A Proofs and Intermediate Results

Proof of lemma 1:

1. If $\mathcal{M} \models p(\bar{a}) : \top$, then $\mathcal{M} \models p(\bar{a}) : \mathbf{t_c}$ and $\mathcal{M} \models p(\bar{a}) : \mathbf{f_c}$. Thus, $\mathcal{M} \not\models \mathcal{T}(\mathbf{IC})$, a contradiction.
2. We know that $\mathcal{M} \models p(\bar{a}) : \mathbf{t_c} \vee p(\bar{a}) : \mathbf{f_c}$ and $\mathcal{M} \models p(\bar{a}) : \mathbf{t_d} \vee p(\bar{a}) : \mathbf{f_d}$ (since $p(\bar{a}) : \mathbf{t_d} \in \mathcal{T}(\mathbf{DB}, \mathbf{IC})$ or $p(\bar{a}) : \mathbf{f_d} \in \mathcal{T}(\mathbf{DB}, \mathbf{IC})$). Thus, one of the following cases must be true: (1) $\mathcal{M} \models p(\bar{a}) : \mathbf{t_c}$ and $\mathcal{M} \models p(\bar{a}) : \mathbf{t_d}$, and therefore $\mathcal{M} \models p(\bar{a}) : \mathbf{t}$, (2) $\mathcal{M} \models p(\bar{a}) : \mathbf{t_c}$ and $\mathcal{M} \models p(\bar{a}) : \mathbf{f_d}$, and therefore $\mathcal{M} \models p(\bar{a}) : \mathbf{t_a}$, (3) $\mathcal{M} \models p(\bar{a}) : \mathbf{f_c}$ and $\mathcal{M} \models p(\bar{a}) : \mathbf{t_d}$, and therefore $\mathcal{M} \models p(\bar{a}) : \mathbf{f_a}$, (4) $\mathcal{M} \models p(\bar{a}) : \mathbf{f_c}$ and $\mathcal{M} \models p(\bar{a}) : \mathbf{f_d}$, and therefore $\mathcal{M} \models p(\bar{a}) : \mathbf{f}$.

□

Proof of lemma 2: We have to prove that $\mathcal{M}(\mathbf{DB}, \mathbf{DB'}) \models \mathcal{T}(\mathbf{DB})$ and $\mathcal{M}(\mathbf{DB}, \mathbf{DB'}) \models \mathcal{T}(\mathbf{IC})$.

1. Let us consider $p(\bar{a}) : \mathbf{a} \in \mathcal{T}(\mathbf{DB})$. If $\mathbf{a} = \mathbf{t_d}$, then $p(\bar{a}) \in \mathbf{DB}$, and then by considering (1) we obtain that $I_P(p(\bar{a})) = \mathbf{t}$ or $I_P(p(\bar{a})) = \mathbf{f_a}$, and therefore $\mathcal{M}(\mathbf{DB}, \mathbf{DB'}) \models p(\bar{a}) : \mathbf{a}$. If $\mathbf{a} = \mathbf{f_d}$, then $p(\bar{a}) \notin \mathbf{DB}$, and then by considering (1) we obtain that $I_P(p(\bar{a})) = \mathbf{f}$ or $I_P(p(\bar{a})) = \mathbf{t_a}$, and therefore $\mathcal{M}(\mathbf{DB}, \mathbf{DB'}) \models p(\bar{a}) : \mathbf{a}$.
2. (a) Let us suppose that $p_1(\bar{T}_1) : \mathbf{t_c} \vee \cdots \vee p_n(\bar{T}_n) : \mathbf{t_c} \vee q_1(\bar{T}_1) : \mathbf{f_c} \vee \cdots \vee q_m(\bar{T}_m) : \mathbf{f_c} \in \mathcal{T}(\mathbf{IC})$, and let us assume that $p_1(\bar{a}_1) : \mathbf{t_c} \vee \cdots \vee p_n(\bar{a}_n) : \mathbf{t_c} \vee q_1(\bar{b}_1) : \mathbf{f_c} \vee \cdots \vee q_m(\bar{a}_m) : \mathbf{f_c}$ was obtained from this constraint by instantiating in the domain of the database. In this case we have that $p_1(\bar{T}_1) \vee \cdots \vee p_n(\bar{T}_n) \vee \neg q_1(\bar{T}_1) \vee \cdots \vee \neg q_m(\bar{T}_m)$ is an element of $\mathbf{IC}$, and therefore we have that $\mathbf{DB'} \models_{\mathrm{DB}} p_1(\bar{a}_1) \vee \cdots \vee p_n(\bar{a}_n) \vee \neg q_1(\bar{b}_1) \vee \cdots \vee \neg q_m(\bar{b}_m)$.
   Firstly, we are going to consider what happens if $\mathbf{DB'} \models_{\mathrm{DB}} p_i(\bar{a}_i)$ $(1 \leq i \leq n)$. If p_i is a built-in predicate, then $I_R(p_i(\bar{a}_i)) = \mathbf{t}$, since $\mathcal{M}(\mathbf{DB}, \mathbf{DB'})$ gives to the built-in predicates in the database the appropriate truth values, and therefore $\mathcal{M}(\mathbf{DB}, \mathbf{DB'}) \models p_i(\bar{a}_i) : \mathbf{t_c}$. If p_i is not a built-in predicate, then $I_P(p_i(\bar{a}_i)) = \mathbf{t}$ or $I_P(p_i(\bar{a}_i)) = \mathbf{t_a}$, since $p_i(\bar{a}_i) \in \mathbf{DB'}$, and therefore $\mathcal{M}(\mathbf{DB}, \mathbf{DB'}) \models p_i(\bar{a}_i) : \mathbf{t_c}$.
   Secondly, we are going to consider what happens if $\mathbf{DB'} \models_{\mathrm{DB}} \neg q_i(\bar{b}_i)$ $(1 \leq i \leq m)$. If q_i is a built-in predicate, then $I_R(q_i(\bar{b}_i)) = \mathbf{f}$, since $\mathcal{M}(\mathbf{DB}, \mathbf{DB'})$ gives to the built-in predicates in the database the appropriate truth values, and therefore $\mathcal{M}(\mathbf{DB}, \mathbf{DB'}) \models q_i(\bar{b}_i) : \mathbf{f_c}$. If q_i is not a built-in predicate, then $I_P(q_i(\bar{b}_i)) = \mathbf{f}$ or $I_P(q_i(\bar{b}_i)) = \mathbf{f_a}$, since $q_i(\bar{b}_i) \notin \mathbf{DB'}$, and therefore $\mathcal{M}(\mathbf{DB}, \mathbf{DB'}) \models q_i(\bar{b}_i) : \mathbf{f_c}$.

   (b) Let us consider a predicate p in P. By considering (1) we know that for every tuple $\bar{a}$ (of appropriate arity) $I_P(p(\bar{a})) = \mathbf{t}$, $I_P(p(\bar{a})) = \mathbf{f}$, $I_P(p(\bar{a})) = \mathbf{t_a}$ or $I_P(p(\bar{a})) = \mathbf{f_a}$, and therefore $\mathcal{M}(\mathbf{DB}, \mathbf{DB'}) \models p(\bar{a}) : \mathbf{t_c} \vee p(\bar{a}) : \mathbf{f_c}$. Thus, we conclude that $\mathcal{M}(\mathbf{DB}, \mathbf{DB'}) \models \forall \bar{x}(p(\bar{x}) :$

$\mathbf{t_c} \vee p(\bar{x}) : \mathbf{f_c})$. Additionally, if $I_P(p(\bar{a})) = \mathbf{t}$ or $I_P(p(\bar{a})) = \mathbf{t_a}$, then $\mathcal{M}(\mathbf{DB}, \mathbf{DB}') \not\models p(\bar{a}) : \mathbf{f_c}$, and if $I_P(p(\bar{a})) = \mathbf{f}$ or $I_P(p(\bar{a})) = \mathbf{f_a}$, then $\mathcal{M}(\mathbf{DB}, \mathbf{DB}') \not\models p(\bar{a}) : \mathbf{t_c}$. Thus, we also conclude that $\mathcal{M}(\mathbf{DB}, \mathbf{DB}') \models \forall \bar{x}(\neg p(\bar{x}) : \mathbf{t_c} \vee \neg p(\bar{x}) : \mathbf{f_c})$.

□

Proof of lemma 3: We are going to prove that $\mathbf{DB}_{\mathcal{M}} \models_{\text{DB}} \mathbf{IC}$. Let us suppose that $p_1(\bar{T}_1) \vee \cdots \vee p_n(\bar{T}_n) \vee \neg q_1(\bar{T}_1) \vee \cdots \vee \neg q_m(\bar{T}_m)$ is an integrity constraint in $\mathbf{IC}$, and let us assume that $p_1(\bar{a}_1) \vee \cdots \vee p_n(\bar{a}_n) \vee \neg q_1(\bar{b}_1) \vee \cdots \vee \neg q_m(\bar{b}_m)$ was obtained from it by instantiated in the domain of the database. In this case we have that $p_1(\bar{a}_1) : \mathbf{t_c} \vee \cdots \vee p_n(\bar{a}_n) : \mathbf{t_c} \vee q_1(\bar{b}_1) : \mathbf{f_c} \vee \cdots \vee q_m(\bar{b}_m) : \mathbf{f_c}$ could be obtained by instantiated an integrity constraint in $\mathcal{T}(\mathbf{IC})$. Thus, we have that $\mathcal{M} \models p_1(\bar{a}_1) : \mathbf{t_c} \vee \cdots \vee p_n(\bar{a}_n) : \mathbf{t_c} \vee q_1(\bar{b}_1) : \mathbf{f_c} \vee \cdots \vee q_m(\bar{b}_m) : \mathbf{f_c}$.

Firstly, we are going to consider what happens if $\mathcal{M} \models p_i(\bar{a}_i) : \mathbf{t_c}$ ($1 \leq i \leq n$). If p_i is a built-in predicate, then $I_R(p_i(\bar{a}_i)) = \mathbf{t}$, since $\mathcal{M}$ gives to the built-in predicates in the database the value $\mathbf{t}$ or $\mathbf{f}$, and if in this case we suppose that $I_R(p_i(\bar{a}_i)) = \mathbf{f}$ then $\mathcal{M} \not\models p_i(\bar{a}_i) : \mathbf{t_c}$, a contradiction. Therefore $\mathbf{DB}_{\mathcal{M}} \models_{\text{DB}} p_i(\bar{a}_i)$. If p_i is not a built-in predicate, then $p_i(\bar{a}_i) : \mathbf{t_d} \in \mathcal{T}(\mathbf{DB})$ or $p_i(\bar{a}_i) : \mathbf{f_d} \in \mathcal{T}(\mathbf{DB})$. In the first case we have that $\mathcal{M} \models p_i(\bar{a}_i) : \mathbf{t}$, and therefore $p_i(\bar{a}_i) \in \mathbf{DB}_{\mathcal{M}}$. In the second case $\mathcal{M} \models p_i(\bar{a}_i) : \mathbf{t_a}$, and therefore $p_i(\bar{a}_i) \in \mathbf{DB}_{\mathcal{M}}$.

Secondly, we are going to consider what happens if $\mathcal{M} \models q_i(\bar{b}_i) : \mathbf{f_c}$ ($1 \leq i \leq m$). If q_i is a built-in predicate, then $I_R(q_i(\bar{b}_i)) = \mathbf{f}$, since $\mathcal{M}$ gives to the built-in predicates in the database the value $\mathbf{t}$ or $\mathbf{f}$, and if in this case we suppose that $I_R(q_i(\bar{b}_i)) = \mathbf{t}$ then $\mathcal{M} \not\models q_i(\bar{b}_i) : \mathbf{f_c}$, a contradiction. Therefore $\mathbf{DB}_{\mathcal{M}} \models_{\text{DB}} \neg q_i(\bar{b}_i)$. If q_i is not a built-in predicate, then $q_i(\bar{b}_i) : \mathbf{t_d} \in \mathcal{T}(\mathbf{DB})$ or $q_i(\bar{b}_i) : \mathbf{f_d} \in \mathcal{T}(\mathbf{DB})$. In the first case we have that $\mathcal{M} \models q_i(\bar{b}_i) : \mathbf{f_a}$, and therefore $q_i(\bar{b}_i) \notin \mathbf{DB}_{\mathcal{M}}$. In the second case $\mathcal{M} \models q_i(\bar{b}_i) : \mathbf{f}$, and therefore $q_i(\bar{b}_i) \notin \mathbf{DB}_{\mathcal{M}}$.
□

Proof of proposition 1:

1. By Lemma 3, we conclude that $\mathbf{DB}_{\mathcal{M}} \models_{\text{DB}} \mathbf{IC}$.
2. Now, we need to prove that $\mathbf{DB}_{\mathcal{M}}$ is minimal. Let us suppose this is not true. Then, there is a database instance $\mathbf{DB}^*$ such that $\mathbf{DB}^* \models_{\text{DB}} \mathbf{IC}$ and $\Delta(\mathbf{DB}, \mathbf{DB}^*) \subsetneq \Delta(\mathbf{DB}, \mathbf{DB}_{\mathcal{M}})$.
   (a) From Lemma 2, we conclude that $\mathcal{M}(\mathbf{DB}, \mathbf{DB}^*) \models \mathcal{T}(\mathbf{DB}, \mathbf{IC})$.
   (b) Now, we are going to prove that $\mathcal{M}(\mathbf{DB}, \mathbf{DB}^*) <_{\Delta} \mathcal{M}$.
   If $\mathcal{M}(\mathbf{DB}, \mathbf{DB}^*) \models p(\bar{a}) : \mathbf{t_a}$, then by considering (1) we can conclude that $p(\bar{a}) \notin \mathbf{DB}$ and $p(\bar{a}) \in \mathbf{DB}^*$, and therefore $p(\bar{a}) \in \Delta(\mathbf{DB}, \mathbf{DB}^*)$. But $\Delta(\mathbf{DB}, \mathbf{DB}^*) \subsetneq \Delta(\mathbf{DB}, \mathbf{DB}_{\mathcal{M}})$, and therefore $p(\bar{a}) \in \mathbf{DB}_{\mathcal{M}}$. Thus, we can conclude that $\mathcal{M} \models p(\bar{a}) : \mathbf{t} \vee p(\bar{a}) : \mathbf{t_a}$. If we suppose that $\mathcal{M} \models p(\bar{a}) : \mathbf{t}$, then $\mathcal{M} \not\models p(\bar{a}) : \mathbf{f_d}$, but we know that $\mathcal{M} \models \mathcal{T}(\mathbf{DB}, \mathbf{IC})$ and $p(\bar{a}) : \mathbf{f_d} \in \mathcal{T}(\mathbf{DB}, \mathbf{IC})$, since $p(\bar{a}) \notin \mathbf{DB}$, a contradiction. Therefore, $\mathcal{M} \models p(\bar{a}) : \mathbf{t_a}$.
   If $\mathcal{M}(\mathbf{DB}, \mathbf{DB}^*) \models p(\bar{a}) : \mathbf{f_a}$, then by considering (1) we can conclude that $p(\bar{a}) \in \mathbf{DB}$ and $p(\bar{a}) \notin \mathbf{DB}^*$, and therefore $p(\bar{a}) \in \Delta(\mathbf{DB}, \mathbf{DB}^*)$.

But $\Delta(\mathbf{DB}, \mathbf{DB}^*) \subsetneqq \Delta(\mathbf{DB}, \mathbf{DB}_{\mathcal{M}})$, and therefore $p(\bar{a}) \notin \mathbf{DB}_{\mathcal{M}}$. Thus, we can conclude that $\mathcal{M} \models p(\bar{a}) : \mathbf{f} \vee p(\bar{a}) : \mathbf{f_a}$. If we suppose that $\mathcal{M} \models p(\bar{a}) : \mathbf{f}$, then $\mathcal{M} \not\models p(\bar{a}) : \mathbf{t_d}$, but we know that $\mathcal{M} \models \mathcal{T}(\mathbf{DB}, \mathbf{IC})$ and $p(\bar{a}) : \mathbf{t_d} \in \mathcal{T}(\mathbf{DB}, \mathbf{IC})$, since $p(\bar{a}) \in \mathbf{DB}$, a contradiction. Therefore, $\mathcal{M} \models p(\bar{a}) : \mathbf{f_a}$. Thus, we can deduce that $\mathcal{M}(\mathbf{DB}, \mathbf{DB}^*) \leq_\Delta \mathcal{M}$.

Finally, we know that there exists $p(\bar{a})$ such that it is not in $\Delta(\mathbf{DB}, \mathbf{DB}^*)$ and it is in $\Delta(\mathbf{DB}, \mathbf{DB}_{\mathcal{M}})$. Thus, $p(\bar{a}) \in \mathbf{DB}$ and $p(\bar{a}) \in \mathbf{DB}^*$, and therefore $\mathcal{M}(\mathbf{DB}, \mathbf{DB}^*) \models p(\bar{a}) : \mathbf{t}$, or $p(\bar{a}) \notin \mathbf{DB}$ and $p(\bar{a}) \notin \mathbf{DB}^*$, and therefore $\mathcal{M}(\mathbf{DB}, \mathbf{DB}^*) \models p(\bar{a}) : \mathbf{f}$. Then, we have that $\mathcal{M}(\mathbf{DB}, \mathbf{DB}^*) \not\models p(\bar{a}) : \mathbf{t_a}$ and $\mathcal{M}(\mathbf{DB}, \mathbf{DB}^*) \not\models p(\bar{a}) : \mathbf{f_a}$. Additionally, since $p(\bar{a}) \in \Delta(\mathbf{DB}, \mathbf{DB}_{\mathcal{M}})$, we can conclude that $p(\bar{a}) \in \mathbf{DB}$ and $p(\bar{a}) \notin \mathbf{DB}_{\mathcal{M}}$, or $p(\bar{a}) \notin \mathbf{DB}$ and $p(\bar{a}) \in \mathbf{DB}_{\mathcal{M}}$. In the first case we can conclude that $\mathcal{M} \models p(\bar{a}) : \mathbf{f_a}$, since $\mathcal{M}$ must be satisfied $p(\bar{a}) : \mathbf{f} \vee p(\bar{a}) : \mathbf{f_a}$, and if we suppose that $\mathcal{M} \models p(\bar{a}) : \mathbf{f}$, then $\mathcal{M} \not\models p(\bar{a}) : \mathbf{t_d}$, but $p(\bar{a}) : \mathbf{t_d} \in \mathcal{T}(\mathbf{DB}, \mathbf{IC})$ in this case, a contradiction. In the second case we can conclude that $\mathcal{M} \models p(\bar{a}) : \mathbf{t_a}$, since $\mathcal{M}$ must be satisfied $p(\bar{a}) : \mathbf{t} \vee p(\bar{a}) : \mathbf{t_a}$, and if we suppose that $\mathcal{M} \models p(\bar{a}) : \mathbf{t}$, then $\mathcal{M} \not\models p(\bar{a}) : \mathbf{f_d}$, but $p(\bar{a}) : \mathbf{f_d} \in \mathcal{T}(\mathbf{DB}, \mathbf{IC})$ in this case, a contradiction. Thus, we can conclude that $\mathcal{M} \models p(\bar{a}) : \mathbf{t_a} \vee p(\bar{a}) : \mathbf{f_a}$. Therefore we can deduce that $\mathcal{M} \not\leq_\Delta \mathcal{M}(\mathbf{DB}, \mathbf{DB}^*)$.

Finally, we deduce that $\mathcal{M}$ is not e-consistent maximal in the class of the models of $\mathcal{T}(\mathbf{DB}, \mathbf{IC})$, with respect to Δ, a contradiction.

□

Proof of proposition 2:

1. By Lemma 2, we conclude that $\mathcal{M}(\mathbf{DB}, \mathbf{DB}') \models \mathcal{T}(\mathbf{DB}, \mathbf{IC})$.
2. Let us suppose that $\mathcal{M}(\mathbf{DB}, \mathbf{DB}')$ is not e-consistent maximal in the class of models of $\mathcal{T}(\mathbf{DB}, \mathbf{IC})$ with respect to Δ. Then, there exists $\mathcal{M} \models \mathcal{T}(\mathbf{DB}, \mathbf{IC})$, such that $\mathcal{M} <_\Delta \mathcal{M}(\mathbf{DB}, \mathbf{DB}')$. By using this it is possible to prove that $\Delta(\mathbf{DB}, \mathbf{DB}_{\mathcal{M}}) \subsetneqq \Delta(\mathbf{DB}, \mathbf{DB}')$.

   (a) Let us suppose that $p(\bar{a}) \in \Delta(\mathbf{DB}, \mathbf{DB}_{\mathcal{M}})$. Then $p(\bar{a}) \in \mathbf{DB}$ and $p(\bar{a}) \notin \mathbf{DB}_{\mathcal{M}}$, or $p(\bar{a}) \notin \mathbf{DB}$ and $p(\bar{a}) \in \mathbf{DB}_{\mathcal{M}}$. In the first case we can conclude that $p(\bar{a}) : \mathbf{t_d} \in \mathcal{T}(\mathbf{DB}, \mathbf{IC})$ and $\mathcal{M} \models p(\bar{a}) : \mathbf{f} \vee p(\bar{a}) : \mathbf{f_a}$. If we suppose that $\mathcal{M} \models p(\bar{a}) : \mathbf{f}$, then $\mathcal{M} \not\models p(\bar{a}) : \mathbf{t_d}$, a contradiction. Thus, we have that $\mathcal{M} \models p(\bar{a}) : \mathbf{f_a}$. But $\mathcal{M} <_\Delta \mathcal{M}(\mathbf{DB}, \mathbf{DB}')$, and therefore $\mathcal{M}(\mathbf{DB}, \mathbf{DB}') \models p(\bar{a}) : \mathbf{f_a}$. Then, by considering (1) we conclude that $p(\bar{a}) \notin \mathbf{DB}'$, and therefore in this case it is possible to conclude that $p(\bar{a}) \in \Delta(\mathbf{DB}, \mathbf{DB}')$. In the second case we can conclude that $p(\bar{a}) : \mathbf{f_d} \in \mathcal{T}(\mathbf{DB}, \mathbf{IC})$ and $\mathcal{M} \models p(\bar{a}) : \mathbf{t} \vee p(\bar{a}) : \mathbf{t_a}$. If we suppose that $\mathcal{M} \models p(\bar{a}) : \mathbf{t}$, then $\mathcal{M} \not\models p(\bar{a}) : \mathbf{f_d}$, a contradiction. Thus, we have that $\mathcal{M} \models p(\bar{a}) : \mathbf{t_a}$. But $\mathcal{M} <_\Delta \mathcal{M}(\mathbf{DB}, \mathbf{DB}')$, and therefore $\mathcal{M}(\mathbf{DB}, \mathbf{DB}') \models p(\bar{a}) : \mathbf{t_a}$. Then, by considering (1) we conclude that $p(\bar{a}) \in \mathbf{DB}'$, and therefore in this case it is possible to conclude that $p(\bar{a}) \in \Delta(\mathbf{DB}, \mathbf{DB}')$. Thus, we can conclude that $\Delta(\mathbf{DB}, \mathbf{DB}_{\mathcal{M}}) \subsetneqq \Delta(\mathbf{DB}, \mathbf{DB}')$.

(b) Since $\mathcal{M}(\mathbf{DB}, \mathbf{DB}') \not\leq_\Delta \mathcal{M}$, there exists $p(\bar{a})$ such that $\mathcal{M}(\mathbf{DB}, \mathbf{DB}') \models p(\bar{a}) : \mathbf{t_a} \vee p(\bar{a}) : \mathbf{f_a}$ and $\mathcal{M} \models p(\bar{a}) : \mathbf{t} \vee p(\bar{a}) : \mathbf{f}$. By using (1) and the first fact it is possible to conclude that $p(\bar{a}) \in \Delta(\mathbf{DB}, \mathbf{DB}')$. If we suppose that $p(\bar{a}) \in \mathbf{DB}$, then $p(\bar{a}) : \mathbf{t_d} \in \mathcal{T}(\mathbf{DB}, \mathbf{IC})$, and therefore by considering the second fact it is possible to deduce that $\mathcal{M}$ must satisfy $p(\bar{a}) : \mathbf{t}$. Thus, we can conclude that in this case $p(\bar{a}) \in \mathbf{DB}_\mathcal{M}$, and therefore $p(\bar{a}) \notin \Delta(\mathbf{DB}, \mathbf{DB}_\mathcal{M})$. By the other hand, if we suppose that $p(\bar{a}) \notin \mathbf{DB}$, then $p(\bar{a}) : \mathbf{f_d} \in \mathcal{T}(\mathbf{DB}, \mathbf{IC})$, and therefore by considering the second fact it is possible to deduce that $\mathcal{M}$ must satisfy $p(\bar{a}) : \mathbf{f}$. Thus, we can conclude that in this case $p(\bar{a}) \notin \mathbf{DB}_\mathcal{M}$, and therefore $p(\bar{a}) \notin \Delta(\mathbf{DB}, \mathbf{DB}_\mathcal{M})$. Finally, we conclude that $\Delta(\mathbf{DB}, \mathbf{DB}_\mathcal{M}) \not\supseteq \Delta(\mathbf{DB}, \mathbf{DB}')$.

We know that $\mathbf{DB}'$ is a database instance, and therefore $\Delta(\mathbf{DB}, \mathbf{DB}')$ must be a finite set. Thus, we can conclude that $\Delta(\mathbf{DB}, \mathbf{DB}_\mathcal{M})$ is a finite set, and therefore $\mathbf{DB}_\mathcal{M}$ is a database instance. With the help of Lemma 3, we deduce that $\mathbf{DB}_\mathcal{M} \models \mathbf{IC}$. But this a contradiction, since $\mathbf{DB}'$ is a repair of $\mathbf{DB}$ with respect to $\mathbf{IC}$ and $\Delta(\mathbf{DB}, \mathbf{DB}_\mathcal{M}) \subsetneq \Delta(\mathbf{DB}, \mathbf{DB}')$.

□

Proof of lemma 4: Let us suppose that

$$\mathcal{T}(\mathbf{DB}, \mathbf{IC}) \vdash r_1(\bar{c}_1) : ?_\mathbf{a} \vee \cdots \vee \cdots \vee r_k(\bar{c}_k) : ?_\mathbf{a}. \tag{5}$$

Because of the form of the clauses in $\mathcal{T}(\mathbf{DB}, \mathbf{IC})$, the above a-clause can be obtained by applying a series of reduction and resolution rules to the clauses in $\mathcal{T}(\mathbf{DB}) \cup \mathcal{T}(\mathcal{B})$ (the database part of $\mathcal{T}(\mathbf{DB}, \mathbf{IC})$ plus builtins) and a clause of the form

$$r_1(\bar{t}_1) : \mathbf{f_c} \vee \cdots \vee r_j(\bar{t}_j) : \mathbf{f_c} \vee r_{j+1}(\bar{t}_{j+1}) : \mathbf{t_c} \vee \cdots \vee r_k(\bar{t}_k) : \mathbf{t_c}, \tag{6}$$

where the latter is a clause obtained from $\mathcal{T}(\mathbf{IC})$ (the constraint part of $\mathcal{T}(\mathbf{DB}, \mathbf{IC})$) by resolution (and factorization) alone.

Furthermore, it is easy to show that resolution applied to a pair of range-restricted constraints yields a range-restricted constraint. Thus, (6) is range-restricted.

Since (5) is obtained from (6) by resolution and reduction with the clauses in $\mathcal{T}(\mathbf{DB})$, there must be clauses $r_i(\bar{c}_i) : \mathbf{t} \vee r_i(\bar{c}_i) : \top \in \mathcal{T}(\mathbf{DB})$, $1 \leq i \leq j$ (which are resolved with (6)), and clauses $r_{i'}(\bar{c}_{i'}) : \mathbf{f} \vee r_{i'}(\bar{c}_{i'}) : \top \in \mathcal{T}(\mathbf{DB})$, $j < i' \leq k$ (which are reduced with (6)), such that there is a substitution θ for which $\bar{t}_i\theta = \bar{c}_i$ $(1 \leq i \leq k)$.

Therefore, due to the range-restrictedness of (6), every constant in $\bar{c}_{i'}$ $(j < i' \leq k)$ occurs in some $\bar{c}_i$ $(1 \leq i \leq j)$. Since every constant in $\bar{c}_i$ is in the active domain of $\mathbf{DB}$, we conclude that every constant mentioned in (5) belongs to the active domain of $\mathbf{DB}$. □

Proof of corollary 1: By Lemma 4, the clauses in $\mathcal{T}^{\mathbf{a}}(\mathbf{DB}, \mathbf{IC})$ can mention only the constants that occur in the active domain of **DB**, which is a finite set. □

Proof of theorem 3: At the end of section 6 we showed that the decision problem is equivalent to the problem of deciding, given a finite collection of sets, and a subset of the union of the family, whether the subset can be extended to a minimal hitting set of the family. In the following lemmas we prove that this is *NP-complete.*

Lemma 5. *Given a finite collection of sets S and a hitting set of it H, H is a minimal hitting set of S if and only if for each $h \in H$ there exists an $A \in S$ such that $A \cap H = \{h\}$.*

Proof
($\Rightarrow$) Let us suppose that the lemma is not true. Then there exists $h \in H$ such that for every $A \in S$, $A \cap H \neq \{h\}$. We are going to prove $H' = H - \{h\}$ is also a hitting set. Let us consider $A \in S$. If $h \in A$, then there exists another $h' \in H$ such that $h' \in A$, since $A \cap H \neq \{h\}$, and therefore $A \cap H' \neq \emptyset$. If $h \notin A$, then there exist $h' \neq h$ such that $h' \in A \cap H$, and therefore $A \cap H' \neq \emptyset$. Thus, we obtain a contradiction.

($\Leftarrow$) If $H' \subsetneqq H$, then there exists $h \in H$ such that $h \notin H'$. But we know that there is a set $A \in S$ such that $A \cap H = \{h\}$, and therefore $A \cap H' = \emptyset$. Thus, H' is not a hitting set of S.

Lemma 6. *Given a finite collection of sets S and a set $H \subseteq \cup S$, the problem of deciding if there exists a minimal hitting set H' of S such that $H \subseteq H'$ is NP*

Proof We are going to reduce our problem to SAT. For each $x \in \cup S$ we introduce a propositional letter x, and we define:

$$f(S, H) = \Big(\bigwedge_{h \in H} \bigvee_{\{A \in S \mid h \in A\}} \bigwedge_{\{a \in A \mid a \neq h\}} \neg a\Big) \wedge \bigwedge_{h \in H} h \wedge \Big(\bigwedge_{\{A \in S \mid A \cap H = \emptyset\}} \bigvee_{a \in A} a\Big).$$

There exists a minimal hitting set H' of S which contains H if and only if $f(H, S)$ is a satisfied formula.
($\Rightarrow$) For every proposition letter x in $f(H, S)$ we define $\sigma(x) = 1$ if and only if $x \in H'$.

1. If $h \in H$, then $h \in H'$, and therefore by lemma 5 we conclude that there exists $A \in S$ such that $A \cap H' = \{h\}$. Thus, for every $a \in A - \{h\}$ we have that $a \notin H'$, and then $\sigma(a) = 0$. We conclude that $\sigma(\bigvee_{\{A \in S \mid h \in A\}} \bigwedge_{\{a \in A \mid a \neq h\}} \neg a) = 1$.
2. $\sigma(\bigwedge_{h \in H} h) = 1$, since $H \subseteq H'$.

3. If $A \in S$ and $A \cap H = \emptyset$, then $A \cap (H' - H) \neq \emptyset$, since H' is a hitting set of S. Thus, there exists $a \in H'$ such that $a \in A$, and therefore $\sigma(a) = 1$. We conclude that $\sigma(\bigvee_{a \in A} a) = 1$.

($\Leftarrow$) Let σ such that $\sigma(f(H, S)) = 1$. We construct $H'' = \{x \mid \sigma(x) = 1\}$. $H \subseteqq H''$, since $\sigma(\bigwedge_{h \in H} h) = 1$. H'' is a hitting set of S. Let us consider $A \in S$. If $A \cap H \neq \emptyset$, then $A \cap H'' \neq \emptyset$. If $A \cap H = \emptyset$, then $\sigma(\bigvee_{a \in A} a) = 1$, and therefore $A \cap (H'' - H) \neq \emptyset$.

H'' is a finite set. Then there exists a minimal hitting set of S such that $H' \subseteqq H''$. We are going to prove that $H \subseteqq H'$. By contradiction, let us suppose that there exists $h \in H$ such that $h \notin H'$. We know that $\sigma(\bigvee_{\{A \in S \mid h \in A\}} \bigwedge_{\{a \in A \mid a \neq h\}} \neg a) = 1$. Then there exists $A \in S$ such that $\sigma(\bigwedge_{\{a \in A \mid a \neq h\}} \neg a) = 1$, and therefore $A \cap H' = \emptyset$, by definition of H' and given that $h \notin H'$. Thus, we conclude a contradiction.

Lemma 7. *Given a finite collection of sets S and a set $H \subseteqq \cup S$, the problem of deciding if there exists a minimal hitting set H' of S such that $H \subseteqq H'$ is NP-hard*

Proof. We are going to reduce SAT(3) to our problem. Given a formula $\varphi = C_1 \wedge \cdots \wedge C_k$, where every C_i is a clause, we define $PL(\varphi)$ as the set of propositional letters mentioned in it. Additionally, for each clause C_i, of the form $p_1 \vee \cdots \vee p_n \vee \neg q_1 \vee \cdots \vee \neg q_m$, we define

$$CH(C_i) = \{p_1\_1, ..., p_n\_1, q_1\_0, ..., q_m\_0\}.$$

After that, we define $f(\varphi) = (S, H)$, where

$$S = \{\{v\_p, p\_0\} \mid p \in PL(\varphi)\} \cup \{\{v\_p, p\_1\} \mid p \in PL(\varphi)\} \cup \{CH(C_i) \mid 1 \leq i \leq k\}$$
$$H = \{v\_p \mid p \in PL(\varphi)\}$$

We are going to prove that φ is consistent if and only if there exists a minimal hitting set H' of S such that $H \subseteqq H'$.

($\Rightarrow$) Let σ that satisfies φ. We define

$$H'' = H \cup \{p\_0 \mid p \in PL(\varphi) \text{ and } \sigma(p) = 0\} \cup \{p\_1 \mid p \in PL(\varphi) \text{ and } \sigma(p) = 1\}$$

H'' is a hitting set of S, and therefore there exists H' minimal hitting set of S such that $H' \subseteqq H''$, since H'' is a finite set. If we suppose that there is $v\_p \in H$ such that $v\_p \notin H'$, then $H' \cap \{v\_p, p\_0\} = \emptyset$ or $H' \cap \{v\_p, p\_1\} = \emptyset$, given that $\sigma(p) = 1$ or $\sigma(p) = 0$. Thus, we conclude a contradiction.

($\Leftarrow$) Let us suppose that there exists H' minimal hitting set of S such that $H \subseteqq H'$. Notice that for every $p \in PL(\varphi)$ we have that $p\_0 \notin H'$ or $p\_1 \notin H'$, since if both elements would be in H', then $H' - \{v\_p\}$ will be a hitting set, a contradiction given that H' is minimal. Thus, we can define a function $\sigma : PL(\varphi) \rightarrow \{0, 1\}$ by means of the rule $\sigma(p) = 1$ if and only if $p\_1 \in H'$. We have that $\sigma(\varphi) = 1$, given that for every clause C_i, $H' \cap CH(C_i) \neq \emptyset$. □