of these atoms belong to $\mathcal{M}$. Now, some of the atoms in $\mathcal{M}$ may have not received an interpretation in terms of $\mathbf{t}^{\star\star}$ and $\mathbf{f}^{\star\star}$, *i.e.* $\mathcal{M}$ is not a minimal model of $(\Pi^{\star}(DB,IC))^{\mathcal{M}}$. Anyway, if we use the interpretation rules over $\mathcal{M}$, we will finish with a model $\mathcal{M}'$ that is a minimal model of $(\Pi^{\star}(DB,IC))^{\mathcal{M}'}$. From $\mathcal{M}$ the model $\mathcal{M}'$ is constructed as follows:

- If $p(\bar{a},\mathbf{t_d}) \in \mathcal{M}$ and $p(\bar{a},\mathbf{f_a}) \notin \mathcal{M}$, then $p(\bar{a},\mathbf{t_d}), p(\bar{a},\mathbf{t}^{\star})$ and $p(\bar{a},\mathbf{t}^{\star\star}) \in \mathcal{M}'$.
- If $p(\bar{a},\mathbf{t_d}) \in \mathcal{M}$ and $p(\bar{a},\mathbf{f_a}) \in \mathcal{M}$, then $p(\bar{a},\mathbf{t_d}), p(\bar{a},\mathbf{t}^{\star}), p(\bar{a},\mathbf{f_a}), p(\bar{a},\mathbf{f}^{\star})$ and $p(\bar{a},\mathbf{f}^{\star\star}) \in \mathcal{M}'$.
- If $p(\bar{a},\mathbf{t_d}) \notin \mathcal{M}$ and $p(\bar{a},\mathbf{t_a}) \notin \mathcal{M}$, then $p(\bar{a},\mathbf{f}^{\star})$ and $p(\bar{a},\mathbf{f}^{\star\star}) \in \mathcal{M}'$.
- If $p(\bar{a},\mathbf{t_d}) \notin \mathcal{M}$ and $p(\bar{a},\mathbf{t_a}) \in \mathcal{M}$, then $p(\bar{a},\mathbf{f}^{\star}), p(\bar{a},\mathbf{t_a}), p(\bar{a},\mathbf{t}^{\star})$ and $p(\bar{a},\mathbf{t}^{\star\star}) \in \mathcal{M}'$.

It is clear that $\mathcal{M}'$ is a coherent and minimal model of $(\Pi^{\star}(DB,IC))^{\mathcal{M}'}$. It just rests to prove that $\Delta(DB,DB_{\mathcal{M}'}) \subsetneqq \Delta(DB,DB')$. First, we will prove $\Delta(DB,DB_{\mathcal{M}'}) \subseteq \Delta(DB,DB')$. Let us suppose $p(\bar{a}) \in \Delta(DB,DB_{\mathcal{M}'})$. Then, either $p(\bar{a}) \in DB$ and $p(\bar{a}) \notin DB_{\mathcal{M}'}$ or $p(\bar{a}) \notin DB$ and $p(\bar{a}) \in DB_{\mathcal{M}'}$. In the first case, $p(\bar{a}, \mathbf{t_d})$, $p(\bar{a}, \mathbf{t}^\star)$, $p(\bar{a}, \mathbf{f_a})$ and $p(\bar{a}, \mathbf{f}^\star)$ are in $\mathcal{M}'$. These atoms are also in $\mathcal{M}$ and, by our assumption, they are also in $\mathcal{M}^\star(DB, DB')$. Hence, $p(\bar{a}) \in \Delta(DB, DB')$. In the second case, $p(\bar{a}, \mathbf{f}^\star)$, $p(\bar{a}, \mathbf{t_a})$ and $p(\bar{a}, \mathbf{t}^\star)$ are in $\mathcal{M}'$. These atoms are also in $\mathcal{M}$ and, by our assumption, these are also in $\mathcal{M}^\star(DB, DB')$. Hence, $p(\bar{a}) \in \Delta(DB, DB')$.

We will now prove $\Delta(DB, DB_{\mathcal{M}'}) \subsetneqq \Delta(DB, DB')$. We know for some fact $p(\bar{a})$ there is an element related to it which is in $\mathcal{M}^\star(DB, DB')$ and which is not in $\mathcal{M}$. One possible case is $p(\bar{a}, \mathbf{f_a})$ and $p(\bar{a}, \mathbf{f}^\star)$ are in $\mathcal{M}^\star(DB, DB')$ and not in $\mathcal{M}$. Then, $p(\bar{a}) \in \Delta(DB, DB')$, but $p(\bar{a}) \notin \Delta(DB, DB_{\mathcal{M}'})$. The other possible case is that $p(\bar{a}, \mathbf{t_a})$ and $p(\bar{a}, \mathbf{t}^\star)$ are in $\mathcal{M}^\star(DB, DB')$ and not in $\mathcal{M}$. Then, $p(\bar{a}) \in \Delta(DB, DB')$, but $p(\bar{a}) \notin \Delta(DB, DB_{\mathcal{M}'})$. □

Proposition 5. *If DB' is a repair of DB with respect to IC, then there is a coherent stable model $\mathcal{M}$ of the program $\Pi^\star(DB, IC)$ such that $DB_{\mathcal{M}} = DB'$. Furthermore, the model $\mathcal{M}$ corresponds to $\mathcal{M}^\star(DB, DB')$.*

Proof: By Lemma 5 we have $\mathcal{M}^\star(DB, DB')$ is a coherent model of the program $\Pi^\star(DB, IC)^{\mathcal{M}^\star(DB,DB')}$. We just have to show it is minimal. Let us suppose first there exists a model $\mathcal{M}$ of $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$ such that it is the case that $\mathcal{M} \subsetneqq \mathcal{M}^\star(DB, DB')$ (it is also coherent since it is contained in $\mathcal{M}^\star(DB, DB')$). Since $\mathcal{M} \subsetneqq \mathcal{M}^\star(DB, DB')$, the model $\mathcal{M}$ contains the atom $p(\bar{a}, \mathbf{t_d})$ iff $p(\bar{a}) \in DB$. Then, we can assume without loss of generality that $\mathcal{M}$ is minimal (if it is not minimal, we can always generate from it a minimal model $\mathcal{M}'$, such that $\mathcal{M}' \subsetneqq \mathcal{M}$, by deleting its non-supported atoms).

By Lemma 7, there exists model $\mathcal{M}'$ such that $\Delta(DB, DB_{\mathcal{M}'}) \subsetneqq \Delta(DB, DB')$ and $\mathcal{M}'$ is a coherent and minimal model of $(\Pi^\star(DB, IC))^{\mathcal{M}'}$. By Lemma 6, $DB_{\mathcal{M}'} \models_\Sigma IC$. This contradicts our fact that DB' is a repair. □

Proposition 6. *If $\mathcal{M}$ is a coherent and minimal model of $(\Pi^\star(DB, IC))^{\mathcal{M}}$ and $DB_{\mathcal{M}}$ is finite, then $DB_{\mathcal{M}}$ is a repair of DB with respect to IC.*

Proof: From Lemma 6, we have $DB_{\mathcal{M}} \models_\Sigma IC$. We just have to show minimality. Let us suppose there is a database instance DB', such that $DB' \models_\Sigma IC$ and $\Delta(DB, DB') \subsetneqq \Delta(DB, DB_{\mathcal{M}})$. Then, by Lemma 5, $\mathcal{M}^\star(DB, DB')$ is a coherent model of $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$. We will first show it is the case that $\mathcal{M}^\star(DB, DB') \subseteqq \mathcal{M}$ and that $\mathcal{M}^\star(DB, DB')$ is a model of $(\Pi^\star(DB, IC))^{\mathcal{M}}$. Notice that since $\mathcal{M}$ is a minimal model of $(\Pi^\star(DB, IC))^{\mathcal{M}}$, this program contains the clause $p(\bar{a}, \mathbf{f}^\star) \leftarrow$ for every $p(\bar{a}) \notin DB$. The rest of the program must look exactly like $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$. This is true because the only other clauses in $\Pi^\star(DB, IC)$ that contain negation in their bodies are the interpretation rules $p(\bar{a}, \mathbf{f}^{\star\star}) \leftarrow \mathit{not}\ p(\bar{a}, \mathbf{t_d}), \mathit{not}\ p(\bar{a}, \mathbf{t_a})$ and $p(\bar{a}, \mathbf{t}^{\star\star}) \leftarrow p(\bar{a}, \mathbf{t_d}), \mathit{not}\ p(\bar{a}, \mathbf{f_a})$. Since $\Delta(DB, DB') \subsetneqq \Delta(DB, DB_{\mathcal{M}})$, if $\mathcal{M}$ does not satisfy $p(\bar{a}, \mathbf{f_a})$ then $\mathcal{M}^\star(DB, DB')$ does not satisfy it either (this is, either both programs, $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$ and $(\Pi^\star(DB, IC))^{\mathcal{M}}$, contain the clause $p(\bar{a}, \mathbf{t}^{\star\star}) \leftarrow p(\bar{a}, \mathbf{t_d})$ or both do not contain it) and if $\mathcal{M}$ does not satisfy $p(\bar{a}, \mathbf{t_a})$ then $\mathcal{M}^\star(DB, DB')$ does not satisfy it either (this is, either both programs, $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$ and $(\Pi^\star(DB, IC))^{\mathcal{M}}$, contain the clause $p(\bar{a}, \mathbf{f}^{\star\star}) \leftarrow$ or both do not contain it). By Definition 11, for an arbitrary atom $p(\bar{a})$ in a model $\mathcal{M}^\star(DB, DB')$, we just have to analyze four cases:

1. Let us suppose just $p(\bar{a}, \mathbf{t}^{\star\star})$, $p(\bar{a}, \mathbf{t}^\star)$ and $p(\bar{a}, \mathbf{t_d})$ belong to $\mathcal{M}^\star(DB, DB')$. Then $p(\bar{a}) \in DB$ and $p(\bar{a}) \in DB'$. Since $p(\bar{a}) \notin \Delta(DB, DB')$, we have two possibilities. The first one saying $p(\bar{a}) \notin \Delta(DB, DB_{\mathcal{M}})$. Then, $p(\bar{a}, \mathbf{t}^\star)$, $p(\bar{a}, \mathbf{t_d})$ and $p(\bar{a}, \mathbf{t}^{\star\star})$ also belong to $\mathcal{M}$ and $\mathcal{M}^\star(DB, DB')$ is clearly a model of the clauses in $(\Pi^\star(DB, IC))^{\mathcal{M}}$ concerning $p(\bar{a})$. The second one saying $p(\bar{a}) \in \Delta(DB, DB_{\mathcal{M}})$. Again, $p(\bar{a}, \mathbf{t}^\star)$, $p(\bar{a}, \mathbf{t_d})$ and $p(\bar{a}, \mathbf{t}^{\star\star})$ belong to $\mathcal{M}$ and $\mathcal{M}^\star(DB, DB')$ is clearly a model of the clauses in $(\Pi^\star(DB, IC))^{\mathcal{M}}$ concerning $p(\bar{a})$.
2. Let us suppose now, just $p(\bar{a}, \mathbf{f}^\star)$ and $p(\bar{a}, \mathbf{f}^{\star\star})$ belong to $\mathcal{M}^\star(DB, DB')$. Again we have two possibilities. The first one says that $p(\bar{a}) \notin \Delta(DB, DB_{\mathcal{M}})$. Then, $p(\bar{a}, \mathbf{f}^\star)$ and $p(\bar{a}, \mathbf{f}^{\star\star})$ also belong to $\mathcal{M}$. The program $(\Pi^\star(DB, IC))^{\mathcal{M}}$ contains (among others) the clause $p(\bar{a}, \mathbf{f}^\star) \leftarrow$, that is satisfied by the program $\mathcal{M}^\star(DB, DB')$. The rest of the clauses concerning $p(\bar{a})$ are satisfied because are also present in $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$. The second one says that $p(\bar{a}) \in \Delta(DB, DB_{\mathcal{M}})$. Again, $p(\bar{a}, \mathbf{f}^\star)$ and $p(\bar{a}, \mathbf{f}^{\star\star})$ belong to $\mathcal{M}$. The program $(\Pi^\star(DB, IC))^{\mathcal{M}}$ contains (among others) the clause $p(\bar{a}, \mathbf{f}^\star) \leftarrow$, that is satisfied by $\mathcal{M}^\star(DB, DB')$. The rest of the clauses concerning $p(\bar{a})$ are satisfied because they are also present in $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$.
3. Let us suppose just $p(\bar{a}, \mathbf{t}^\star)$, $p(\bar{a}, \mathbf{t_d})$, $p(\bar{a}, \mathbf{f_a})$, $p(\bar{a}, \mathbf{f}^\star)$ and $p(\bar{a}, \mathbf{f}^{\star\star})$ belong to the model $\mathcal{M}^\star(DB, DB')$. Then $p(\bar{a}) \in DB$ and $p(\bar{a}) \notin DB'$. Hence, $p(\bar{a}) \in \Delta(DB, DB')$, and due to our assumption $p(\bar{a}) \in \Delta(DB, DB_{\mathcal{M}})$. Therefore, $p(\bar{a}, \mathbf{t}^\star)$, $p(\bar{a}, \mathbf{t_d})$, $p(\bar{a}, \mathbf{f_a})$, $p(\bar{a}, \mathbf{f}^\star)$ and $p(\bar{a}, \mathbf{f}^{\star\star})$ belong to $\mathcal{M}$. Moreover, $\mathcal{M}^\star(DB, DB')$ is clearly a model of the clauses in $(\Pi^\star(DB, IC))^{\mathcal{M}}$ concerning $p(\bar{a})$.
4. Finally, we will suppose just $p(\bar{a}, \mathbf{f}^\star)$, $p(\bar{a}, \mathbf{t_a})$, $p(\bar{a}, \mathbf{t}^\star)$ and $p(\bar{a}, \mathbf{t}^{\star\star})$ belong to the model $\mathcal{M}^\star(DB, DB')$. Then, $p(\bar{a}) \notin DB$ and $p(\bar{a}) \in DB'$. Hence, $p(\bar{a}) \in \Delta(DB, DB')$, and due to our assumption $p(\bar{a}) \in \Delta(DB, DB_{\mathcal{M}})$. Therefore, $p(\bar{a}, \mathbf{f}^\star)$, $p(\bar{a}, \mathbf{t}^\star)$, $p(\bar{a}, \mathbf{t_a})$ and $p(\bar{a}, \mathbf{t}^{\star\star})$ belong to $\mathcal{M}$. The program $(\Pi^\star(DB, IC))^{\mathcal{M}}$ contains (among others) the clause $p(\bar{a}, \mathbf{f}^\star) \leftarrow$, that is satisfied by $\mathcal{M}^\star(DB, DB')$. The rest of the clauses concerning $p(\bar{a})$ are satisfied because are also present in $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$.

We will now show $\mathcal{M}^\star(DB, DB') \subsetneq \mathcal{M}$. We have assumed there is an element of $\Delta(DB, DB_{\mathcal{M}})$ that is not an element of $\Delta(DB, DB')$. Thus, for some element $p(\bar{a})$, either $p(\bar{a}) \in DB$, $p(\bar{a}) \in DB'$ and $p(\bar{a}) \notin DB_{\mathcal{M}}$, or $p(\bar{a}) \notin DB$, $p(\bar{a}) \notin DB'$ and $p(\bar{a}) \in DB_{\mathcal{M}}$. For the first one we have $\mathcal{M}^\star(DB, DB')$ satisfies $p(\bar{a}, \mathbf{t_d})$ and $p(\bar{a}, \mathbf{t}^\star)$, and $\mathcal{M}$ satisfies $p(\bar{a}, \mathbf{t_d})$ and $p(\bar{a}, \mathbf{t}^\star)$, but also satisfies $p(\bar{a}, \mathbf{f_a})$ and $p(\bar{a}, \mathbf{f}^\star)$. In the second one, $\mathcal{M}^\star(DB, DB')$ satisfies $p(\bar{a}, \mathbf{f}^\star)$ and $\mathcal{M}$ satisfies $p(\bar{a}, \mathbf{f}^\star)$, but also $p(\bar{a}, \mathbf{t_a})$ and $p(\bar{a}, \mathbf{t}^\star)$. Then, $\mathcal{M}$ is not a minimal model; a contradiction. □

Proof of of Theorem 1: From Propositions 5 and 6. $\square$

Proofs for Section 7

The following is an extension of Lemma 4, considering the introduction of *null* values.

Lemma 8. *If $\mathcal{M}$ is a coherent stable model of $\Pi^\star(DB, IC)$, i.e. a coherent minimal model of $(\Pi^\star(DB, IC))^{\mathcal{M}}$, then exactly one of the following cases holds:*

- $p(\bar{a}, \mathbf{t_d})$, $p(\bar{a}, \mathbf{t}^\star)$ *and* $p(\bar{a}, \mathbf{t}^{\star\star})$ *belong to* $\mathcal{M}$*, and no other* $p(\bar{a}, v)$*, for* v *an annotation value, belongs to* $\mathcal{M}$*.*
- $p(\bar{a}, \mathbf{t_d})$, $p(\bar{a}, \mathbf{t}^\star)$, $p(\bar{a}, \mathbf{f_a})$, $p(\bar{a}, \mathbf{f}^\star)$ *and* $p(\bar{a}, \mathbf{f}^{\star\star})$ *belong to* $\mathcal{M}$*, and no other* $p(\bar{a}, v)$*, for* v *an annotation value, belongs to* $\mathcal{M}$*.*
- $p(\bar{a}, \mathbf{t_a})$, $p(\bar{a}, \mathbf{f}^\star)$, $p(\bar{a}, \mathbf{t}^\star)$ *and* $p(\bar{a}, \mathbf{t}^{\star\star})$ *belong to* $\mathcal{M}$*, and no other* $p(\bar{a}, v)$*, for* v *an annotation value, belongs to* $\mathcal{M}$*.*
- $p(\bar{a}, \mathbf{f}^\star)$ *and* $p(\bar{a}, \mathbf{f}^{\star\star})$ *belongs to* $\mathcal{M}$*, and no other* $p(\bar{a}, v)$*, for* v *an annotation value, belongs to* $\mathcal{M}$*.*
- $p(\bar{a}, null, \mathbf{t_d})$ *and* $p(\bar{a}, null, \mathbf{t}^{\star\star})$ *belongs to* $\mathcal{M}$*, and no other* $p(\bar{a}, null, v)$ *for* v *an annotation value, belongs to* $\mathcal{M}$*.*
- $p(\bar{a}, null, \mathbf{t_a})$, $p(\bar{a}, null, \mathbf{t}^{\star\star})$ *belongs to* $\mathcal{M}$*, and no other* $p(\bar{a}, null, v)$*, for* v *an annotation value, belongs to* $\mathcal{M}$*.*
- $\nexists v\, p(\bar{a}, null, v)$ *for* v *an annotation value.*

Proof: The first four cases where already proven in Lemma 4. The two new cases are deduced directly considering the new rules involving the referential ICs and the inclusion of *null* values. $\square$

Definition 11 is extended to consider the atoms with *null* values as follows:

Definition 12. *For two database instances DB_1 and DB_2 over the same schema and domain, $\mathcal{M}^\star(DB_1, DB_2)$ is the Herbrand structure $\langle D, I_P, I_B\rangle$, where D is the domain of the database*[7] *and I_P, I_B are the interpretations for the database predicates (extended with annotation arguments) and the built-ins, respectively. I_P is defined as follows:*

- *If* $p(\bar{a}) \in DB_1$ *and* $p(\bar{a}) \in DB_2$*, then* $p(\bar{a}, \mathbf{t_d})$, $p(\bar{a}, \mathbf{t}^\star)$ *and* $p(\bar{a}, \mathbf{t}^{\star\star}) \in I_P$*.*
- *If* $p(\bar{a}) \in DB_1$ *and* $p(\bar{a}) \notin DB_2$*, then* $p(\bar{a}, \mathbf{t_d})$, $p(\bar{a}, \mathbf{t}^\star)$, $p(\bar{a}, \mathbf{f_a})$, $p(\bar{a}, \mathbf{f}^\star)$ *and* $p(\bar{a}, \mathbf{f}^{\star\star}) \in I_P$*.*
- *If* $p(\bar{a}) \notin DB_1$ *and* $p(\bar{a}) \notin DB_2$*, then* $p(\bar{a}, \mathbf{f}^\star)$ *and* $p(\bar{a}, \mathbf{f}^{\star\star}) \in I_P$*.*
- *If* $p(\bar{a}) \notin DB_1$ *and* $p(\bar{a}) \in DB_2$*, then* $p(\bar{a}, \mathbf{f}^\star)$, $p(\bar{a}, \mathbf{t_a})$, $p(\bar{a}, \mathbf{t}^\star)$ *and* $p(\bar{a}, \mathbf{t}^{\star\star}) \in I_P$*.*
- *If* $p(\bar{a}, null) \in DB_1$ *and* $p(\bar{a}, null) \in DB_2$*, then* $p(\bar{a}, null, \mathbf{t_d})$ *and* $p(\bar{a}, null, \mathbf{t}^{\star\star}) \in I_P$*.*
- *If* $p(\bar{a}, null) \notin DB_1$ *and* $p(\bar{a}, null) \in DB_2$*, then* $p(\bar{a}, null, \mathbf{t_a})$ *and* $p(\bar{a}, null, \mathbf{t}^{\star\star}) \in I_P$*.*

[7] Strictly speaking, the domain D now also contains the annotations values.

The interpretation I_B is defined as expected: if q is a built-in, then $q(\bar{a}) \in I_B$ iff $q(\bar{a})$ is true in classical logic, and $q(\bar{a}) \notin I_B$ iff $q(\bar{a})$ is false. □

Notice that, as before, the database associated to $\mathcal{M}^\star(DB_1, DB_2)$ corresponds exactly to DB_2, i.e. $DB_{\mathcal{M}^\star(DB_1,DB_2)} = DB_2$. The next lemma states that Lemma 6 still holds when considering universal and referential ICs.

Lemma 9. *If $\mathcal{M}$ is a coherent stable model of the program $\Pi^\star(DB, IC)$ and $DB_{\mathcal{M}}$ is finite, then $DB_{\mathcal{M}} \models_\Sigma IC$.*

Proof: As in Lemma 6 it was already proven that universal constraints are satisfied. As $\mathcal{M}$ satisfies: $\{aux(\bar{x}') \leftarrow q(\bar{x}', y, \mathbf{t_d}) \wedge$ *not* $q(\bar{x}', y, \mathbf{f_a})$; $aux(\bar{x}') \leftarrow q(\bar{x}', y, \mathbf{t_a})$; $p(\bar{x}, \mathbf{f_a}) \vee q(\bar{x}', null, \mathbf{t_a}) \leftarrow p(\bar{x}, \mathbf{t}^\star) \wedge$ *not* $aux(\bar{x}')$, *not* $q(\bar{x}', null, \mathbf{t_d})\}$ we have that it can be proved, as in Lemma 6 that the RICs of the form $p(\bar{x}) \rightarrow \exists y(q(\bar{x}', y))$ are satisfied by $\mathcal{M}$. □

The next lemma is a variation of Lemma 5 that considers universal and referential ICs and the fact that a database that is inconsistent wrt a RIC of the form $p(\bar{x}) \rightarrow \exists y(q(\bar{x}', y))$ can be repaired only deleting a tuple or inserting a tuple with the *null* value.

Lemma 10. *If DB' is a repair of DB, then there is a model $\mathcal{M}$ of $\Pi^\star(DB, IC)^{\mathcal{M}}$ such that $DB_{\mathcal{M}} = DB'$.*

Proof: This lemma is proved like Lemma 5, but instead of considering that $\mathcal{M} = \mathcal{M}^\star(DB, DB')$, it considers $\mathcal{M} = \mathcal{M}^\star(DB, DB') \cup \{aux_i(\bar{a}') \mid IC_i \in IC$ and IC_i is of the form $p(\bar{x}) \rightarrow \exists y\, q(\bar{x}', y)$ and $\exists y\, ((q(\bar{a}', y, \mathbf{t_d}) \in \mathcal{M}^\star(DB, DB')$ and $q(\bar{a}', y, \mathbf{f_a}) \notin \mathcal{M}^\star(DB, DB'))$ or $q(\bar{a}', y, \mathbf{t_a}) \in \mathcal{M}^\star(DB, DB'))\}$. □

The next proposition shows that Proposition 5 holds also for $\Pi^\star(DB, IC)$ extended for RICs.

Proposition 7. *If DB' is a repair of DB with respect to IC, then there is a coherent stable model $\mathcal{M}$ of $\Pi^\star(DB, IC)$ such that $DB_{\mathcal{M}} = DB'$.*

Proof: By Lemma 10 we have that $\mathcal{M} = \mathcal{M}^\star(DB, DB') \cup \{aux_i(\bar{a}') \mid IC_i \in IC$ and IC_i is of the form $p(\bar{x}) \rightarrow \exists y\, q(\bar{x}', y)$ and $\exists y\, ((q(\bar{a}', y, \mathbf{t_d}) \in \mathcal{M}^\star(DB, DB')$ and $q(\bar{a}', y, \mathbf{f_a}) \notin \mathcal{M}^\star(DB, DB'))$**or**$q(\bar{a}', y, \mathbf{t_a}) \in \mathcal{M}^\star(DB, DB'))\}$ is a coherent model of the program $\Pi^\star(DB, IC)^{\mathcal{M}}$. Its minimality can be proved as done for $\mathcal{M}^\star(DB, DB')$ in Lemma 5. □

Proposition 8. *If $\mathcal{M}$ is a coherent and stable model of $\Pi^\star(DB, IC)$, and $DB_{\mathcal{M}}$ is finite, then $DB_{\mathcal{M}}$ is a repair of DB with respect to IC.*

Proof: From Lemma 9, we have $DB_{\mathcal{M}} \models_\Sigma IC$. We only need to prove that it is $\leq_{DB}$-*minimal.* This is proven in a similar way as it was done in Proposition 6, but considering $\leq_{DB}$ instead of minimality under set inclusion. □

Proof of of Theorem 2: From Propositions 7 and 8. □

Proofs for Section 8

Proof of of Theorem 3: ($\Leftarrow$) If the set of *ground*(IC) does not have a pair of bilateral literals in the same *IC*, we want to prove that the program $\Pi^\star(DB, IC)$ is HCF for any *DB*.

We will suppose that the program $\Pi^\star(DB, IC)$ is *not* HCF. Then the program *ground*($\Pi(DB, IC)$) has a directed cycle that goes through two literals that belong to the head of the same rule from *ground*($\Pi(DB, IC)$). The only rules with more than one literal in the head are the rules capturing the ICs, i.e. those of the form $\bigvee_{i=1}^{n} p_i(\bar{a}_i, \mathbf{f_a}) \vee \bigvee_{j=1}^{m} q_j(\bar{b_j}, \mathbf{t_a}) \leftarrow \bigwedge_{i=1}^{n} p_i(\bar{a}_i, \mathbf{t}^\star) \wedge \bigwedge_{j=1}^{m} q_j(\bar{b_j}, \mathbf{f}^\star) \wedge \bar{\varphi}$.

For the program no to be HCF there has to be a cycle involving:

- $P_1(\bar{a}_1, \mathbf{f_a})$ and $P_2(\bar{a}_2, \mathbf{f_a})$ or
- $Q_1(\bar{b}_1, \mathbf{t_a})$ and $Q_2(\bar{b}_2, \mathbf{t_a})$ or
- $P_1(\bar{a}_1, \mathbf{f_a})$ and $Q_1(\bar{b}_1, \mathbf{t_a})$

If we analyze the first case, we can consider that only $P_1(\bar{a}_1, \mathbf{f_a})$ might be bilateral. Figure 2 shows that no directed cycle involving $P_1(\bar{a}_1, \mathbf{f_a})$ and $P_2(\bar{a}_2, \mathbf{f_a})$ is possible. The dependency graph of the other two cases is analogous, and it is not possible to have cycles involving to literals of the head of a rule. So the program can not be HCF.

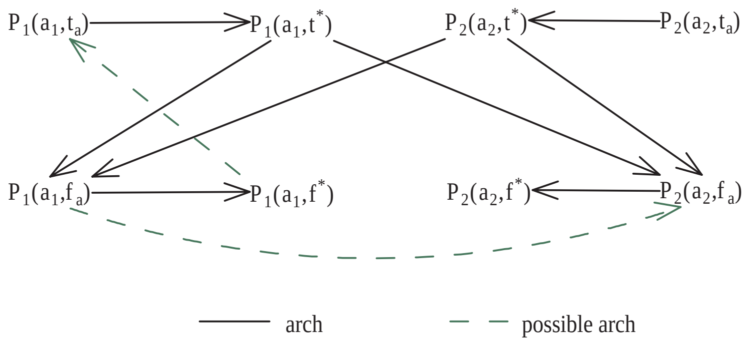

Fig. 2. Dependency Graph of P_1 and P_2

($\Rightarrow$) If the program $\Pi^\star(DB, IC)$ is HCF for any *DB*then the set of instantiated ICs do not have a pair of bilateral literals in the same IC.

Let us suppose there is a pair of bilateral literals, $P_1(\bar{a}_1)$ and $Q_1(\bar{b}_1)$, in the same IC. As $P_1(\bar{a}_1)$ and $Q_1(\bar{b}_1)$ are in the same IC, there are three different cases to study. Note that P and Q can be the same predicate.

1. $P_1(\bar{a}_1)$ and $Q_1(\bar{b}_1)$ are in the head of the *IC*. In this case, $P_1(\bar{a}_1, \mathbf{f_a})$ and $Q_1(\bar{b}_1, \mathbf{f_a})$ are in the head of a rule of $\Pi^\star(DB, IC)$, and as it can be seen in Figure 3 there is a cycle that includes them, so the program is not HCF.

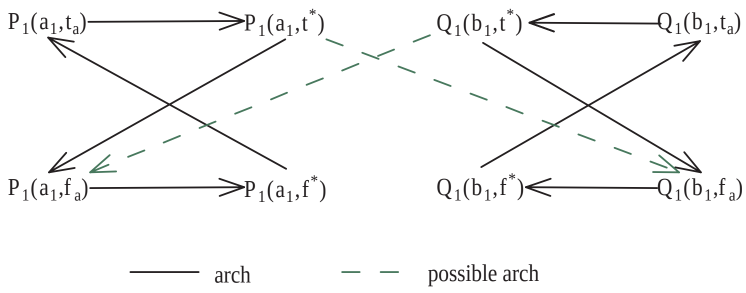

Fig. 3. Dependency Graph of P_1 and Q_1 with both of them in the head of an IC

2. $P_1(\bar{a}_1)$ and $Q_1(\bar{b}_1)$ are in the body of the IC. Analogous to first case.
3. $P_1(\bar{a}_1)$ is in the head and $Q_1(\bar{b}_1)$ is in the body of the IC. Analogous to the first case.

So, if there is a pair of bilateral literals in the same IC, the program can not be HCF, i.e. if the program is HCF, then it can not have a pair of bilateral literals in the same IC. □