Proofs for PADL'03

Lemma 1. *If $\mathcal{M}$ is a coherent and minimal model of $(\Pi^\star(DB, IC))^{\mathcal{M}}$, then exactly one of the following cases holds:*

- $p(\bar{a}, \mathbf{t_d})$, $p(\bar{a}, \mathbf{t}^\star)$ *and* $p(\bar{a}, \mathbf{t}^{\star\star})$ *belong to* $\mathcal{M}$, *and no other* $p(\bar{a}, v)$, *for* v *an annotation value, belongs to* $\mathcal{M}$.
- $p(\bar{a}, \mathbf{t_d})$, $p(\bar{a}, \mathbf{t}^\star)$, $p(\bar{a}, \mathbf{f_a})$, $p(\bar{a}, \mathbf{f}^\star)$ *and* $p(\bar{a}, \mathbf{f}^{\star\star})$ *belong to* $\mathcal{M}$, *and no other* $p(\bar{a}, v)$, *for* v *an annotation value, belongs to* $\mathcal{M}$.
- $p(\bar{a}, \mathbf{t_a})$, $p(\bar{a}, \mathbf{f}^\star)$, $p(\bar{a}, \mathbf{t}^\star)$ *and* $p(\bar{a}, \mathbf{t}^{\star\star})$ *belong to* $\mathcal{M}$, *and no other* $p(\bar{a}, v)$, *for* v *an annotation value, belongs to* $\mathcal{M}$.
- $p(\bar{a}, \mathbf{f}^\star)$ *and* $p(\bar{a}, \mathbf{f}^{\star\star})$ *belongs to* $\mathcal{M}$, *and no other* $p(\bar{a}, v)$, *for* v *an annotation value, belongs to* $\mathcal{M}$.

Proof. For an atom $p(\bar{a})$ we have two possibilities:

1. $p(\bar{a}, \mathbf{t_d}) \in \mathcal{M}$. Then, $p(\bar{a}, \mathbf{t}^\star) \in \mathcal{M}$. Two cases are possible now: $p(\bar{a}, \mathbf{f_a}) \in \mathcal{M}$ or $p(\bar{a}, \mathbf{f_a}) \notin \mathcal{M}$. For the first one we also have $p(\bar{a}, \mathbf{f}^\star) \in \mathcal{M}$ and $p(\bar{a}, \mathbf{t_a}) \notin \mathcal{M}$ (because $\mathcal{M}$ is coherent). For the second one, $p(\bar{a}, \mathbf{f}^\star) \notin \mathcal{M}$ (since $\mathcal{M}$ is minimal) and $p(\bar{a}, \mathbf{t_a}) \notin \mathcal{M}$ (because $p(\bar{a}, \mathbf{f}^\star) \notin \mathcal{M}$ and $\mathcal{M}$ is minimal). This covers the first two items in the lemma.
2. $p(\bar{a}, \mathbf{t_d}) \notin \mathcal{M}$. Then, $p(\bar{a}, \mathbf{f}^\star) \in \mathcal{M}$. Two cases are possible now: $p(\bar{a}, \mathbf{t_a}) \in \mathcal{M}$ or $p(\bar{a}, \mathbf{t_a}) \notin \mathcal{M}$. For the first one we also have $p(\bar{a}, \mathbf{t}^\star) \in \mathcal{M}$ and $p(\bar{a}, \mathbf{f_a}) \notin \mathcal{M}$ (because $\mathcal{M}$ is coherent). For the second one, $p(\bar{a}, \mathbf{t}^\star) \notin \mathcal{M}$ (since $\mathcal{M}$ is minimal) and $p(\bar{a}, \mathbf{f_a}) \notin \mathcal{M}$ (because $p(\bar{a}, \mathbf{t}^\star) \notin \mathcal{M}$ and $\mathcal{M}$ is minimal). This covers the last two items in the lemma.

□

From two database instances we can define a structure.

Definition 4. *From two database instances* DB_1 *and* DB_2 *over the same domain,* $\mathcal{M}^\star(DB_1, DB_2)$ *is the Herbrand structure* $< D, I_P, I_B >$, *where* D *is the domain of the database* [1] *and* I_P, I_B *are the interpretations for the database predicates (extended with annotation arguments) and the built-ins, respectively.* I_P *is defined as follows:*

- *If* $p(\bar{a}) \in DB_1$ *and* $p(\bar{a}) \in DB_2$, *then* $p(\bar{a}, \mathbf{t_d})$, $p(\bar{a}, \mathbf{t}^\star)$ *and* $p(\bar{a}, \mathbf{t}^{\star\star}) \in I_P$.
- *If* $p(\bar{a}) \in DB_1$ *and* $p(\bar{a}) \notin DB_2$, *then* $p(\bar{a}, \mathbf{t_d})$, $p(\bar{a}, \mathbf{t}^\star)$, $p(\bar{a}, \mathbf{f_a})$, $p(\bar{a}, \mathbf{f}^\star)$ *and* $p(\bar{a}, \mathbf{f}^{\star\star}) \in I_P$.
- *If* $p(\bar{a}) \notin DB_1$ *and* $p(\bar{a}) \notin DB_2$, *then* $p(\bar{a}, \mathbf{f}^\star)$ *and* $p(\bar{a}, \mathbf{f}^{\star\star}) \in I_P$.
- *If* $p(\bar{a}) \notin DB_1$ *and* $p(\bar{a}) \in DB_2$, *then* $p(\bar{a}, \mathbf{f}^\star)$, $p(\bar{a}, \mathbf{t_a})$, $p(\bar{a}, \mathbf{t}^\star)$ *and* $p(\bar{a}, \mathbf{t}^{\star\star}) \in I_P$.

The interpretation I_B *is defined as expected: if* q *is a built-in, then* $q(\bar{a}) \in I_B$ *iff* $q(\bar{a})$ *is true in classical logic, and* $q(\bar{a}) \notin I_B$ *iff* $q(\bar{a})$ *is false.* □

From an interpretation model we can obtain a database instance.

Definition 5. *Let* $\mathcal{M}$ *be a coherent stable model of program* $\Pi^\star(DB, IC)$. *The database associated to* $\mathcal{M}$ *is* $DB_{\mathcal{M}} = \{p(\bar{a}) \mid p(\bar{a}, \mathbf{t}^{\star\star}) \in \mathcal{M}\}$. □

[1] Strictly speaking, the domain D now also contains the annotations values.

The next lemma shows that if $\mathcal{M}$ is a coherent and minimal model of the program $(\Pi^\star(DB,IC))^{\mathcal{M}}$, such that $\mathcal{M}$ represents a finite database instance, then that instance satisfies the constraints.

Lemma 2. *Let us suppose $\mathcal{M}$ is a coherent and minimal model of the program $(\Pi^\star(DB,IC))^{\mathcal{M}}$ and $DB_{\mathcal{M}}$ is finite, then $DB_{\mathcal{M}} \models_\Sigma IC$.*

Proof. We want to show $DB_{\mathcal{M}} \models_\Sigma \bigvee_{i=1}^{n} \neg p_i(\bar{x_i}) \vee \bigvee_{j=1}^{m} q_j(\bar{y_j}) \vee \varphi$, for every constraint in IC. Since $\mathcal{M}$ is a model of $(\Pi^\star(DB,IC))^{\mathcal{M}}$, we have that $\mathcal{M} \models \bigvee_{i=1}^{n} p_i(\bar{x_i},\mathbf{f_a}) \vee \bigvee_{j=1}^{m} q_j(\bar{y_j},\mathbf{t_a}) \leftarrow \bigwedge_{i=1}^{n} p_i(\bar{x_i},\mathbf{t}^\star) \wedge \bigwedge_{j=1}^{m} q_j(\bar{y_j},\mathbf{f}^\star) \wedge \bar{\varphi}$. Then, at least one of the following cases is satisfied:

- $\mathcal{M} \models p_i(\bar{a},\mathbf{f_a})$. Then, $\mathcal{M} \models p_i(\bar{a},\mathbf{f}^{\star\star})$ and $p(\bar{a}) \notin DB_{\mathcal{M}}$ (by lemma 1). Hence, $DB_{\mathcal{M}} \models_\Sigma \neg p_i(\bar{a})$. Since the analysis was done for an arbitrary value $\bar{a}$, $DB_{\mathcal{M}} \models_\Sigma \bigvee_{i=1}^{n} \neg p_i(\bar{x_i}) \vee \bigvee_{j=1}^{m} q_j(\bar{y_j}) \vee \varphi$ holds.
- $\mathcal{M} \models q_j(\bar{a},\mathbf{t_a})$. It is symmetrical to the previous one.
- It is not true that $\mathcal{M} \models \bar{\varphi}$. Then $\mathcal{M} \models \varphi$. Hence, φ is true, and $DB_{\mathcal{M}} \models_\Sigma \bigvee_{i=1}^{n} \neg p_i(\bar{x_i}) \vee \bigvee_{j=1}^{m} q_j(\bar{y_j}) \vee \varphi$ holds.
- $\mathcal{M} \not\models p_i(\bar{a},\mathbf{t}^\star)$. Given the model is coherent and minimal, just the last item in lemma 1 holds. This means $\mathcal{M} \models p_i(\bar{a},\mathbf{f}^{\star\star})$, $p_i(\bar{a}) \notin DB_{\mathcal{M}}$ and $DB_{\mathcal{M}} \models_\Sigma \neg p_i(\bar{a})$. Since the analysis was done for an arbitrary value $\bar{a}$, $DB_{\mathcal{M}} \models_\Sigma \bigvee_{i=1}^{n} \neg p_i(\bar{x_i}) \vee \bigvee_{j=1}^{m} q_j(\bar{y_j}) \vee \varphi$ holds.
- $\mathcal{M} \not\models q_j(\bar{a},\mathbf{f}^\star)$. Given the model is coherent and minimal, just the first item in lemma 1 holds. Then, $\mathcal{M} \models q_j(\bar{a},\mathbf{t}^{\star\star})$, $q_j(\bar{a}) \in DB_{\mathcal{M}}$ and $DB_{\mathcal{M}} \models_\Sigma q_j(\bar{a})$. Since the analysis was done for an arbitrary value $\bar{a}$, $DB_{\mathcal{M}} \models_\Sigma \bigvee_{i=1}^{n} \neg p_i(\bar{x_i}) \vee \bigvee_{j=1}^{m} q_j(\bar{y_j}) \vee \varphi$ holds.

□

Lemma 3. *If $DB' \models_\Sigma IC$, then $\mathcal{M}^\star(DB,DB')$ is a coherent model of the program $(\Pi^\star(DB,IC))^{\mathcal{M}^\star(DB,DB')}$.*

Proof. We will first show $\mathcal{M}^\star(DB,DB')$ is a model of $(\Pi^\star(DB,IC))^{\mathcal{M}^\star(DB,DB')}$. Since $DB' \models_\Sigma \bigvee_{i=1}^{n} \neg p_i(\bar{a_i}) \vee \bigvee_{j=1}^{m} q_j(\bar{b_j}) \vee \varphi$, we have three possibilities to analyze with respect to the satisfaction of this clause. The first possibility is $DB' \models_\Sigma \neg p_i(\bar{a})$. Then, two cases arise

- $p_i(\bar{a}) \in DB$. Then, $p_i(\bar{a},\mathbf{f}^\star)$, $p_i(\bar{a},\mathbf{t_d})$, $p_i(\bar{a},\mathbf{f_a})$, $p_i(\bar{a},\mathbf{t}^\star)$ and $p_i(\bar{a},\mathbf{f}^{\star\star})$ belong to $\mathcal{M}^\star(DB,\ DB')$, and the program $(\Pi^\star(DB,IC))^{\mathcal{M}^\star(DB,DB')}$ contains the following clauses: $p_i(\bar{a},\mathbf{t_d}) \leftarrow$, $p_i(\bar{a},\mathbf{t}^\star) \leftarrow p_i(\bar{a},\mathbf{t_d})$, $p_i(\bar{a},\mathbf{t}^\star) \leftarrow p_i(\bar{a},\mathbf{t_a})$, $p_i(\bar{a},\mathbf{f}^\star) \leftarrow p_i(\bar{a},\mathbf{f_a})$ and $p_i(\bar{a},\mathbf{f}^{\star\star}) \leftarrow p_i(\bar{a},\mathbf{f_a})$. Then, all these formulas are satisfied by $\mathcal{M}^\star(DB,DB')$. The program also contains the clause $\bigvee_{i=1}^{n} p_i(\bar{a},\mathbf{f_a}) \vee \bigvee_{j=1}^{m} q_j(\bar{a},\mathbf{t_a}) \leftarrow \bigwedge_{i=1}^{n} p_i(\bar{a},\mathbf{t}^\star) \wedge \bigwedge_{j=1}^{m} q_j(\bar{a},\mathbf{f}^\star) \wedge \bar{\varphi}$, which is satisfied since $p_i(\bar{a},\mathbf{f_a})$ belongs to $\mathcal{M}^\star(DB,DB')$.
- $p_i(\bar{a}) \notin DB$. Then, $p_i(\bar{a},\mathbf{f}^\star)$ and $p_i(\bar{a},\mathbf{f}^{\star\star}) \in \mathcal{M}^\star(DB,DB')$, and $p_i(\bar{a},\mathbf{f}^\star)$, $p_i(\bar{a},\mathbf{t}^\star) \leftarrow p_i(\bar{a},\mathbf{t_d})$, $p_i(\bar{a},\mathbf{t}^\star) \leftarrow p_i(\bar{a},\mathbf{t_a})$, $p_i(\bar{a},\mathbf{f}^\star) \leftarrow p_i(\bar{a},\mathbf{f_a})$ and $p_i(\bar{a},\mathbf{f}^{\star\star}) \leftarrow$ are in $(\Pi^\star(DB,IC))^{\mathcal{M}^\star(DB,DB')}$. All these are satisfied by the model considered. Also in the program is the clause $\bigvee_{i=1}^{n} p_i(\bar{a},\mathbf{f_a}) \vee \bigvee_{j=1}^{m} q_j(\bar{a},\mathbf{t_a}) \leftarrow \bigwedge_{i=1}^{n} p_i(\bar{a},\mathbf{t}^\star) \wedge \bigwedge_{j=1}^{m} q_j(\bar{a},\mathbf{f}^\star) \wedge \bar{\varphi}$, and it is trivially satisfied since $p_i(\bar{a},\mathbf{t}^\star) \notin \mathcal{M}^\star(DB,DB')$.

The second possibility is $DB' \models_\Sigma q_j(\bar{a})$. The following cases arise:

- $q_j(\bar{a}) \in DB$. Then, $\mathcal{M}^\star(DB, DB')$ contains $q_j(\bar{a}, \mathbf{t_d})$, $q_j(\bar{a}, \mathbf{t}^\star)$ and $q_j(\bar{a}, \mathbf{t}^{\star\star})$, and program $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$ contains the formulas $q_j(\bar{a}, \mathbf{t_d}) \leftarrow$, $q_j(\bar{a}, \mathbf{t}^\star) \leftarrow q_j(\bar{a}, \mathbf{t_d})$, $q_j(\bar{a}, \mathbf{t}^\star) \leftarrow q_j(\bar{a}, \mathbf{t_a})$, $q_j(\bar{a}, \mathbf{f}^\star) \leftarrow q_j(\bar{a}, \mathbf{f_a})$ and $q_j(\bar{a}, \mathbf{t}^{\star\star}) \leftarrow q_j(\bar{a}, \mathbf{t_d})$. The structure $\mathcal{M}^\star(DB, DB')$ satisfies all these clauses. The clause $\bigvee_{i=1}^n p_i(\bar{a}, \mathbf{f_a}) \vee \bigvee_{j=1}^m q_j(\bar{a}, \mathbf{t_a}) \leftarrow \bigwedge_{i=1}^n p_i(\bar{a}, \mathbf{t}^\star) \wedge \bigwedge_{j=1}^m q_j(\bar{a}, \mathbf{f}^\star) \wedge \bar{\varphi}$ is also in the program, and is satisfied trivially since it holds that $q_j(\bar{a}, \mathbf{f}^\star)$ does not belong to $\mathcal{M}^\star(DB, DB')$.
- $q_j(\bar{a}) \notin DB$. Then, $q_j(\bar{a}, \mathbf{f}^\star)$, $q_j(\bar{a}, \mathbf{t_a})$, $q_j(\bar{a}, \mathbf{t}^\star)$ and $q_j(\bar{a}, \mathbf{t}^{\star\star})$ are in the structure $\mathcal{M}^\star(DB, DB')$, and the following formulas are in the program $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$: $q_j(\bar{a}, \mathbf{f}^\star) \leftarrow$, $q_j(\bar{a}, \mathbf{t}^\star) \leftarrow q_j(\bar{a}, \mathbf{t_d})$, $q_j(\bar{a}, \mathbf{t}^\star) \leftarrow q_j(\bar{a}, \mathbf{t_a})$, $q_j(\bar{a}, \mathbf{f}^\star) \leftarrow q_j(\bar{a}, \mathbf{f_a})$ and $q_j(\bar{a}, \mathbf{t}^{\star\star}) \leftarrow q_j(\bar{a}, \mathbf{t_a})$. These are satisfied by $\mathcal{M}^\star(DB, DB')$. Also in the program is the clause $\bigvee_{i=1}^n p_i(\bar{a}, \mathbf{f_a}) \vee \bigvee_{j=1}^m q_j(\bar{a}, \mathbf{t_a}) \leftarrow \bigwedge_{i=1}^n p_i(\bar{a}, \mathbf{t}^\star) \wedge \bigwedge_{j=1}^m q_j(\bar{a}, \mathbf{f}^\star) \wedge \bar{\varphi}$, which is satisfied since $q_j(\bar{a}, \mathbf{t_a})$ belongs to $\mathcal{M}^\star(DB, DB')$.

The third possibility is $DB' \models_\Sigma \varphi$. Then, φ is true. The clause $\bigvee_{i=1}^n p_i(\bar{a}, \mathbf{f_a}) \vee \bigvee_{j=1}^m q_j(\bar{a}, \mathbf{t_a}) \leftarrow \bigwedge_{i=1}^n p_i(\bar{a}, \mathbf{t}^\star) \wedge \bigwedge_{j=1}^m q_j(\bar{a}, \mathbf{f}^\star) \wedge \bar{\varphi}$ is in $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$, and it is satisfied since $\mathcal{M}^\star(DB, DB') \not\models \bar{\varphi}$.

As the analysis was done for an arbitrary value $\bar{a}$, it holds that the Herbrand structure $\mathcal{M}^\star(DB, DB')$ is a model of $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$. Moreover, it is also coherent, since $\mathcal{M}^\star(DB, DB')$ was defined for not containig both $p(\bar{a}, \mathbf{t_a})$ and $p(\bar{a}, \mathbf{f_a})$. □

The following theorem establishes the one-to-one correspondence between coherent stable models of the program and the repairs of the original instance.

Theorem 1. *If $\mathcal{M}$ is a coherent stable model of $\Pi^\star(DB, IC)$, and $DB_\mathcal{M}$ is finite, then $DB_\mathcal{M}$ is a repair of DB with respect to IC. Furthermore, the repairs obtained in this way are all the repairs of DB.* □

Proof. From propositions 1 and 2 below. □

Proposition 1. *If $\mathcal{M}$ is a coherent and minimal model of $(\Pi^\star(DB, IC))^\mathcal{M}$, and $DB_\mathcal{M}$ is finite, then $DB_\mathcal{M}$ is a repair of DB with respect to IC.*

Proof. From Lemma 2, we have $DB_\mathcal{M} \models_\Sigma IC$. We just have to show minimality. Let us suppose there is a database instance DB', such that $DB' \models_\Sigma IC$ and $\Delta(DB, DB') \subsetneq \Delta(DB, DB_\mathcal{M})$. Then, by lemma 3, $\mathcal{M}^\star(DB, DB')$ is a coherent model of $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$. We will first show $\mathcal{M}^\star(DB, DB') \subseteq \mathcal{M}$ and that $\mathcal{M}^\star(DB, DB')$ is a model of $(\Pi^\star(DB, IC))^\mathcal{M}$. Notice that since $\mathcal{M}$ is a minimal model of $(\Pi^\star(DB, IC))^\mathcal{M}$, this program contains the clause $p(\bar{a}, \mathbf{f}^\star) \leftarrow$ for every $p(\bar{a}) \notin DB$. The rest of the program must look exactly like $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$, except probably for the interpretation clauses. By definition 4, for an arbitrary atom $p(\bar{a})$ in a model $\mathcal{M}^\star(DB, DB')$, we just have to analyze four cases:

1. Let us suppose just $p(\bar{a}, \mathbf{t^{\star\star}})$, $p(\bar{a}, \mathbf{t^\star})$ and $p(\bar{a}, \mathbf{t_d})$ belong to $\mathcal{M}^\star(DB, DB')$. Then $p(\bar{a}) \in DB$ and $p(\bar{a}) \in DB'$. Since $p(\bar{a}) \notin \Delta(DB, DB')$, we have two possibilities. The first one saying $p(\bar{a}) \notin \Delta(DB, DB_\mathcal{M})$. Then, $p(\bar{a}, \mathbf{t^\star})$, $p(\bar{a}, \mathbf{t_d})$ and $p(\bar{a}, \mathbf{t^{\star\star}})$ also belong to $\mathcal{M}$ and $\mathcal{M}^\star(DB, DB')$ is clearly a model of $(\Pi^\star(DB, IC))^\mathcal{M}$. The second one saying $p(\bar{a}) \in \Delta(DB, DB_\mathcal{M})$. Again, $p(\bar{a}, \mathbf{t^\star})$, $p(\bar{a}, \mathbf{t_d})$ and $p(\bar{a}, \mathbf{t^{\star\star}})$ belong to $\mathcal{M}$ and $\mathcal{M}^\star(DB, DB')$ is clearly a model of the program $(\Pi^\star(DB, IC))^\mathcal{M}$.
2. Let us suppose now, just $p(\bar{a}, \mathbf{f^\star})$ and $p(\bar{a}, \mathbf{f^{\star\star}})$ belong to $\mathcal{M}^\star(DB, DB')$. Again we have two possibilities. The first one says that $p(\bar{a}) \notin \Delta(DB, DB_\mathcal{M})$. Then, $p(\bar{a}, \mathbf{f^\star})$ and $p(\bar{a}, \mathbf{f^{\star\star}})$ also belong to $\mathcal{M}$. The program $(\Pi^\star(DB, IC))^\mathcal{M}$ contains (among others) the clause $p(\bar{a}, \mathbf{f^\star}) \leftarrow$, that is satisfied by the program $\mathcal{M}^\star(DB, DB')$. The rest of the clauses concerning $p(\bar{a})$ are satisfied because are also present in $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$. The second one says that $p(\bar{a}) \in \Delta(DB, DB^\Pi_{i(\mathcal{M})})$. Again, $p(\bar{a}, \mathbf{f^\star})$ and $p(\bar{a}, \mathbf{f^{\star\star}})$ belong to $\mathcal{M}$. The program $(\Pi^\star(DB, IC))^\mathcal{M}$ contains (among others) the clause $p(\bar{a}, \mathbf{f^\star}) \leftarrow$, that is satisfied by $\mathcal{M}^\star(DB, DB')$. The rest of the clauses concerning $p(\bar{a})$ are satisfied because are also present in $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$.
3. Let us suppose just $p(\bar{a}, \mathbf{t^\star})$, $p(\bar{a}, \mathbf{t_d})$, $p(\bar{a}, \mathbf{f_a})$, $p(\bar{a}, \mathbf{f^\star})$ and $p(\bar{a}, \mathbf{f^{\star\star}})$ belong to the model $\mathcal{M}^\star(DB, DB')$. Then $p(\bar{a}) \in DB$ and $p(\bar{a}) \notin DB'$. Hence, $p(\bar{a}) \in \Delta(DB, DB')$, and due to our assumption $p(\bar{a}) \in \Delta(DB, DB_\mathcal{M})$. Therefore, $p(\bar{a}, \mathbf{t^\star})$, $p(\bar{a}, \mathbf{t_d})$, $p(\bar{a}, \mathbf{f_a})$, $p(\bar{a}, \mathbf{f^\star})$ and $p(\bar{a}, \mathbf{f^{\star\star}})$ belong to $\mathcal{M}$. Moreover, $\mathcal{M}^\star(DB, DB')$ is clearly a model of the program $(\Pi^\star(DB, IC))^\mathcal{M}$.
4. Finally, we will suppose just $p(\bar{a}, \mathbf{f^\star})$, $p(\bar{a}, \mathbf{t_a})$, $p(\bar{a}, \mathbf{t^\star})$ and $p(\bar{a}, \mathbf{t^{\star\star}})$ belong to the model $\mathcal{M}^\star(DB, DB')$. Then, $p(\bar{a}) \notin DB$ and $p(\bar{a}) \in DB'$. Hence, $p(\bar{a}) \in \Delta(DB, DB')$, and due to our assumption $p(\bar{a}) \in \Delta(DB, DB_\mathcal{M})$. Therefore, $p(\bar{a}, \mathbf{f^\star})$, $p(\bar{a}, \mathbf{t^\star})$, $p(\bar{a}, \mathbf{t_a})$ and $p(\bar{a}, \mathbf{t^{\star\star}})$ belong to $\mathcal{M}$. The program $(\Pi^\star(DB, IC))^\mathcal{M}$ contains (among others) the clause $p(\bar{a}, \mathbf{f^\star}) \leftarrow$, that is satisfied by $\mathcal{M}^\star(DB, DB')$. The rest of the clauses concerning $p(\bar{a})$ are satisfied because are also present in $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$.

We will now show $\mathcal{M}^\star(DB, DB') \subsetneqq \mathcal{M}$. We have assumed there is an element of $\Delta(DB, DB^\Pi_{i(\mathcal{M})})$ that is not an element of $\Delta(DB, DB')$. Thus, for some element $p(\bar{a})$, either $p(\bar{a}) \in DB$, $p(\bar{a}) \in DB'$ and $p(\bar{a}) \notin DB^\Pi_{i(\mathcal{M})}$, or $p(\bar{a}) \notin DB$, $p(\bar{a}) \notin DB'$ and $p(\bar{a}) \in DB^\Pi_{i(\mathcal{M})}$. For the first one we have $\mathcal{M}^\star(DB, DB')$ satisfies $p(\bar{a}, \mathbf{t_d})$ and $p(\bar{a}, \mathbf{t^\star})$, and $\mathcal{M}$ satisfies $p(\bar{a}, \mathbf{t_d})$ and $p(\bar{a}, \mathbf{t^\star})$, but also satisfies $p(\bar{a}, \mathbf{f_a})$ and $p(\bar{a}, \mathbf{f^\star})$. In the second one, $\mathcal{M}^\star(DB, DB')$ satisfies $p(\bar{a}, \mathbf{f^\star})$ and $\mathcal{M}$ satisfies $p(\bar{a}, \mathbf{f^\star})$, but also $p(\bar{a}, \mathbf{t_a})$ and $p(\bar{a}, \mathbf{t^\star})$.

Then, $\mathcal{M}$ is not a minimal model; a contradiction. □

Proposition 2. *If DB' is a repair of DB with respect to IC, then $\mathcal{M}^\star(DB, DB')$ is a coherent and minimal model of $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$.*

Proof. By Lemma 3 we have $\mathcal{M}^\star(DB, DB')$ is a coherent model of the program $\Pi^\star(DB, IC)^{\mathcal{M}^\star(DB,DB')}$. We just have to show it is minimal. Let us suppose first there exists a model $\mathcal{M}$ of $(\Pi^\star(DB, IC))^{\mathcal{M}^\star(DB,DB')}$ such that it is the case that $\mathcal{M} \subsetneqq \mathcal{M}^\star(DB, DB')$ (it is also coherent since it is contained in $\mathcal{M}^\star(DB, DB')$).

It can be assumed, without loss of generality, that $\mathcal{M}$ is minimal (if it is not minimal, we can always generate from it a minimal model $\mathcal{M}'$, such that $\mathcal{M}' \subsetneqq \mathcal{M}$, by deleting its non-supported atoms). We will prove $\Delta(DB, DB_{\mathcal{M}}) \subsetneqq \Delta(DB, DB')$.

At first, we will prove $\Delta(DB, DB_{\mathcal{M}}) \subseteq \Delta(DB, DB')$. Let us suppose $p(\bar{a}) \in \Delta(DB, DB_{\mathcal{M}})$. Then, either $p(\bar{a}) \in DB$ and $p(\bar{a}) \notin DB_{\mathcal{M}}$ or $p(\bar{a}) \notin DB$ and $p(\bar{a}) \in DB_{\mathcal{M}}$. In the first case, $p(\bar{a}, \mathbf{t_d})$, $p(\bar{a}, \mathbf{t^\star})$, $p(\bar{a}, \mathbf{f_a})$, $p(\bar{a}, \mathbf{f^\star})$ and $p(\bar{a}, \mathbf{f^{\star\star}})$ are in $\mathcal{M}$. By our assumption these are also in $\mathcal{M}^\star(DB, DB')$. Hence, $p(\bar{a}) \in \Delta(DB, DB')$. In the second case, $p(\bar{a}, \mathbf{f^\star})$, $p(\bar{a}, \mathbf{t_a})$, $p(\bar{a}, \mathbf{t^\star})$ and $p(\bar{a}, \mathbf{t^{\star\star}})$ are in $\mathcal{M}$. By our assumption these are also in $\mathcal{M}^\star(DB, DB')$. Hence, $p(\bar{a}) \in \Delta(DB, DB')$.

We will now prove $\Delta(DB, DB_{\mathcal{M}}) \subsetneqq \Delta(DB, DB')$. We know for some fact $p(\bar{a})$ there is an element related to it which is in $\mathcal{M}^\star(DB, DB')$ and which is not in $\mathcal{M}$. One possible case is $p(\bar{a}, \mathbf{f_a})$ and $p(\bar{a}, \mathbf{f^\star})$ are in $\mathcal{M}^\star(DB, DB')$ and not in $\mathcal{M}$. Then, $p(\bar{a}) \in \Delta(DB, DB')$, but $p(\bar{a}) \notin \Delta(DB, DB_{\mathcal{M}}$. The other possible case $p(\bar{a}, \mathbf{t_a})$ and $p(\bar{a}, \mathbf{t^\star})$ are in $\mathcal{M}^\star(DB, DB')$ and not in $\mathcal{M}$. Then, $p(\bar{a}) \in \Delta(DB, DB')$, but $p(\bar{a}) \notin \Delta(DB, DB_{\mathcal{M}})$.

By Lemma 2, we have $DB_{\mathcal{M}} \models_\Sigma IC$. Also, $DB_{\mathcal{M}}$ is finite. This contradicts our fact that DB' is a repair. □