Coverage and Search Algorithms

Chapter 10

Objectives

- To investigate some simple algorithms for covering **the area** in an environment
- To understand how to break down an environment into simple convex pieces
- To understand how to consider **searching** environments with a limited range and limited direction sensor.

What's in Here ?

- Complete Coverage Algorithms
  - Difficulties and Issues
  - Boustrophedon Coverage
  - Other Coverage Ideas
- Search Algorithms
  - Searching and Visibility
  - Guard Placement
  - Traveling Salesman Problem
  - Visibility Search Paths
  - Searching With Limited Range Visibility

Complete Coverage Algorithms

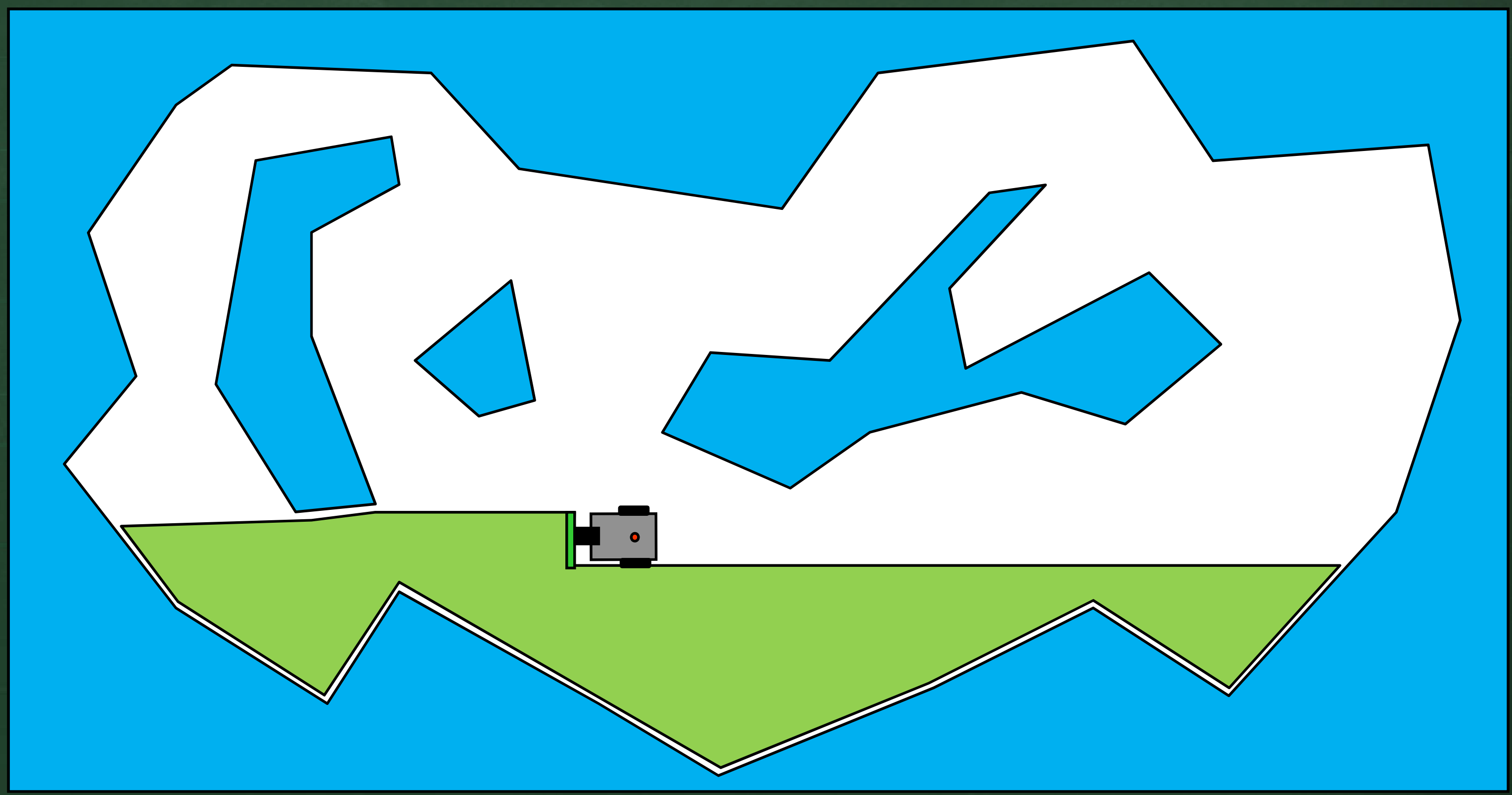
Coverage Algorithms

- A *complete coverage algorithm* produces a path that a robot must travel on in order to "cover" or travel over the entire surface of its environment.

- Applications include:
  – vacuum and sweeping
  – painting
  – searching
  – security patrolling
  – map verification
  – etc…

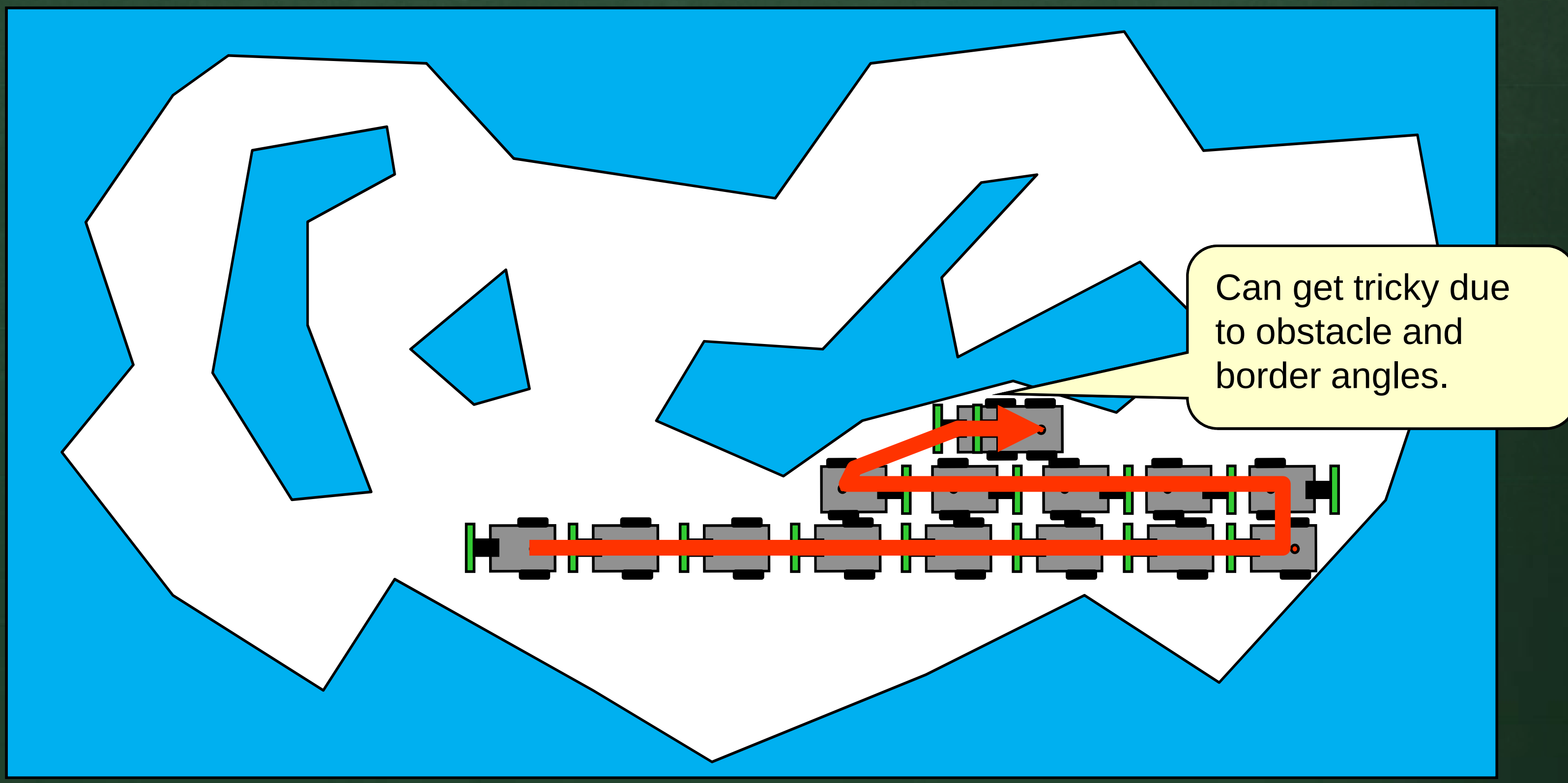
Coverage Algorithms

- How can we determine a **valid** path that the robot can take to cover the whole environment ?



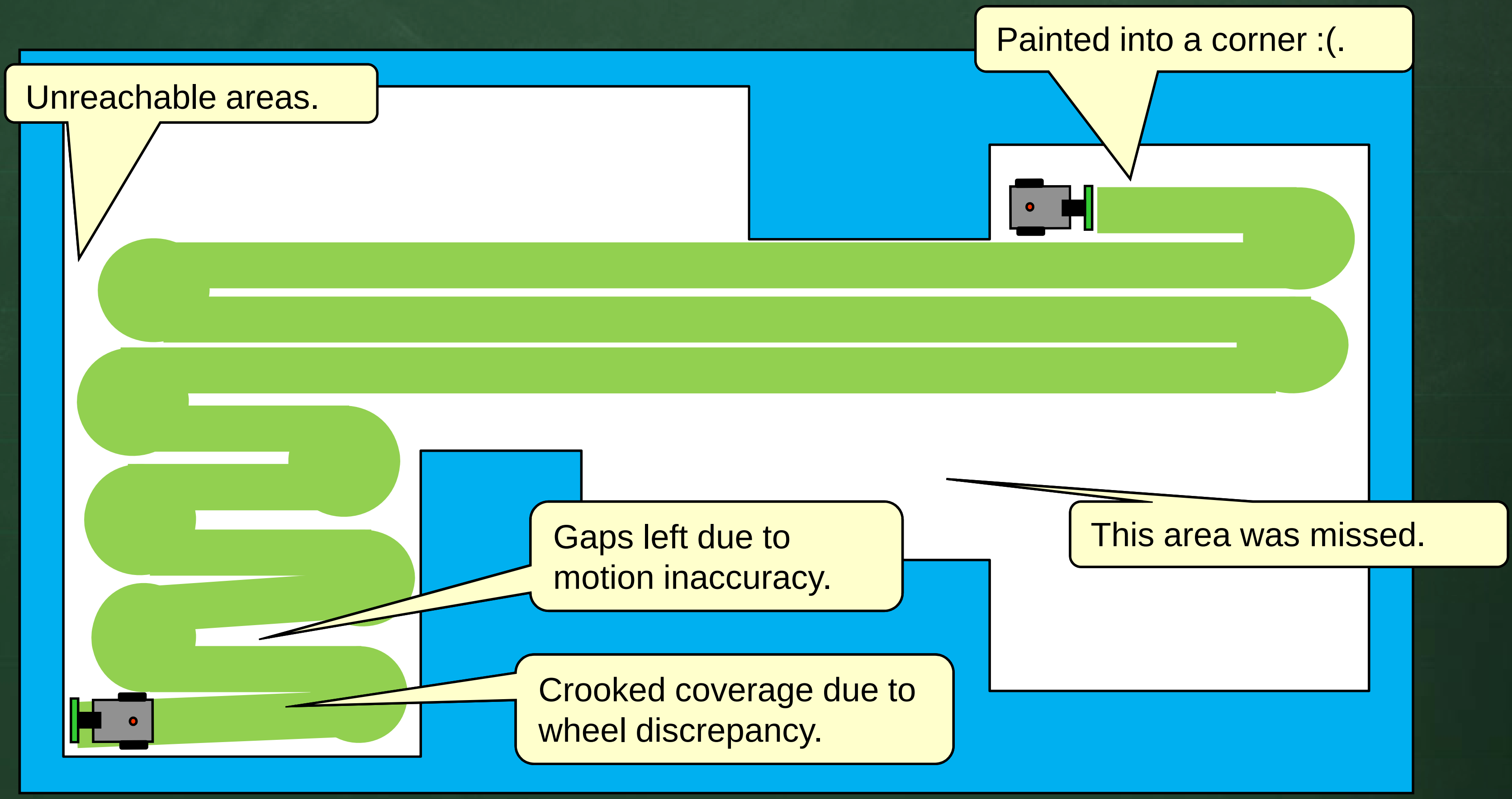
Coverage Algorithms

- One approach is to simply travel in some fixed direction (e.g., North) until an obstacle is encountered, then turn around…cover in strips:



Coverage Algorithms

- Even in rectilinear environments, many problems may arise:

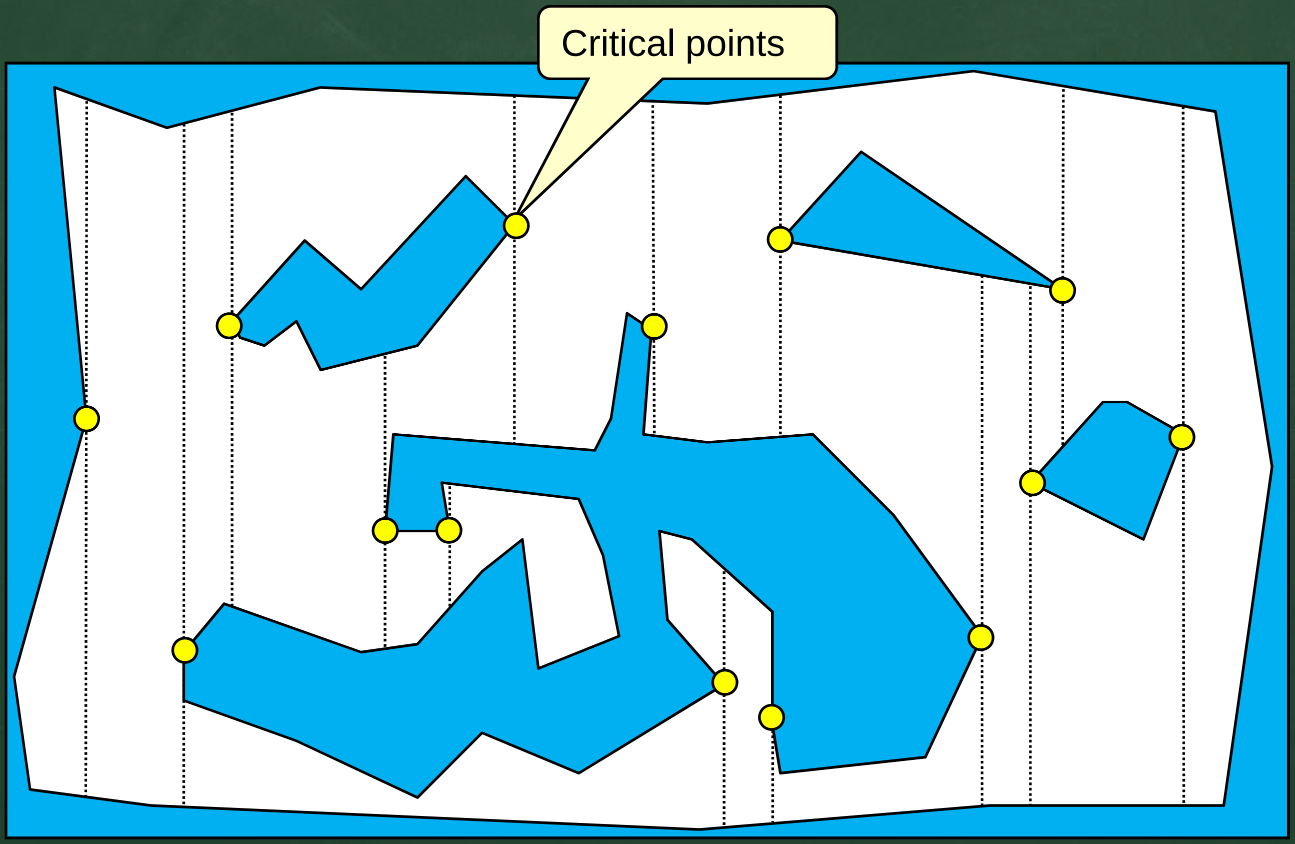


Coverage Algorithms

- Is there any hope ?
  - there will always be some error in terms of coverage.
    - may still miss close to edges and in corners
  - allowing overlapping coverage will help
  - dividing environment into smaller "chunks" will help
- For most applications (not painting the floor) being "close enough" to the obstacles is sufficient.
  - sensors can "pick-up"/detect from a certain distance away.
  - sometimes, a rough coverage is enough.

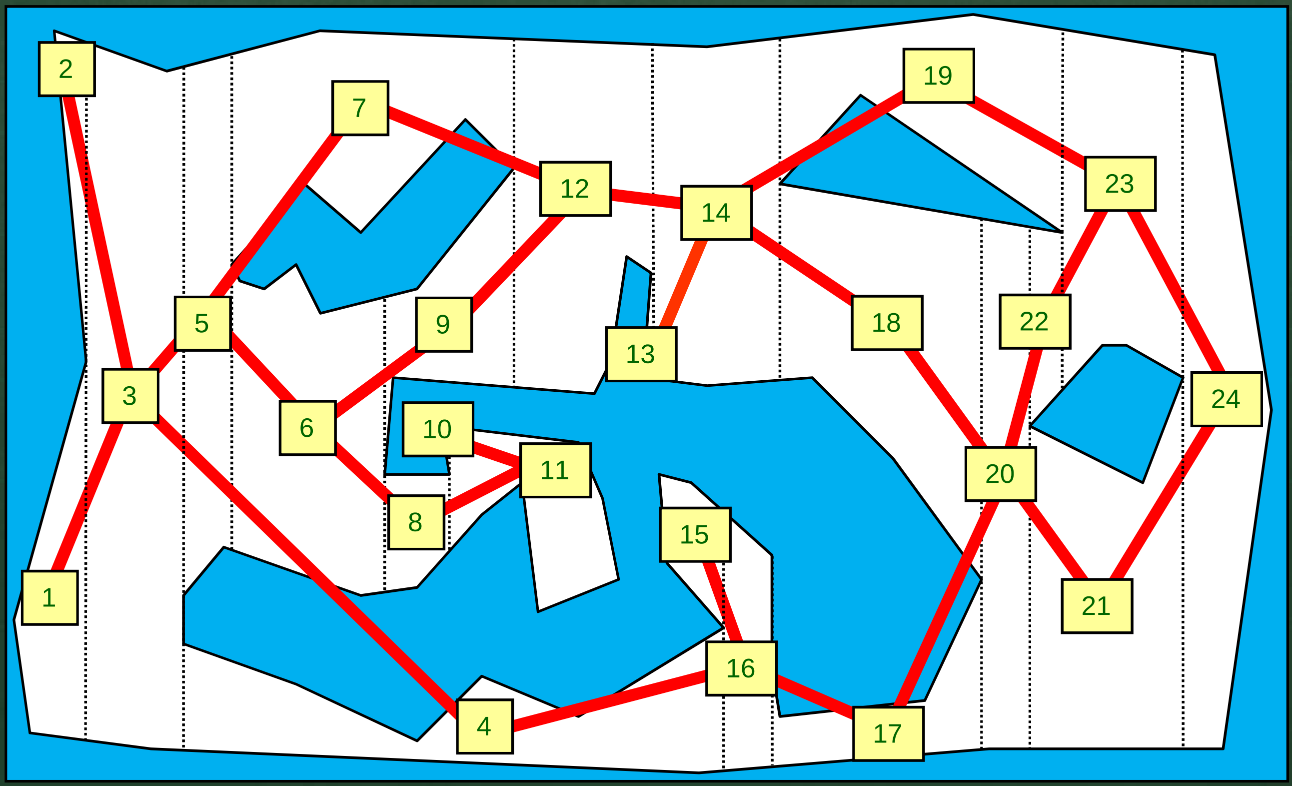
Boustrophedon Coverage

- Recall the Boustrophedon cell decomposition of a polygonal environment:



Boustrophedon Coverage

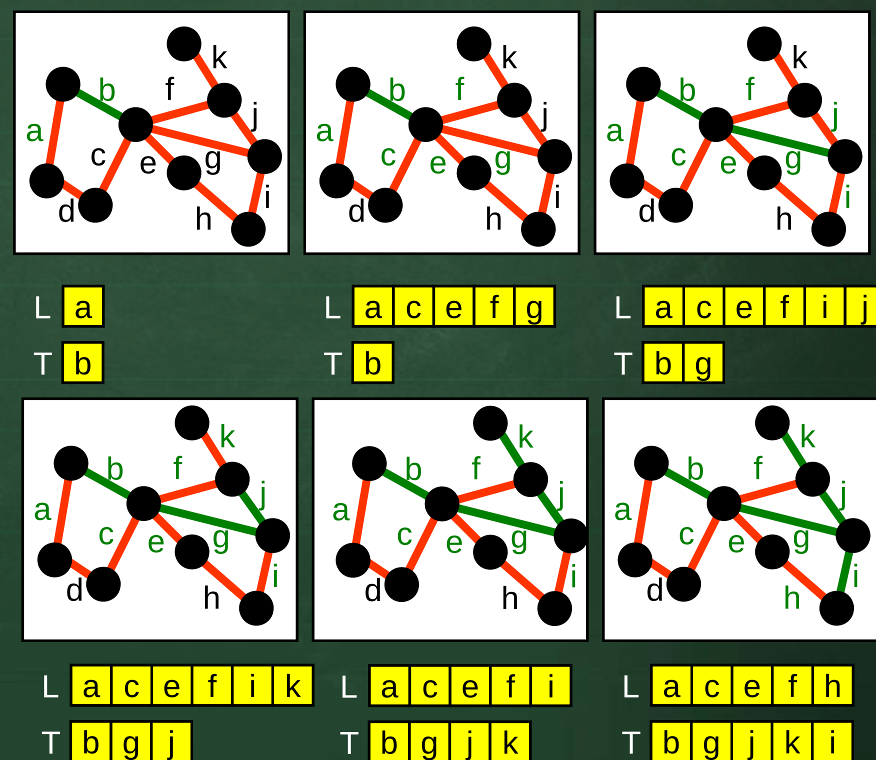
- Now connect adjacent cells to form a graph and consider an arbitrary ordering of the cells:
  - (e.g., from left to right)



Finding a Path

- Perform a *depth-first-search (DFS) on the graph to* determine an exhaustive walk through the cells:

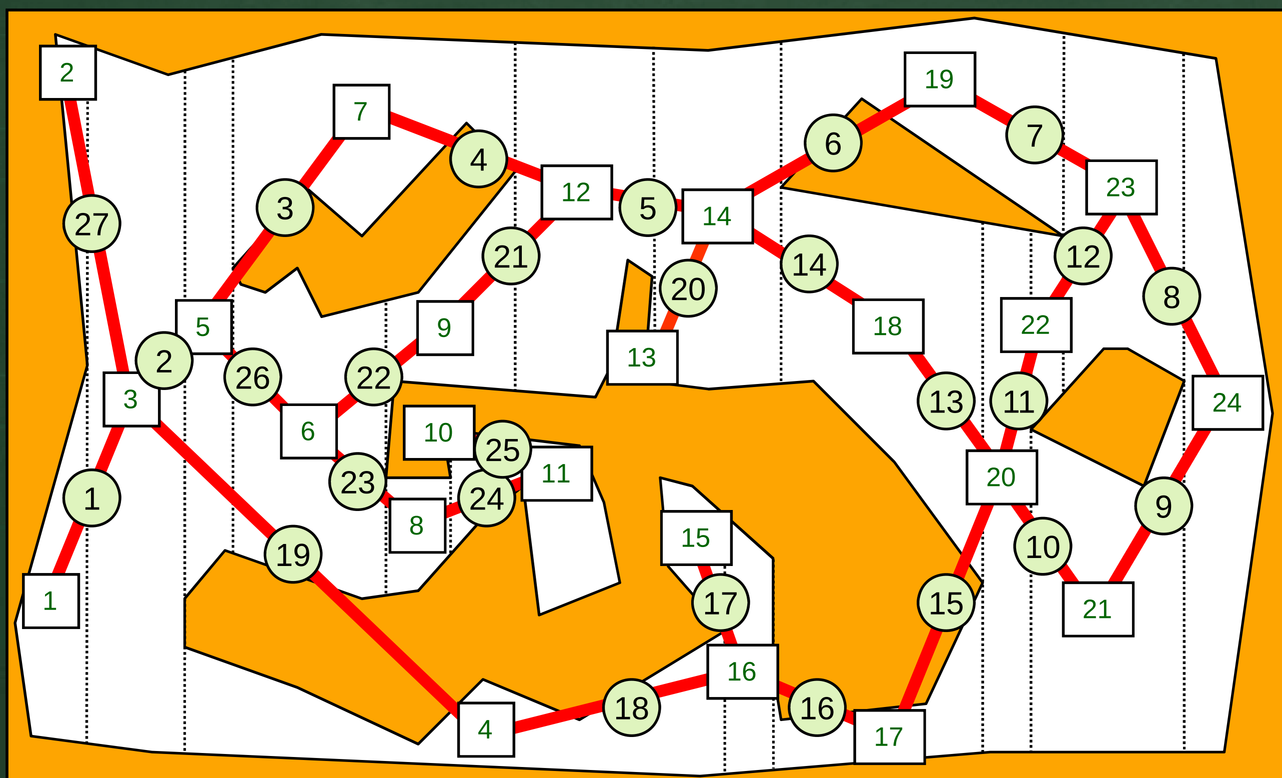
```
dfs(G) {
  list L = empty
  tree T = empty
  choose a starting vertex x
  search(x)
  WHILE (L nonempty) DO
    remove edge (v,w) from end of L
    IF (w not yet visited) THEN
      add (v,w) to T
      search(w)
}

search(vertex v) {
  visit(v)
  FOR (each edge (v,w)) DO
    add edge (v,w) to end of L
}
```



etc…

Finding a Path

- Here is what the DFS ordering may have produced in our example:

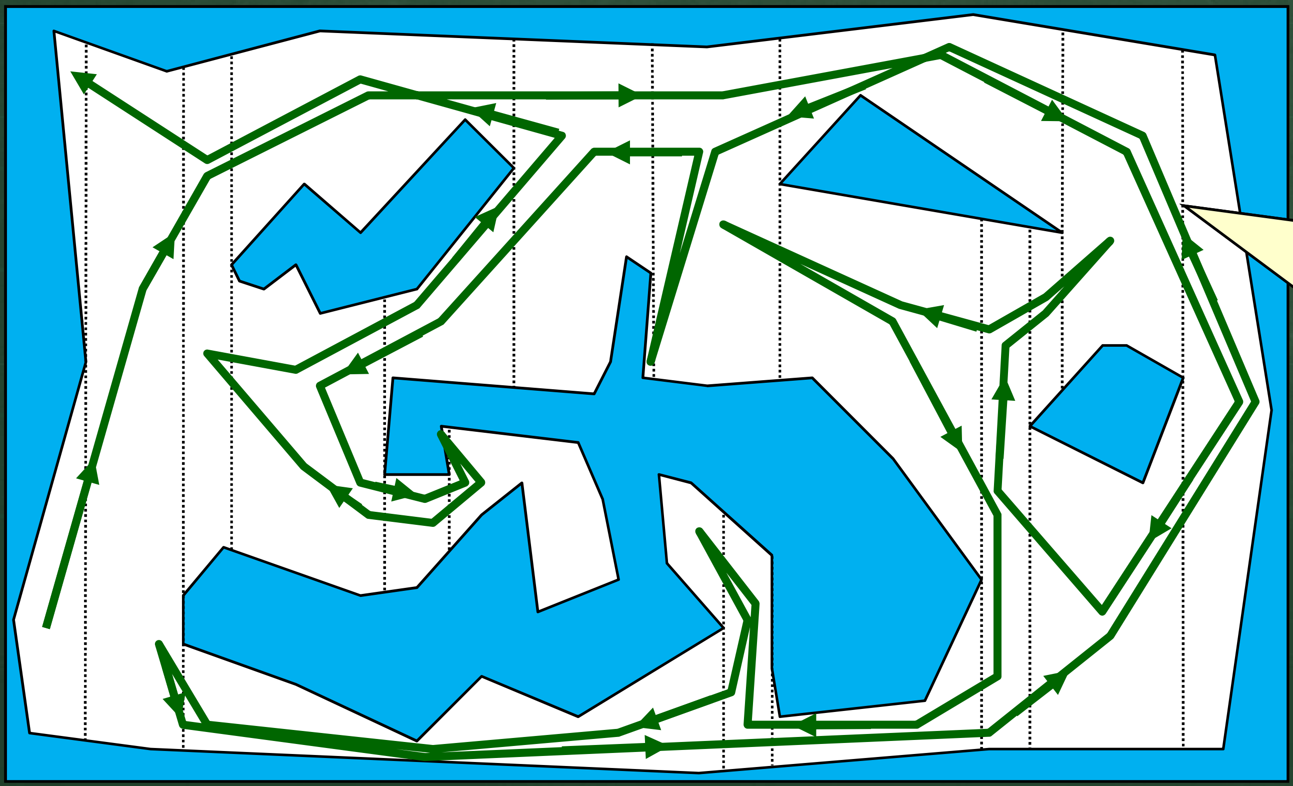


Cells visited in the following order (blue numbers indicate backtracking):

1-3-5-7-12-14-19-23-24-21-20-22-23-22-20-18-14-18-20-17-16-15-16-4-3-4-16-17-20-21-24-23-19-14-13-14-12-9-6-8-11-10-11-8-6-5-6-9-12-7-5-3-2

Coverage Along a Path

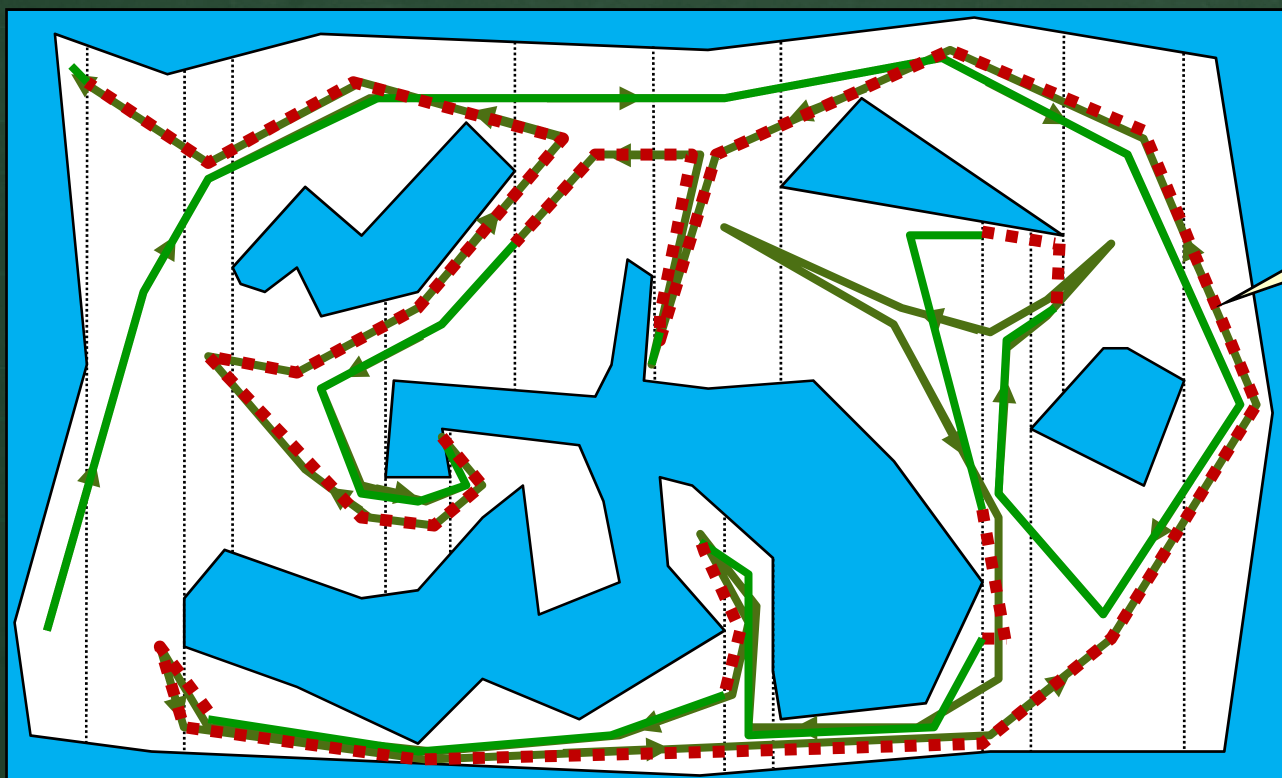
- Once a path is found, the robot visits all of these cells in that order:



Visitation order may not be the most efficient. There are other ways to traverse besides the DFS.

Coverage Along a Path

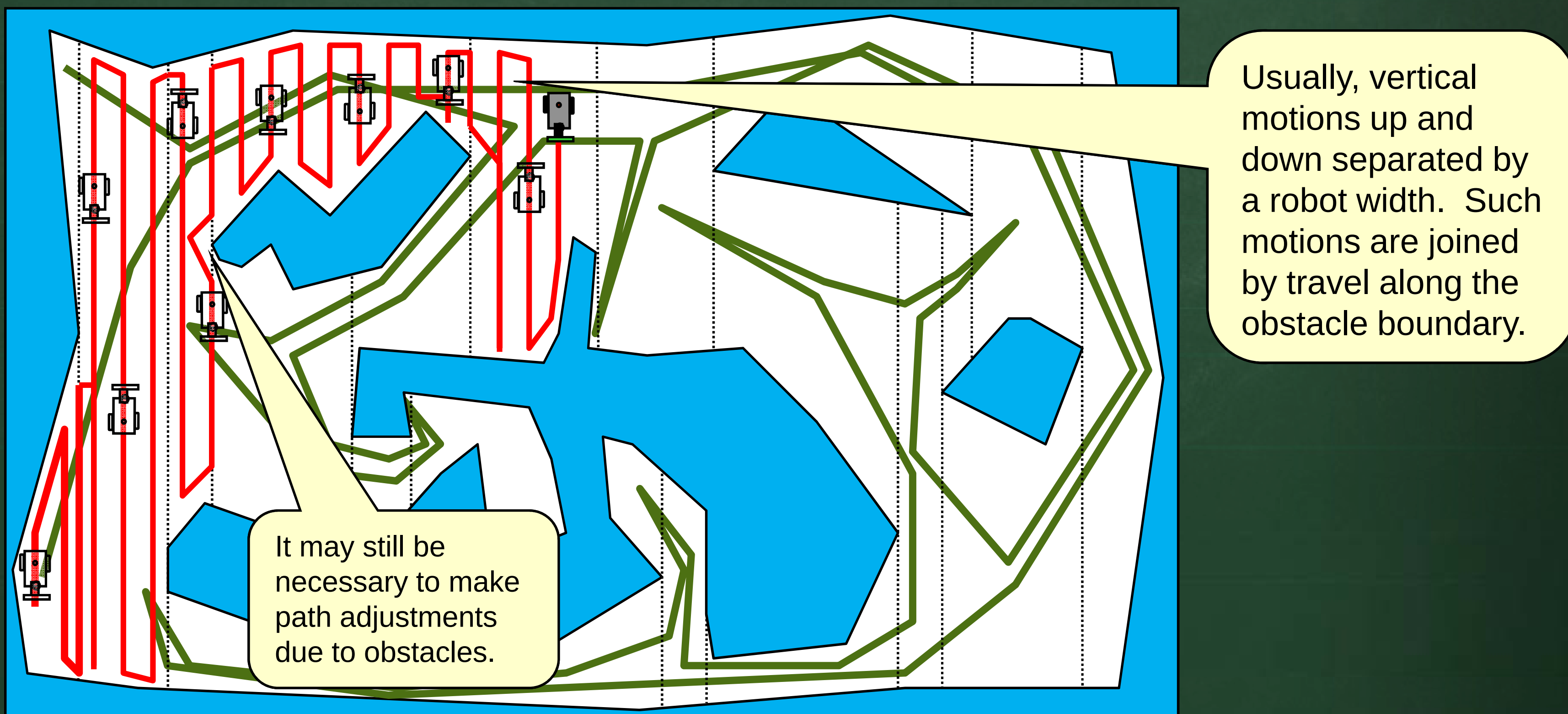
- When coming back to cells already visited, it is not necessary to re-cover the cell again:



Need to compute path back to previous cells.

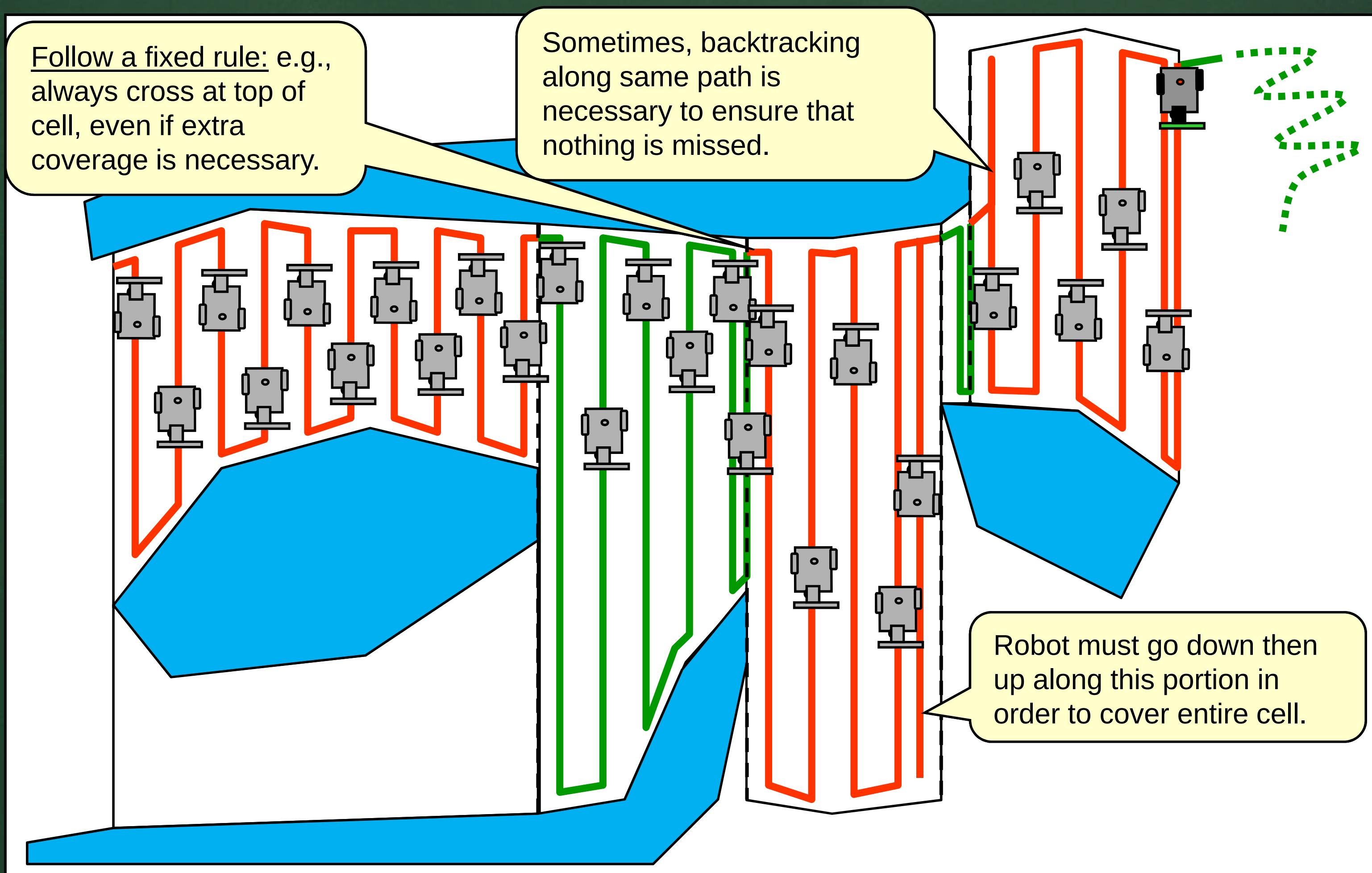
Coverage Along a Path

- When entering a cell, the robot performs some simple maneuvers to cover the cell's entire area:



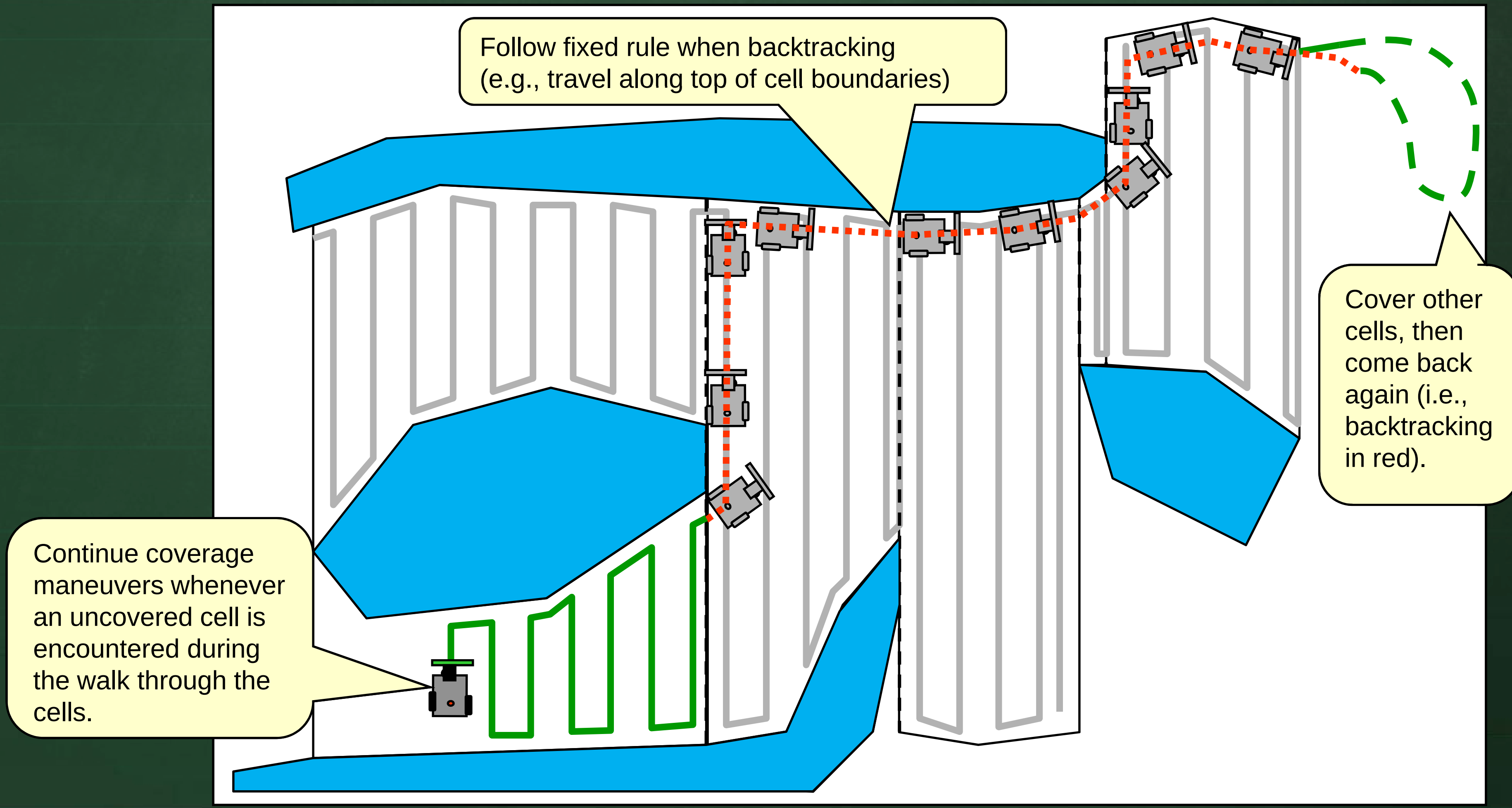
Coverage Along a Path

- Must take care when crossing one cell to another:



Coverage Along a Path

- When backtracking, follow along cell boundaries:



Other Coverage Ideas

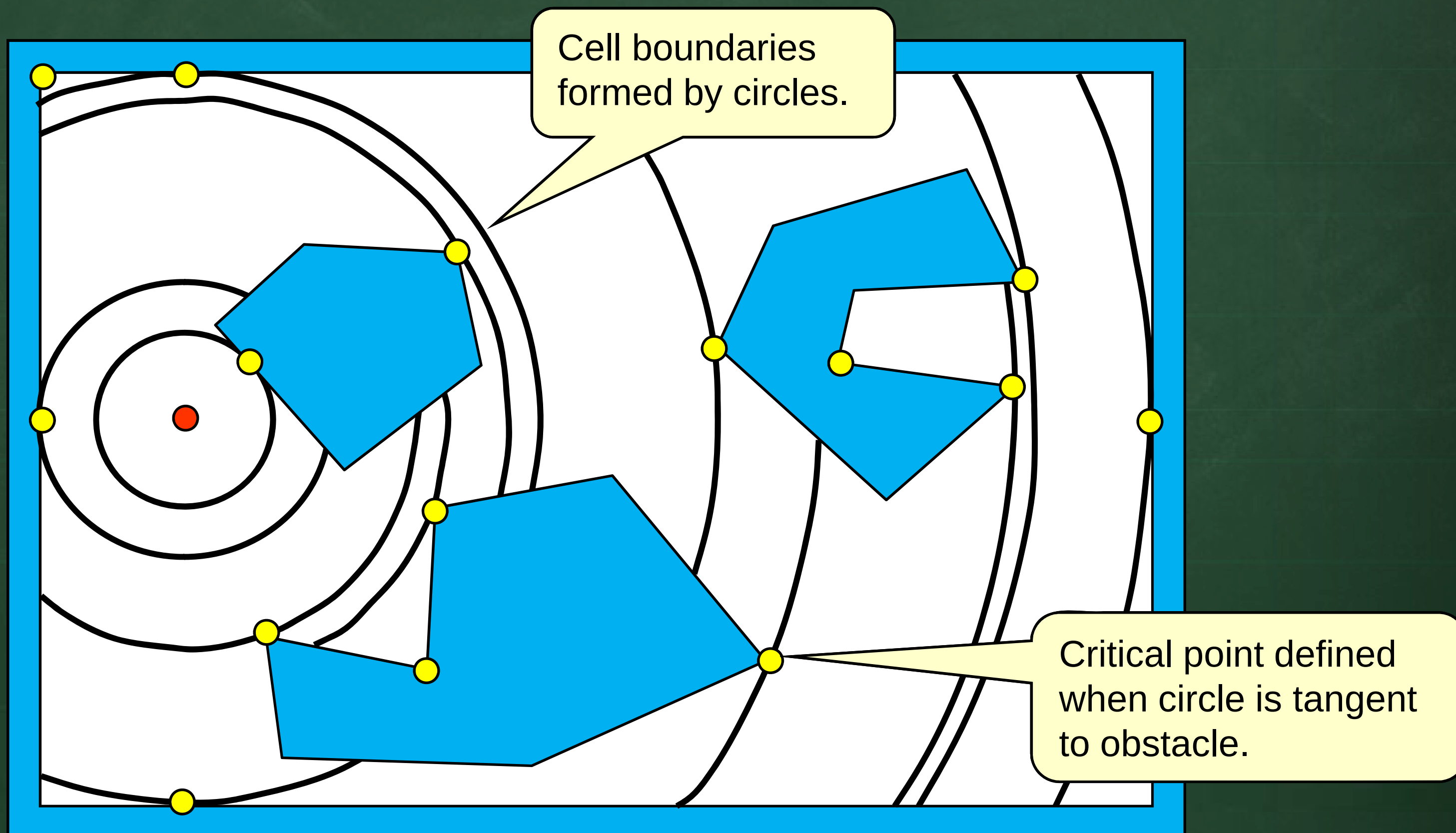
- There are other ways to decompose the environment into cells and compute a coverage path. For example:
  - circular or diamond-shaped spiral cells
  - spike cells
  - brushfire decomposition cells (like GVD)

We will look very briefly at these two

- Each of these, however, may require different traversal techniques.
- Their choice should depend on the robot's sensor characteristics.

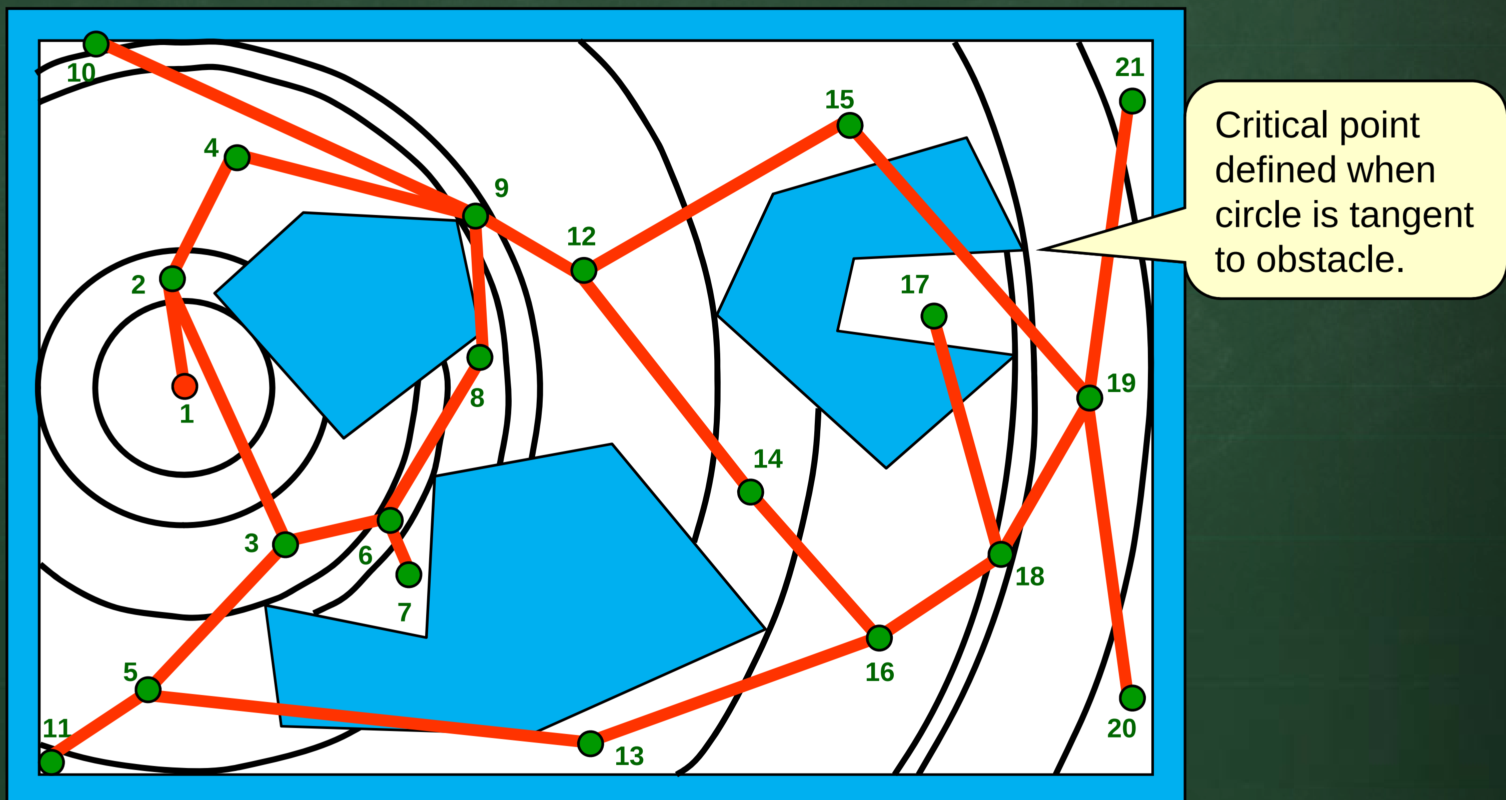
Circular Coverage Patterns

- We can alternatively create circular cells defined by circles extending outwards from the start location:



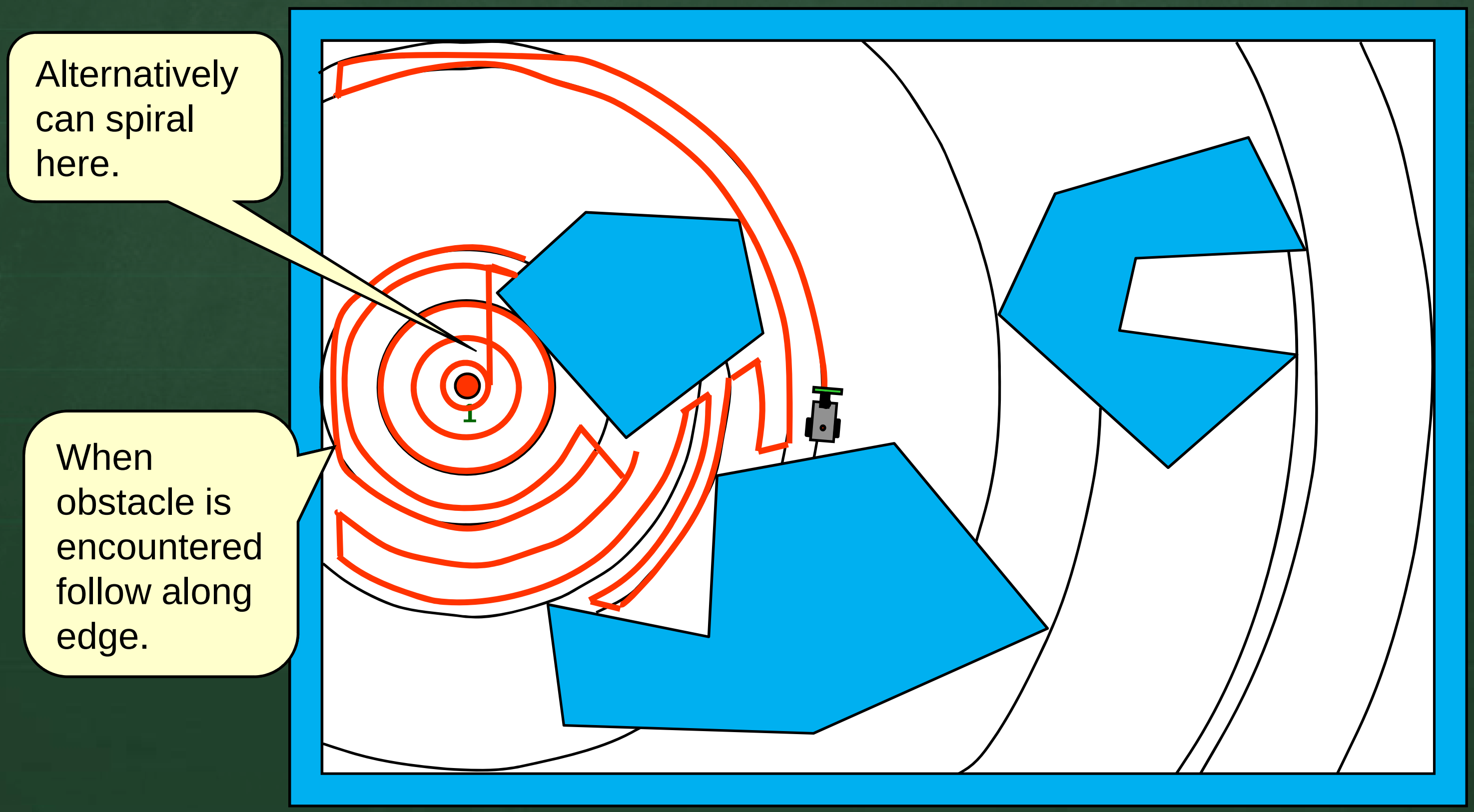
Circular Coverage Patterns

- Once again, interconnect cells and do DFS to find path in graph:



Circular Coverage Patterns

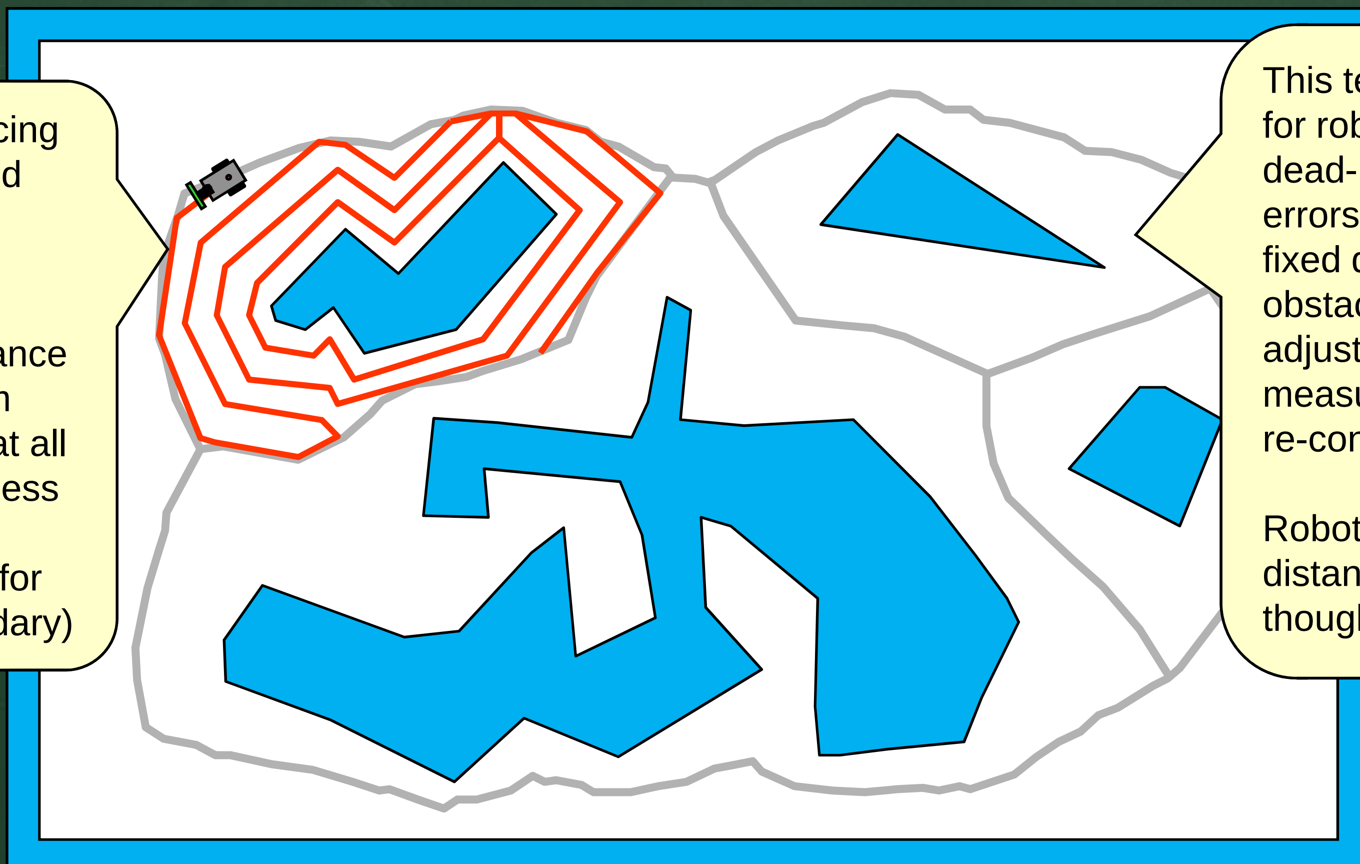
- Traverse each cell by making "laps" around the cell where each lap is separated by the robot width:



Brushfire Decomposition

- Can even break down into regions based on GVD and then traverse cells around obstacles:

When tracing cell around obstacle, robot maintains fixed distance away from obstacle at all times (unless when adjusting for cell boundary)



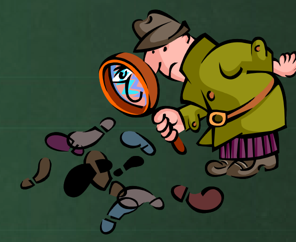
This technique is best for robots that have dead-reckoning errors. By maintaining fixed distance from obstacle, robot can re-adjust its measurements and re-confirm its position.

Robot needs long distance range sensor though.

Search Algorithms

Searching

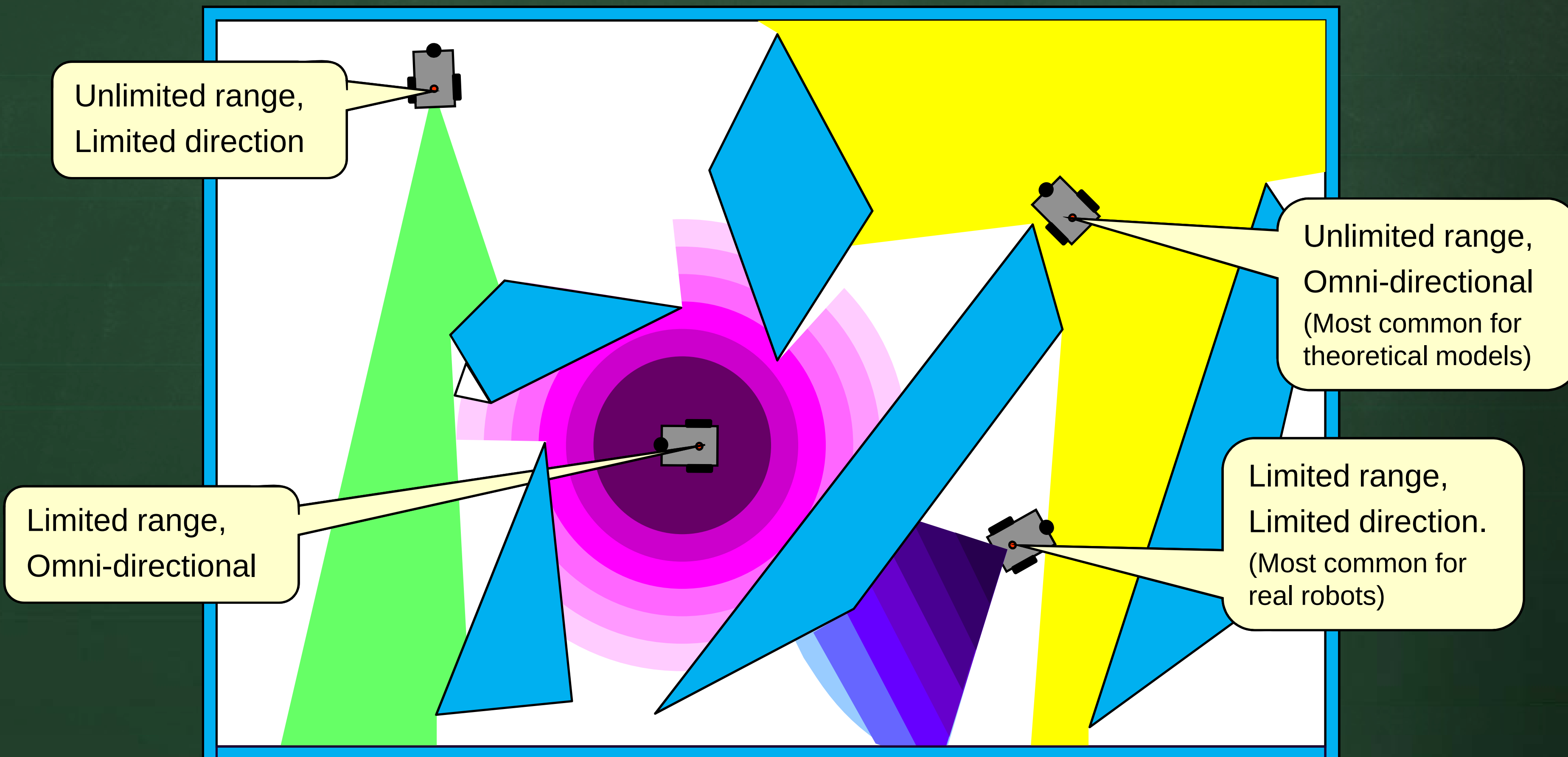
- Consider covering an environment for the purpose of searching for other robots, fire, intruders, any identifiable object etc…



- Robot is equipped with one or more search sensors of some kind which have either:
  - unlimited or limited detection range
  - omni-directional (i.e, 360°) or limited direction detection capabilities

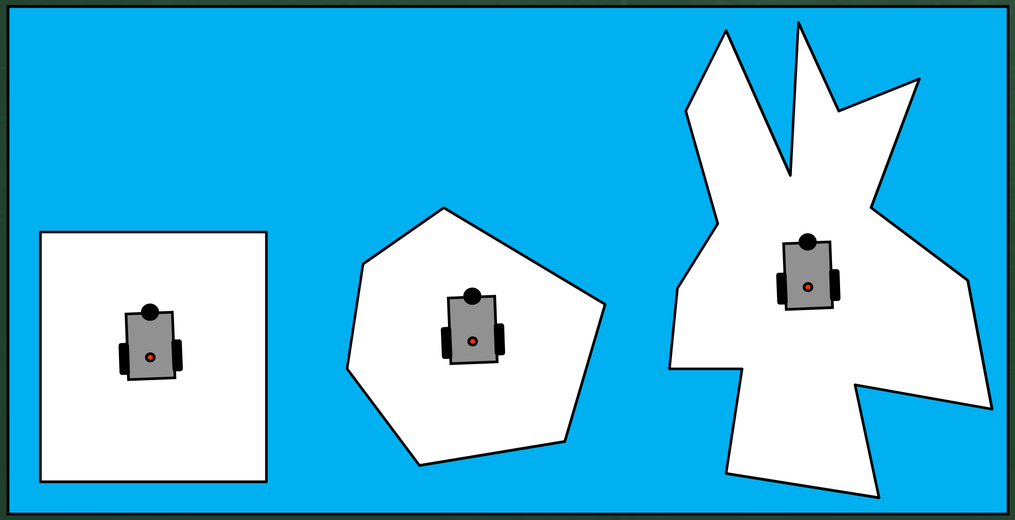
Searching

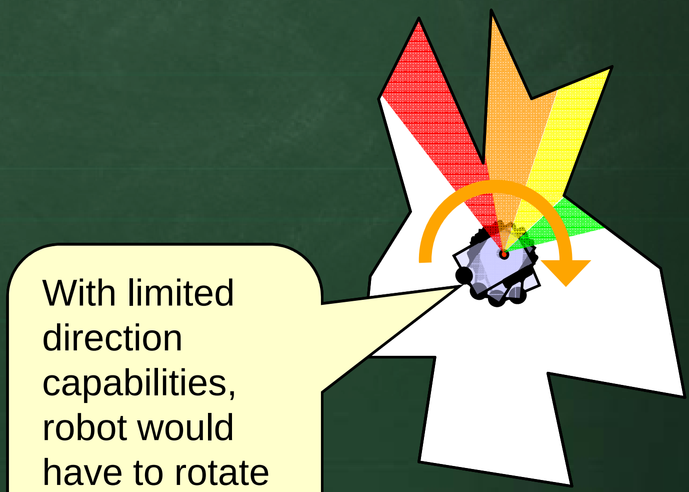
- As the robot moves around in the environment, it is able to search based on its current *visibility*:



Visibility

- Consider a simple environment with no obstacles and a robot with omni-directional sensing with unlimited range capabilities.
- Which environments can it search (i.e., see) completely without moving ?

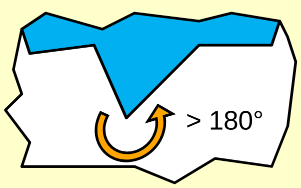


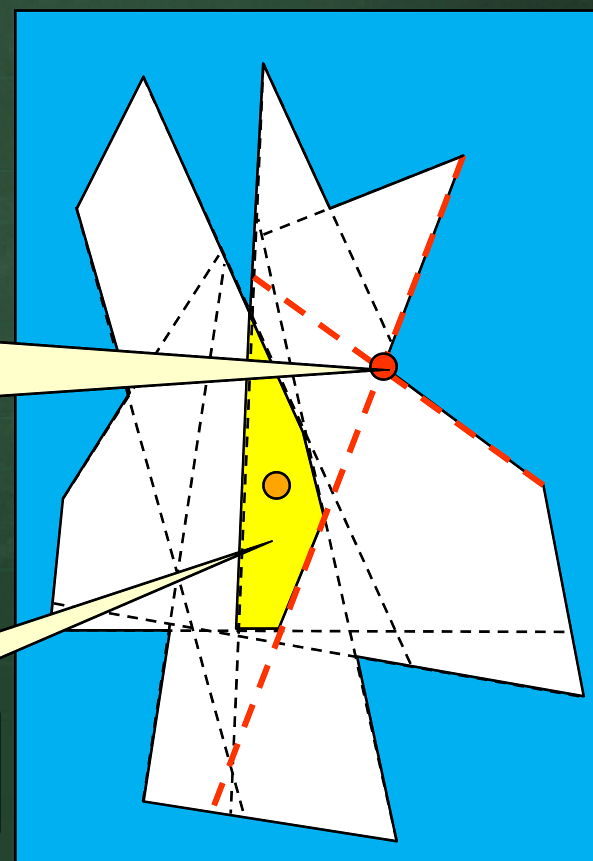


Visibility

- The *kernel* of a star-shaped polygon is the area of the polygon from which the robot can "see" the entire boundary of the environment:

Extend lines from each ***reflex*** vertex parallel to edges containing that vertex. A ***reflex*** vertex is one which forms an inside angle > 180°.

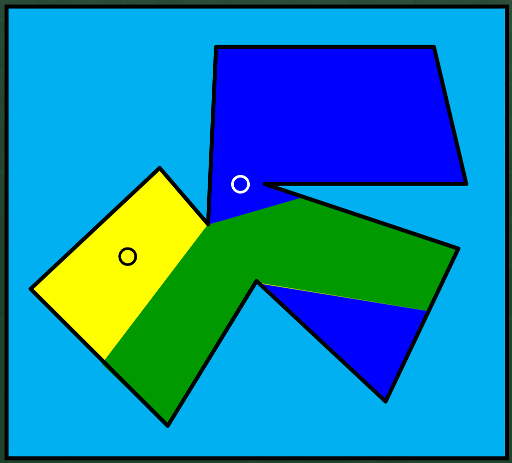


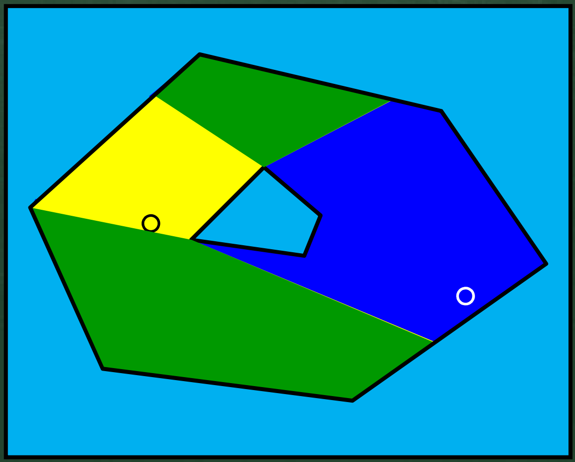


Kernel formed as intersection of the resulting half planes.

Visibility

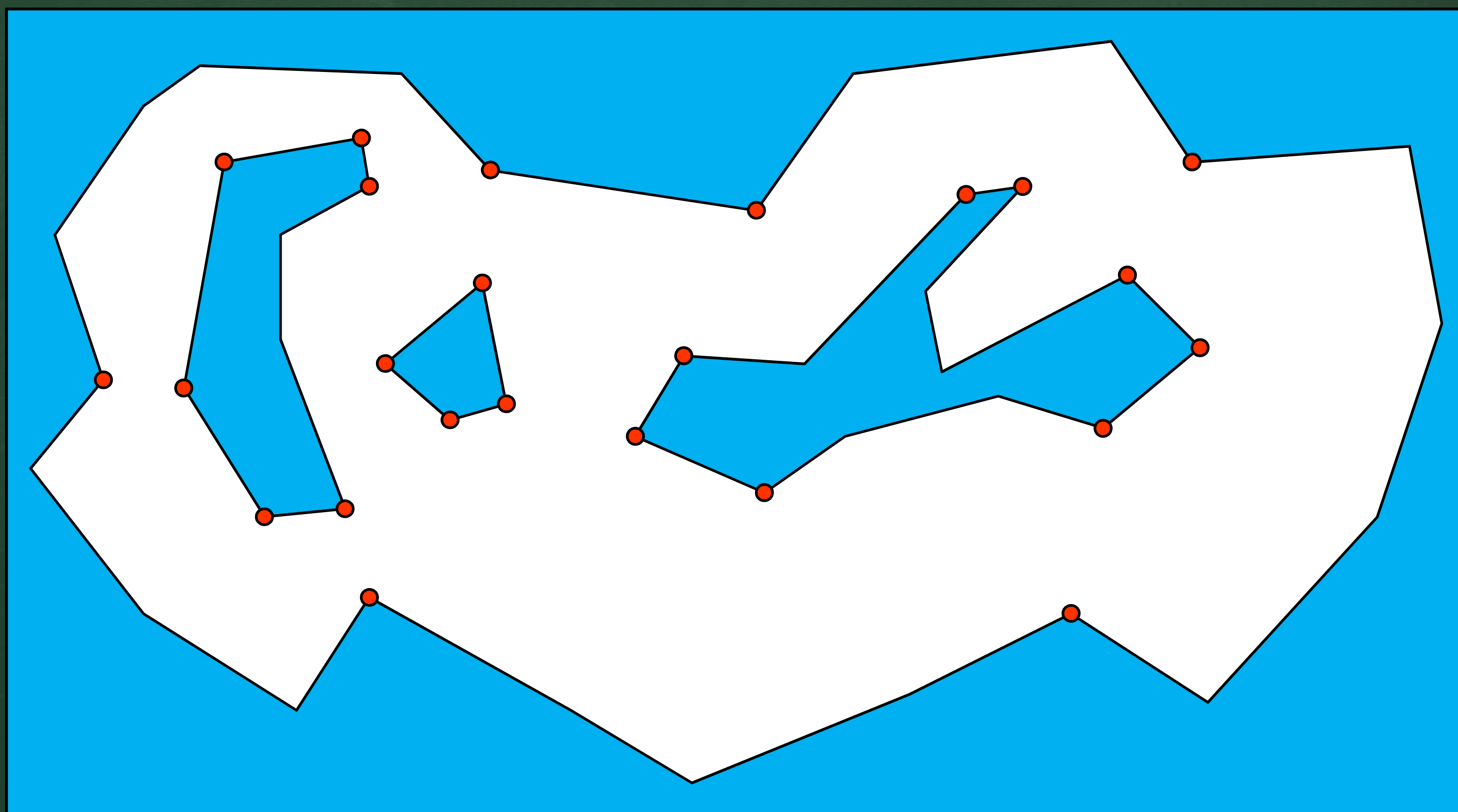
- What if environment is not star-shaped or has obstacles ?
  - kernel is empty (i.e., can't see whole environment from one location)
  - need to determine a set of locations (i.e., view points) that cover the entire environment





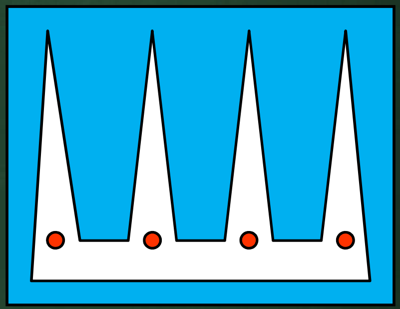
Visibility

- Placing robot at each reflex vertex will ensure complete visibility coverage. Do you know why ?



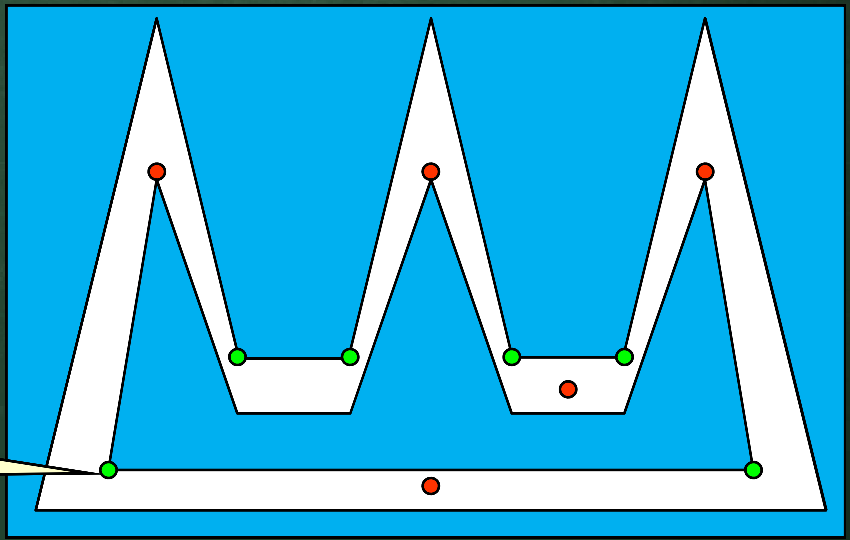
Guard Placement

- Can we cover with less locations ?
- This problem is called the *Guard Placement* problem or *Art Gallery* problem.
- For a simple polygon environment with n vertices:
  - $\lfloor n/3 \rfloor$ locations are occasionally necessary and always sufficient to have every point in the polygon visible from at least one of the locations:
  - e.g., n = 12 and 12 // 3 = 4 locations are necessary and sufficient



Guard Placement

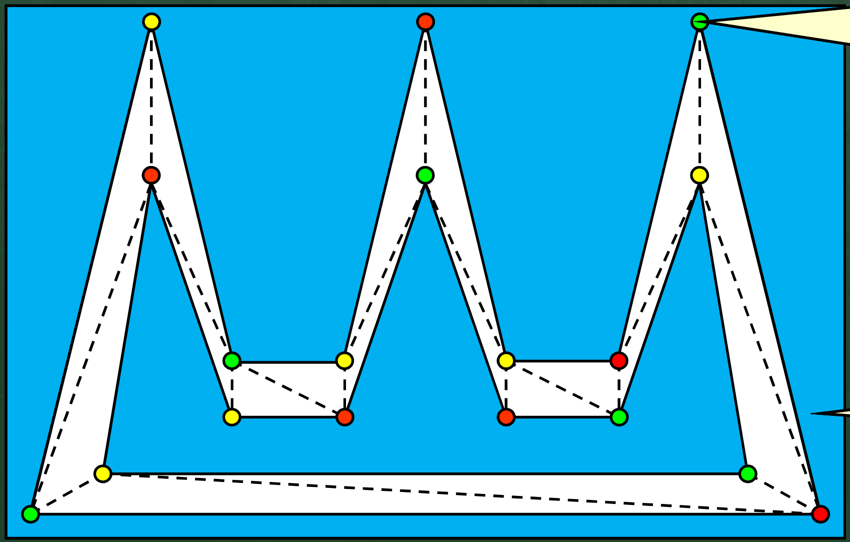
- When the environment contains **h** obstacles and has **n** edges (including obstacle edges), it can be shown that $\lfloor (n+h)/3 \rfloor$ locations are sometimes necessary and always sufficient to cover the entire environment:
  - e.g., n = 19, h = 1
    then (19+1)//3 = 20//3 = 6
    locations are necessary
    and sufficient.



There are many possible placements, here are two …

Guard Placement

- How do we compute these locations ?
- Can do a *3-coloring* of the triangulation:
  - color each vertex of the triangulation with one of 3 colors
  - no two vertices sharing a triangulation edge should have the same color



Coloring is done through a DFS, but in some cases the straight forward approach does not always work…it can be tricky.

Each color here indicates a possible set of robot locations.

Search Paths

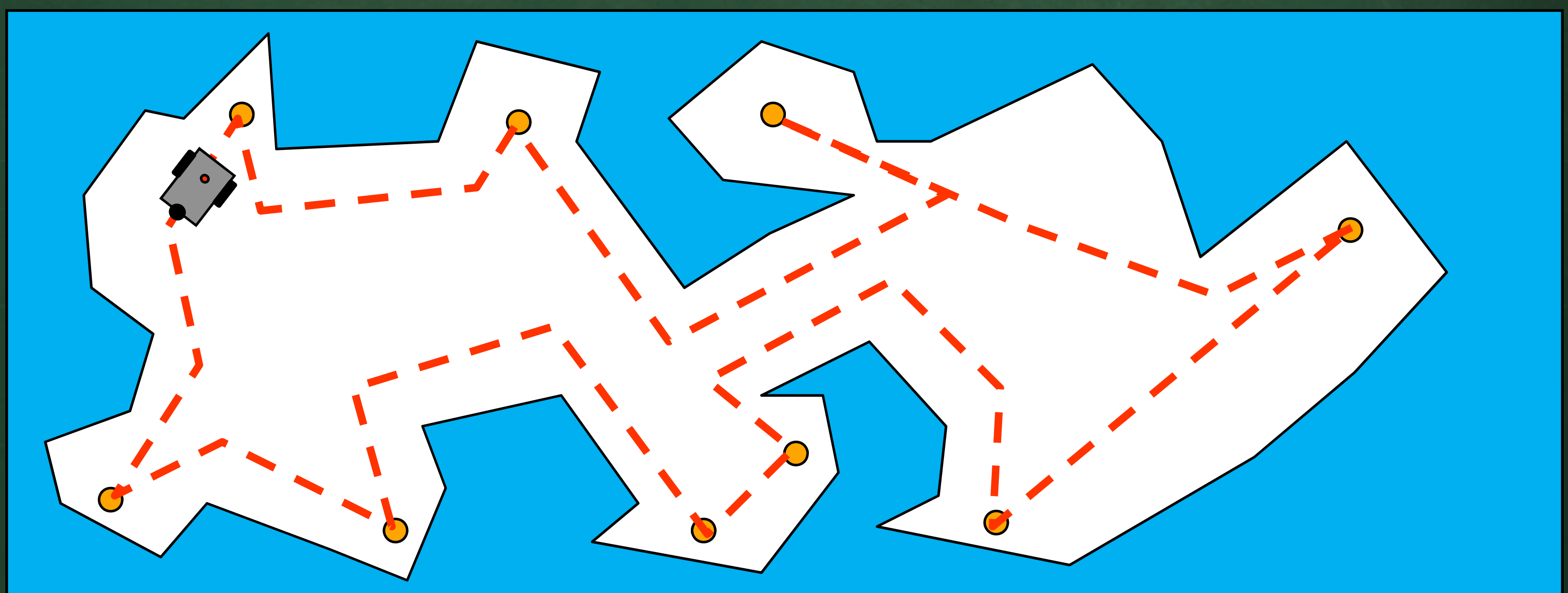
- To perform an exhaustive search, the robot must move around in the environment
  - *shortest watchman route* – the shortest possible path in the environment such that the robot covers (i.e., sees) all areas in the environment.
  - difficult to find exact solution, approximations are usually simpler and acceptable

- Can solve this problem by finding guard placement locations and then connect them with an efficient path (i.e., travel between multiple goal locations).

Traveling Salesman Problem

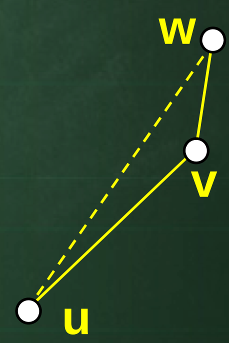
Traveling Salesman Problem

- Given a number of locations that the robot must travel to, what is the cheapest round-trip route that visits each location once and then returns to the starting location ?
  - (e.g., visiting stations in a building for security checks).



Traveling Salesman Problem

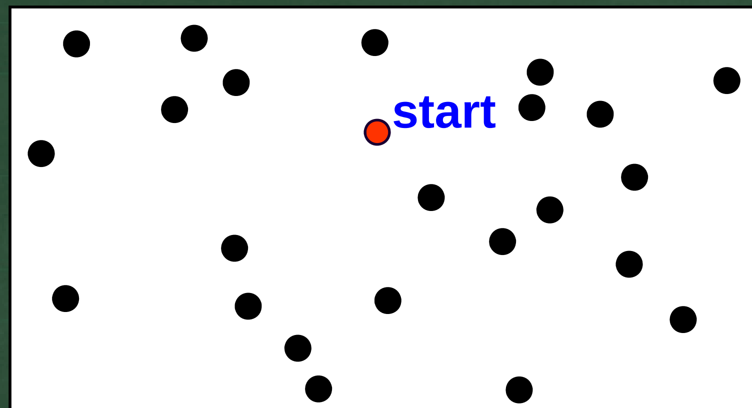
- Most direct solution:
  - try all permutations (ordered combinations) and see which one is cheapest
  - number of permutations is *n!* for *n* locations … *impractical !!*
- There are many approaches to this problem
  - many use heuristics and approximations
- If we don't need the "optimal" path, we can compromise for some simpler algorithms.



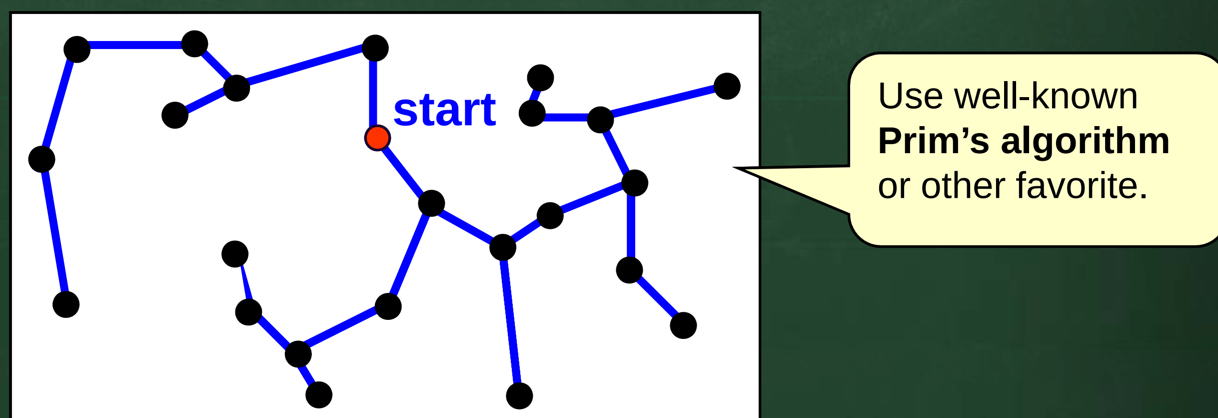
- Assume triangle inequality holds: $|\overline{uw}| \leq |\overline{uv}| + |\overline{vw}|$

Traveling Salesman Problem

- Consider the locations that robot must travel to.



- Approximate tour is based on ***minimum spanning tree*** from the start location:



Traveling Salesman Problem

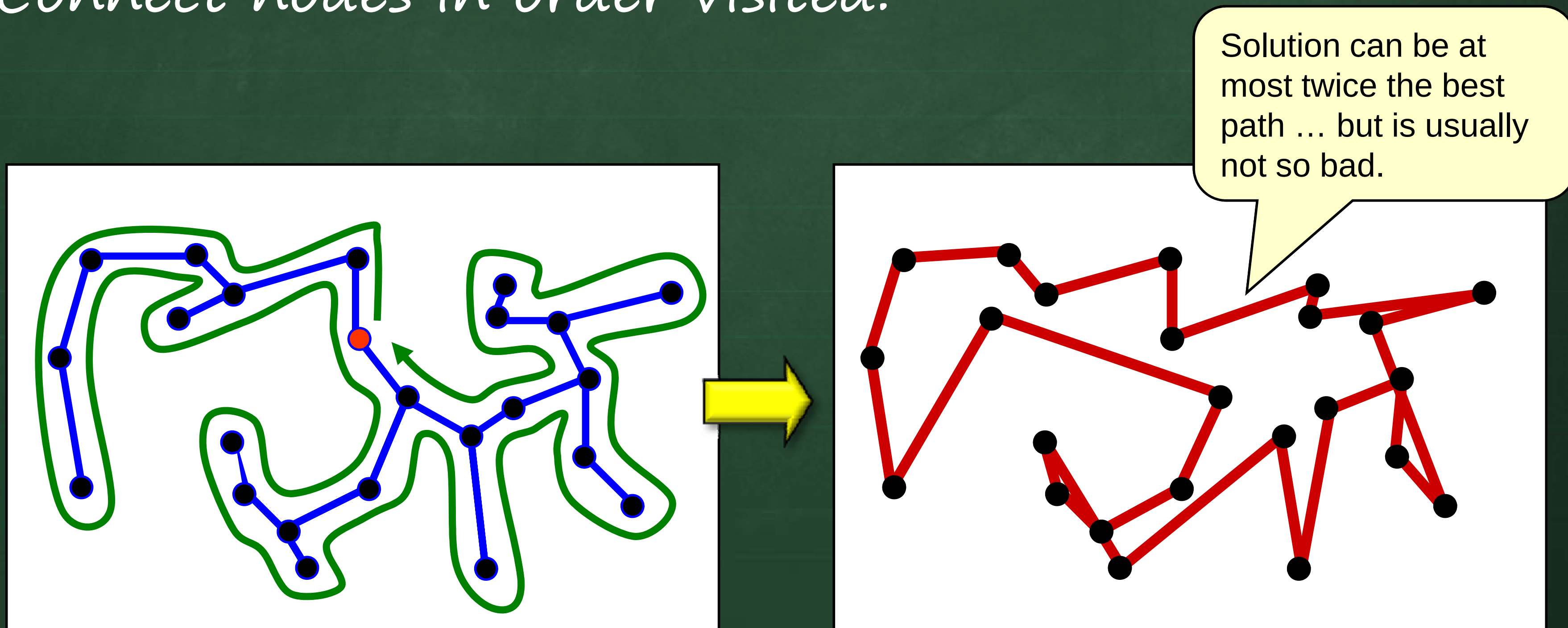
- Consider a complete graph of the locations
  - i.e., each location connects to every other location
- The minimum spanning tree is a subset of the complete graph's edges that forms a tree that includes every location, where the total length of all the edges in the tree is minimized.

```
1.   Create a tree containing a single arbitrarily chosen location
2.   Create a set S containing all the edges in the graph
3.   WHILE (any edge in S does not connect two locations in the tree) DO
4.        Remove the shortest edge from S that connects a
          location in the tree with a location not in the tree
5.        Add that edge to the tree
```

Use simple heap data structure.

Traveling Salesman Problem

- From the root of the minimum spanning tree perform a pre-order traversal of the tree.
- Connect nodes in order visited.



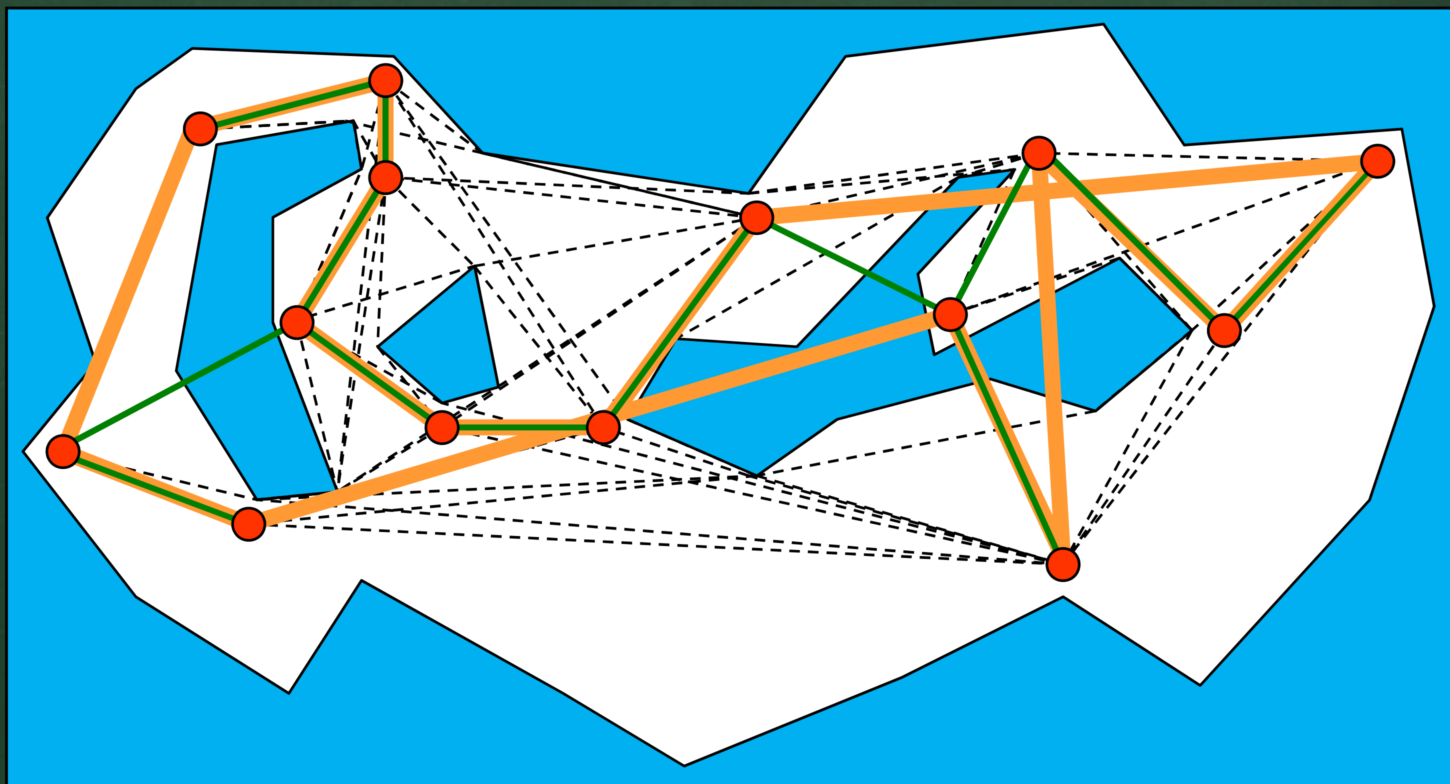
Traveling Salesman Problem

- Running time is $O(n^2)$ for n locations.
- There are other variations of this problem … we could spend a whole course discussing these types of problems.

- Can we use this algorithm practically ?
  - the triangle inequality may not hold since obstacles are often in the way.
  - can still do a minimum spanning tree, but must replace straight line paths with weighted shortest path links.

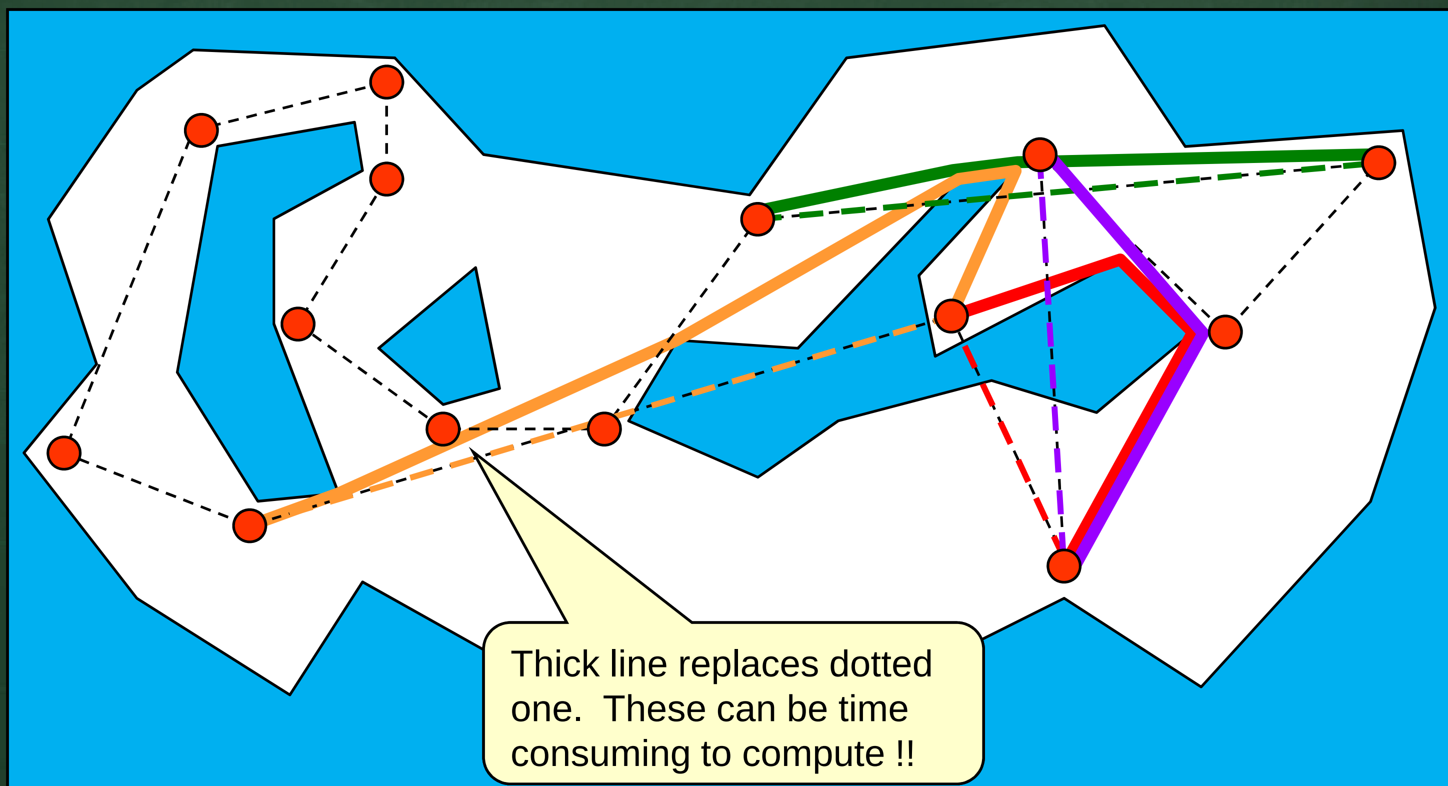
Traveling Salesman Problem

- Solution to TSP may yield invalid paths.
  - would have to replace point-to-point costs with shortest path costs



Traveling Salesman Problem

- Can replace invalid segments with shortest path segments:



Traveling Salesman Problem

- The solution to the traveling salesman problem does not directly apply to our problem since paths may be invalid.

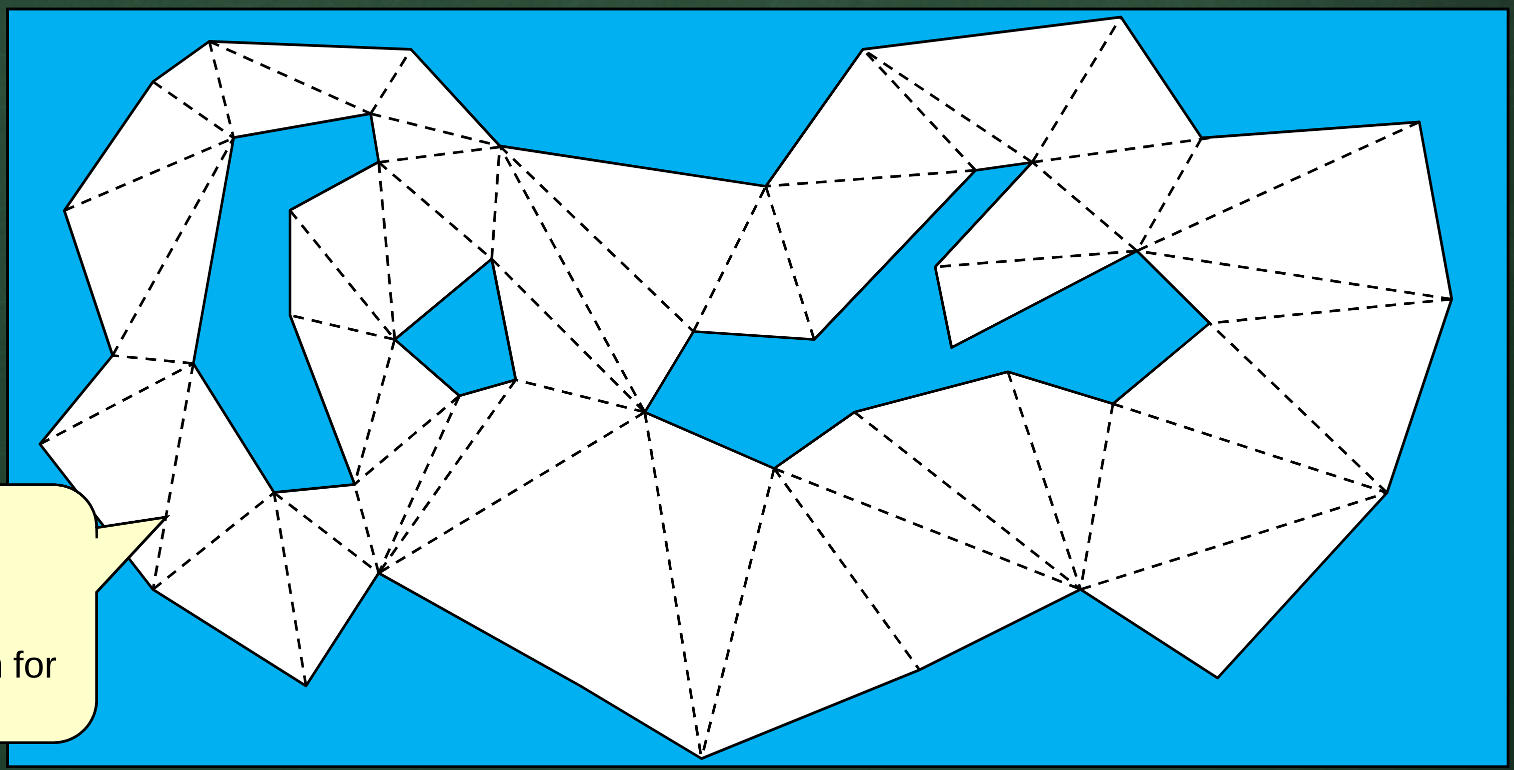
- A simpler, more practical approach is often better
  - + easier to compute
  - + can ensure complete coverage
  - – may end up with longer path

- Simplest, most practical approach is to use the dual graph of the triangulation.

Visibility Search Paths

Dual Paths

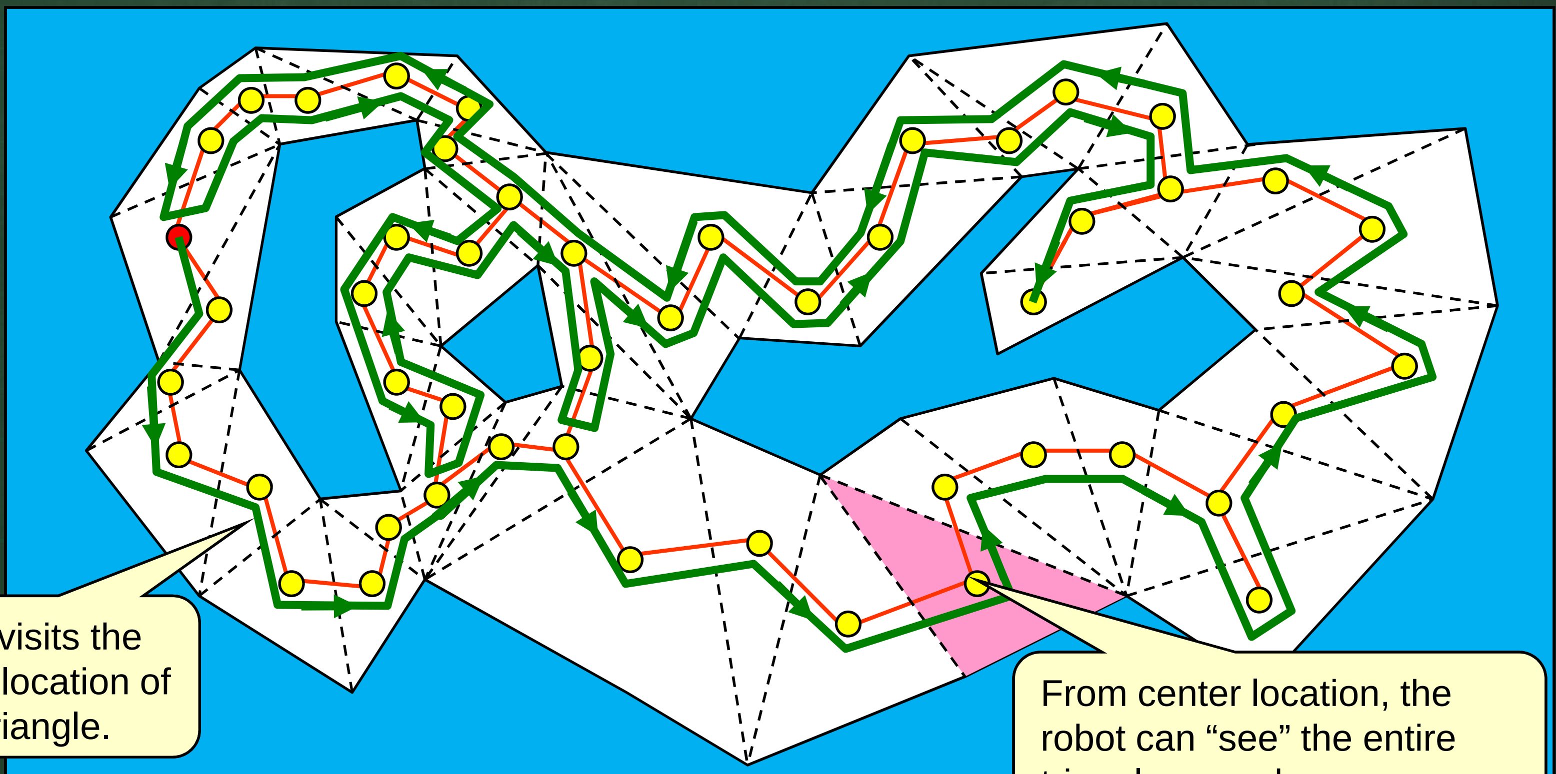
- Consider a robot with an unlimited range, omni-directional sensor.
  - First, triangulate the polygon with holes:



Use a constrained Delauney Triangulation for best results.

Dual Paths

- We can *traverse the dual graph* (using a Depth First Search) as a rough path around all obstacles:



Robot visits the center location of each triangle.

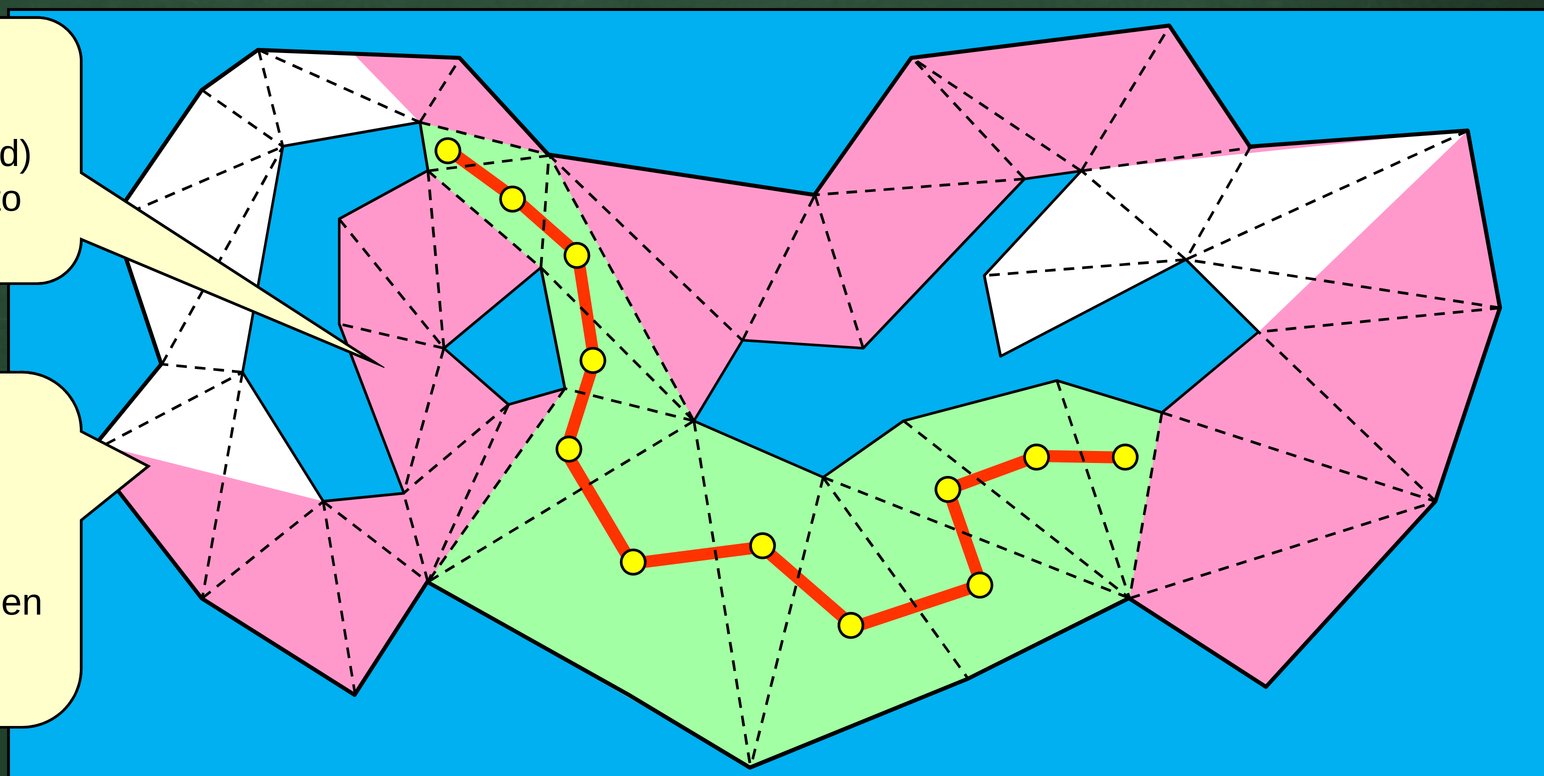
From center location, the robot can "see" the entire triangle … and more.

Visibility Path

- As the robot travels along the dual graph, it can actually "see" (i.e., search) a much larger area than the triangles it passes through:

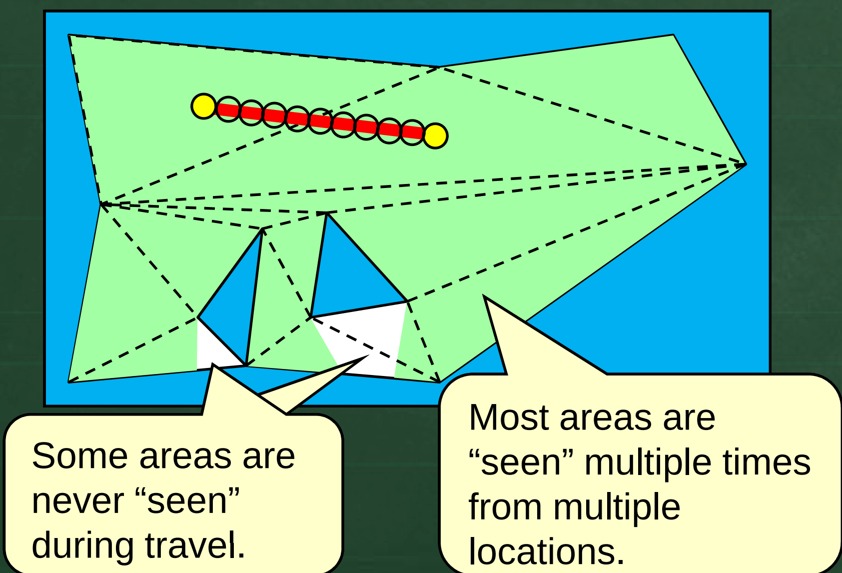
Many triangles can be "seen" (hence searched) without having to enter them.

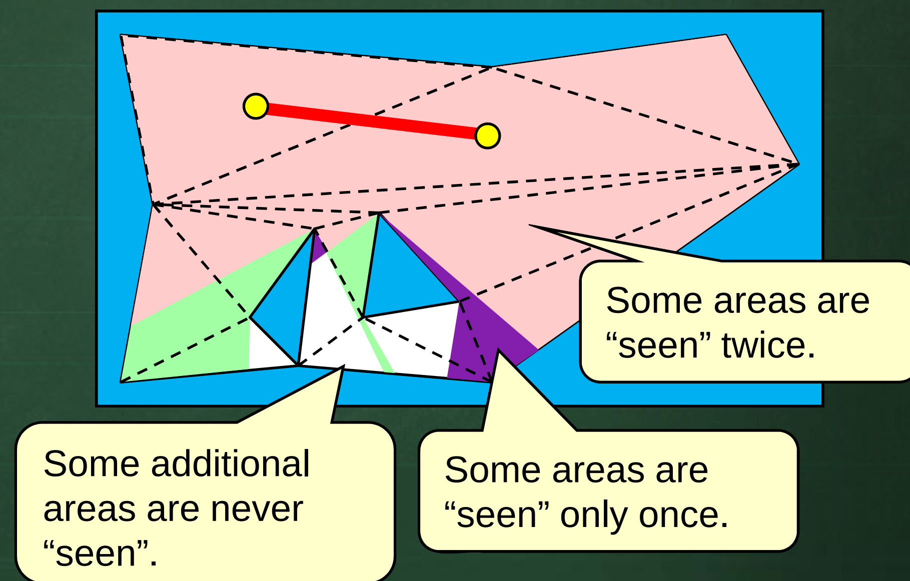
However, it is difficult to keep track of which "parts" of the triangles are seen from other triangles.



Visibility Path

- When robot travels between triangles it can:
  - search along the way while traveling
  - search only when arriving at the triangle centers



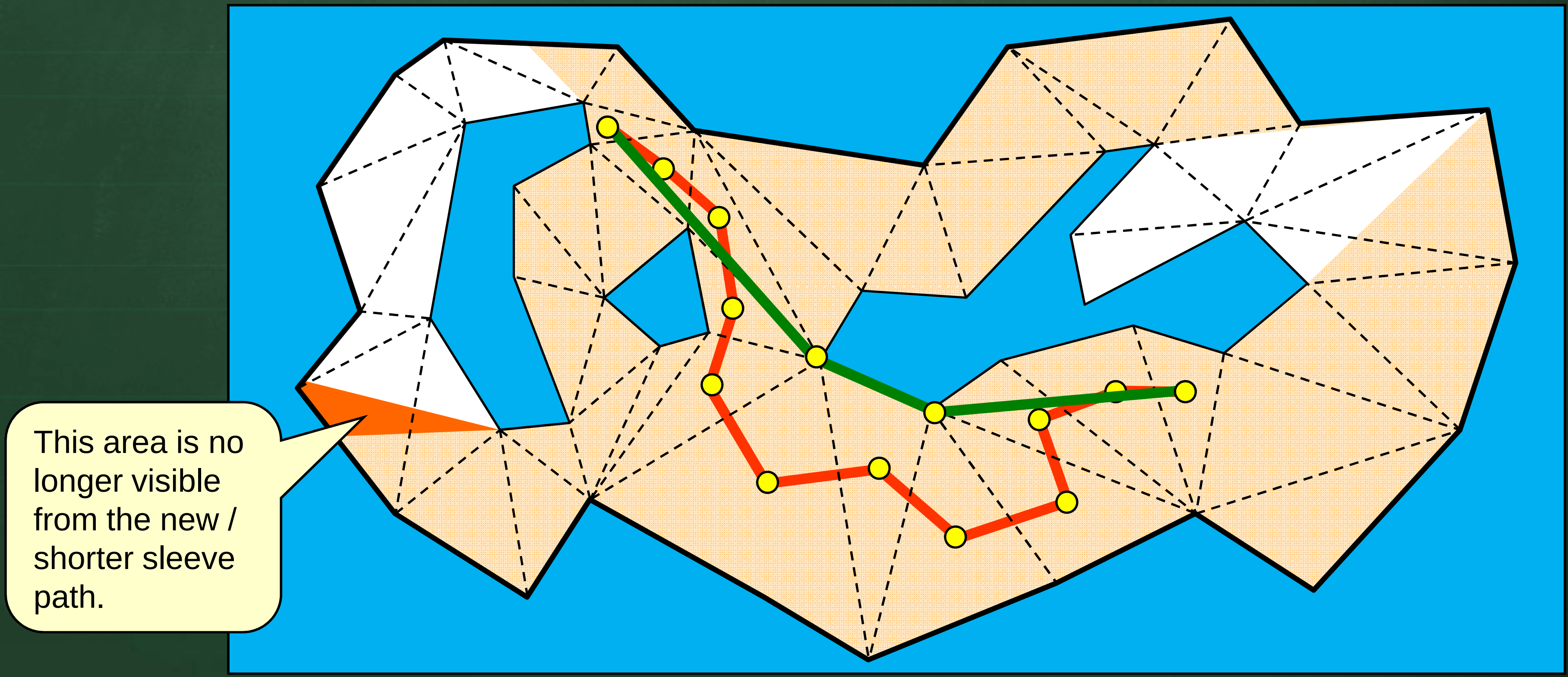


+ more coverage
– more computation

+ less computation
– less coverage

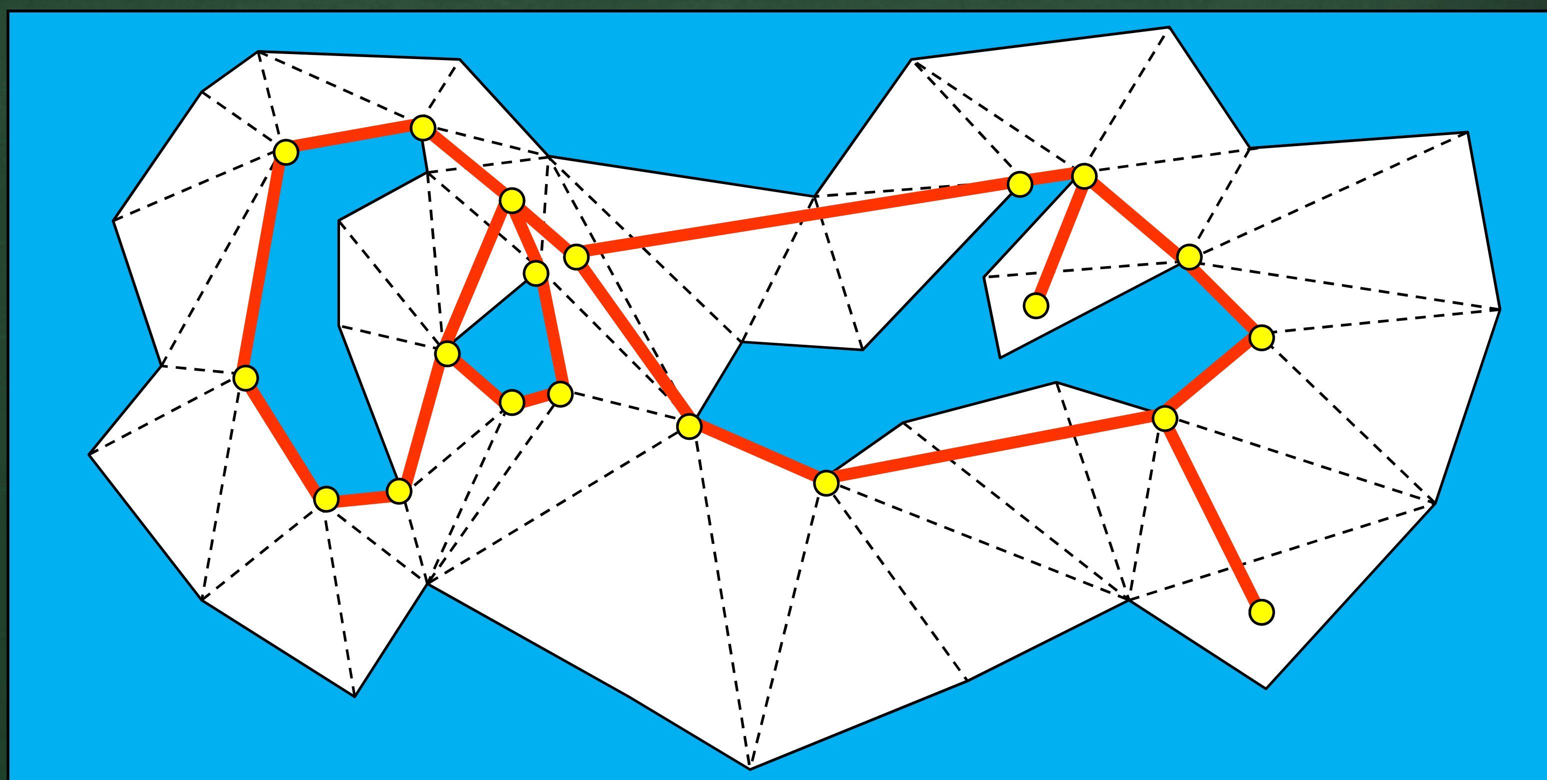
Refining Paths

- Recall that dual graph paths can be refined by computing a shortest sleeve path:
  - results in a slightly modified area coverage



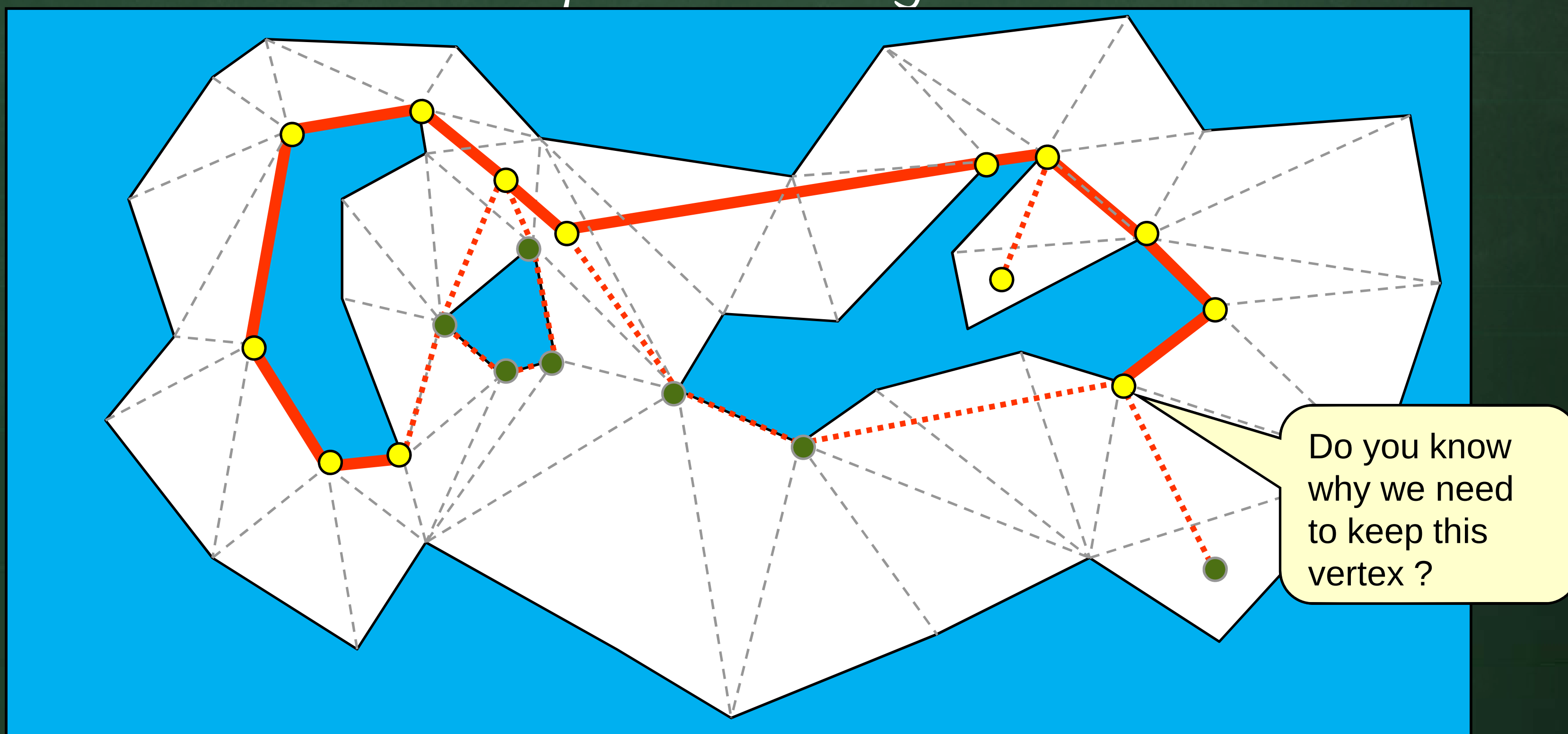
Refining Paths

- Combining all such ***refined paths*** leads to an efficient path that will guarantee visibility of the entire environment:



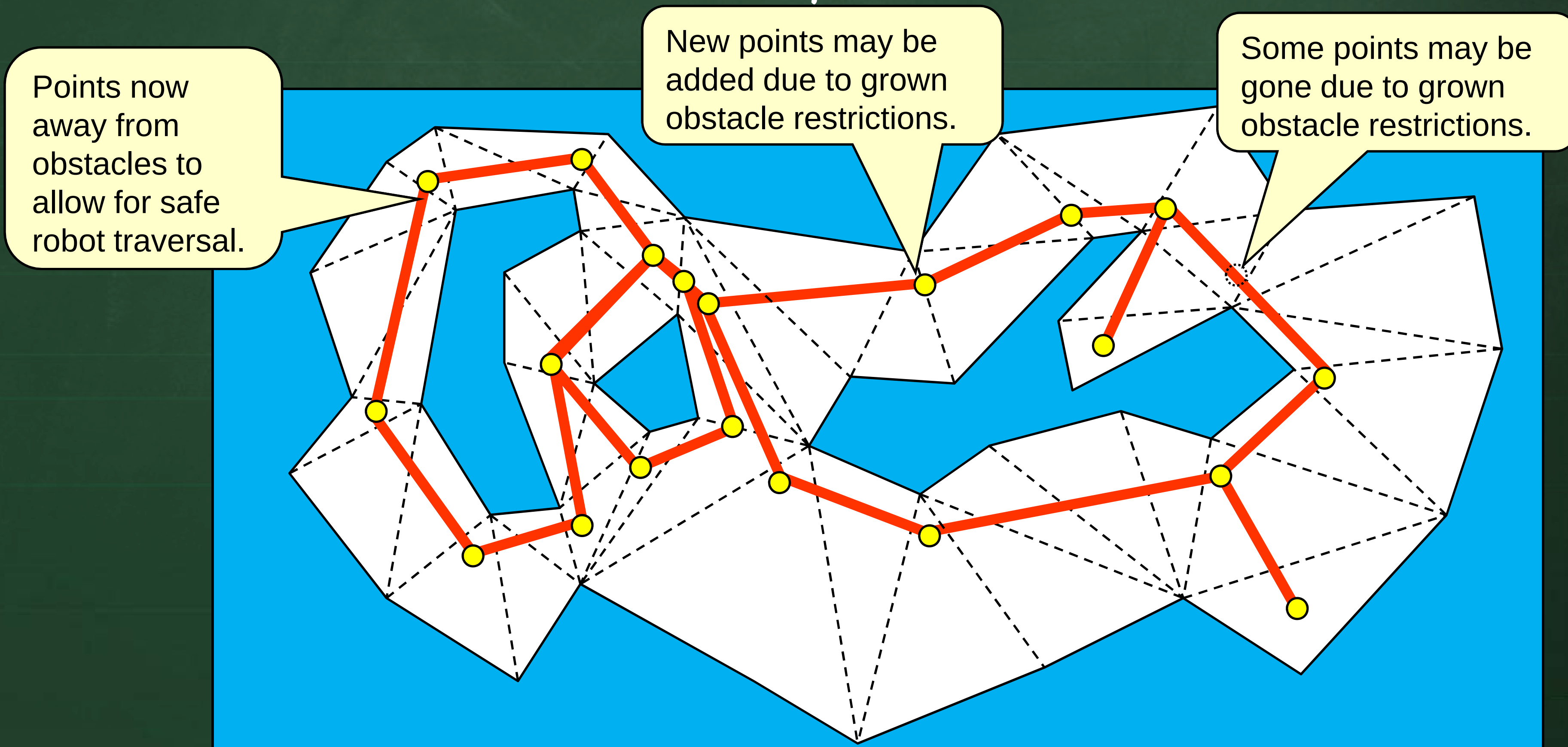
Refining Paths

- Can even *trim* (i.e., remove edges from) the path:
  - walk through the path, keeping track of which triangles are completely covered along the way. Eliminate edges/vertices that do not add to the path's coverage.



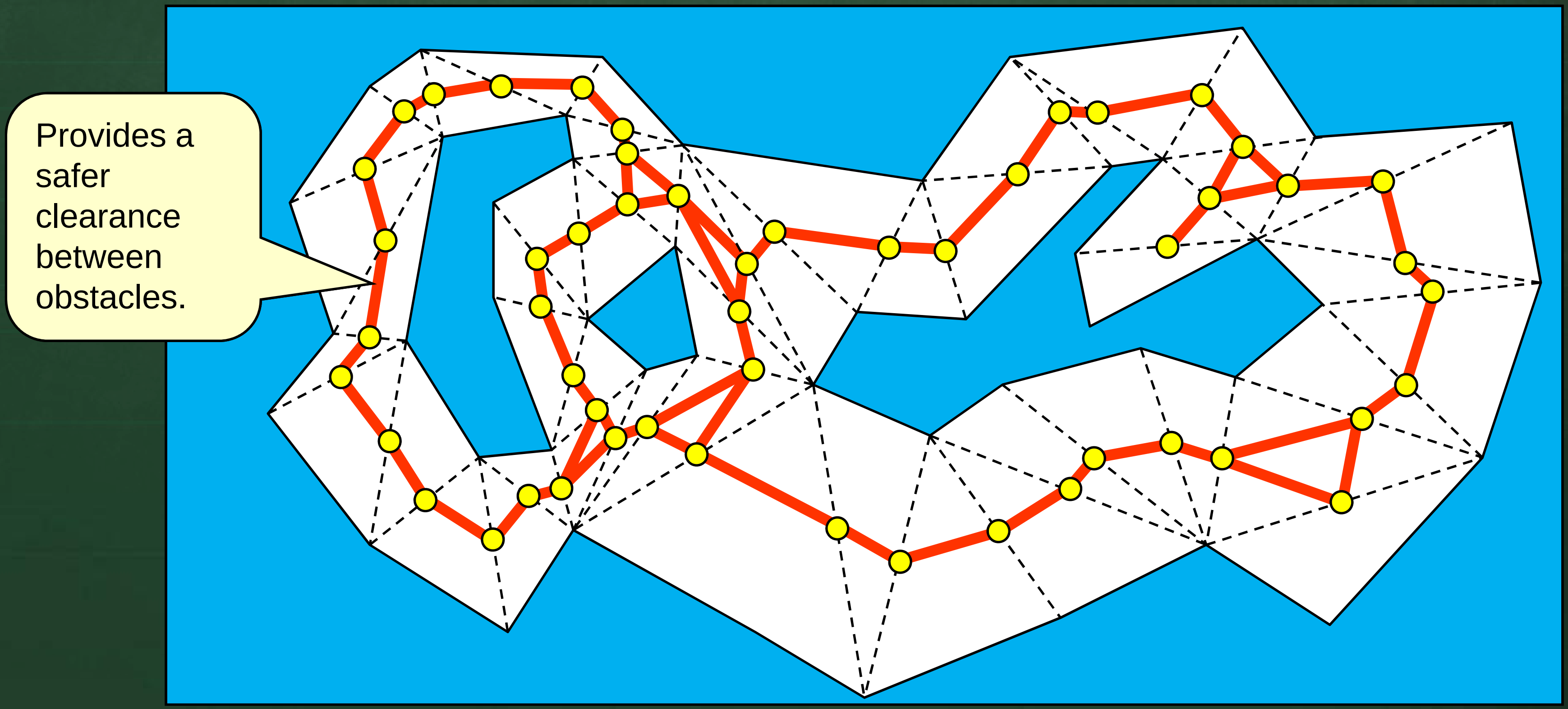
Safer Paths

- For safety, we can first grow obstacles according to robot model to allow valid paths that do not collide:



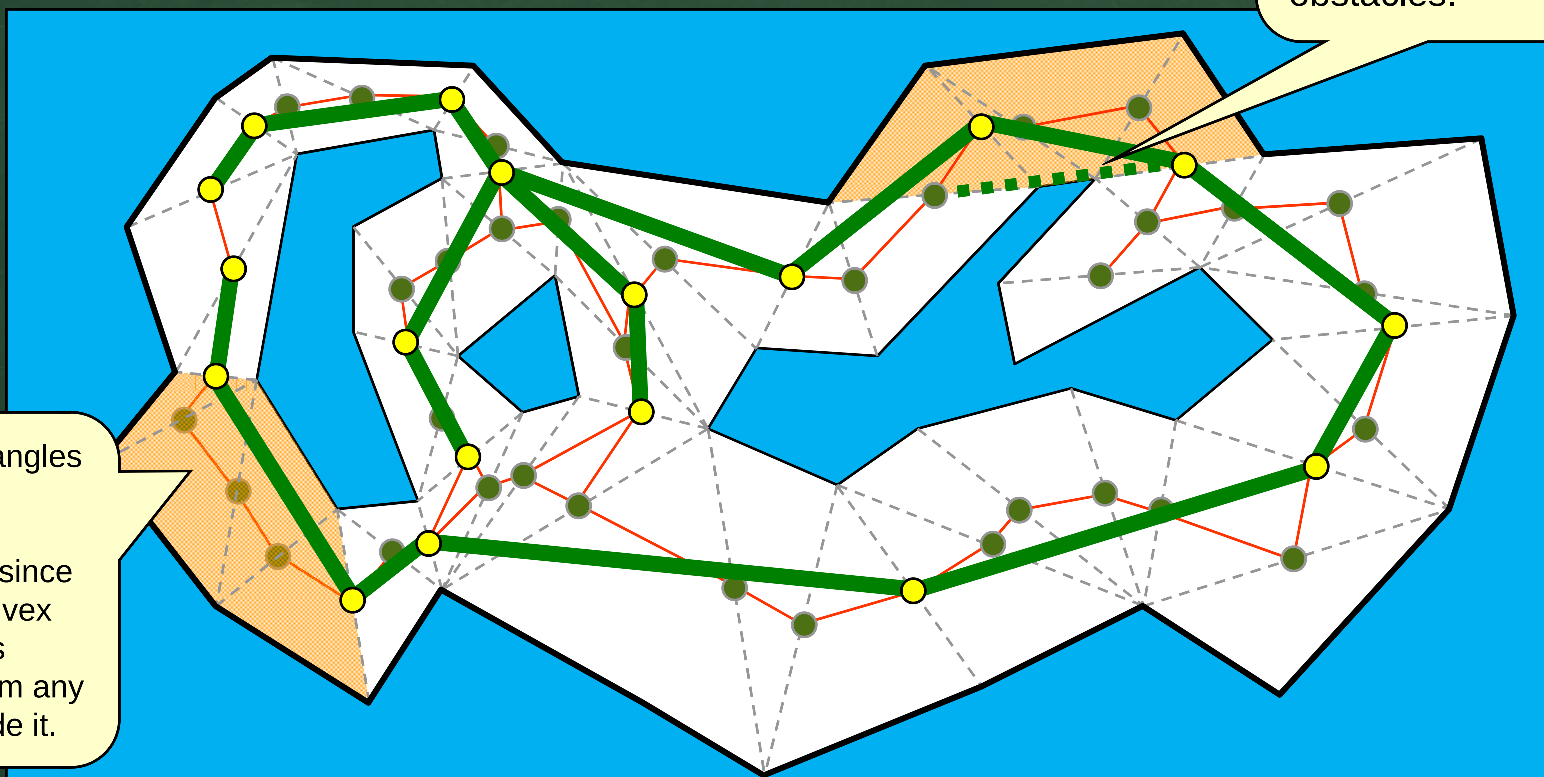
Safer Paths

- A safer/simpler approach:
  - place view locations at midpoints of triang. diagonal edges
  - connect viewpoints from edges on the same triangle



Safer Paths

- Once again, trim edges by removing ones that do not add to the coverage:

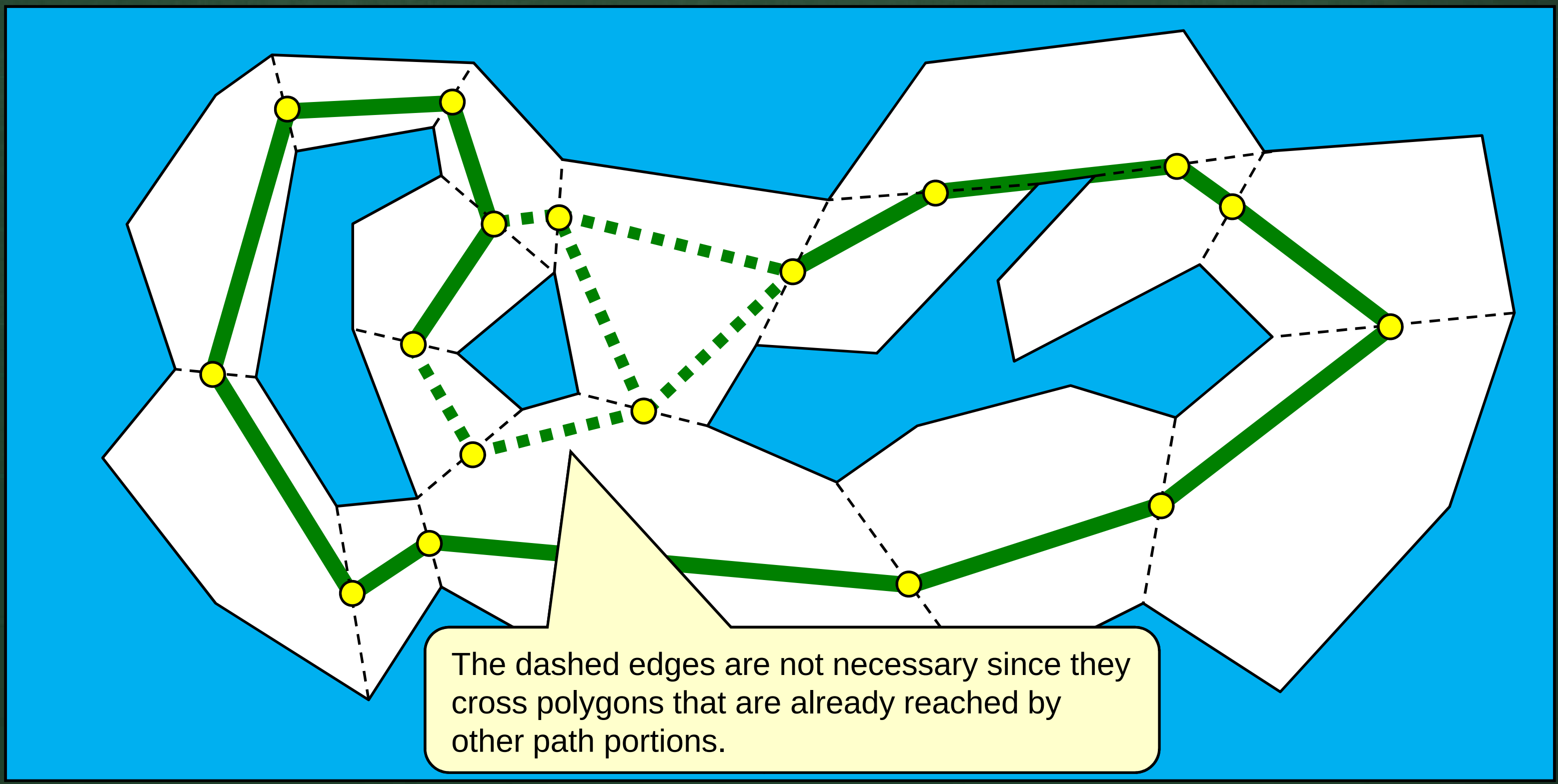


May also put constraint that edge must have certain clearance from obstacles.

Merge triangles that form convex polygons since entire convex polygon is visible from any point inside it.

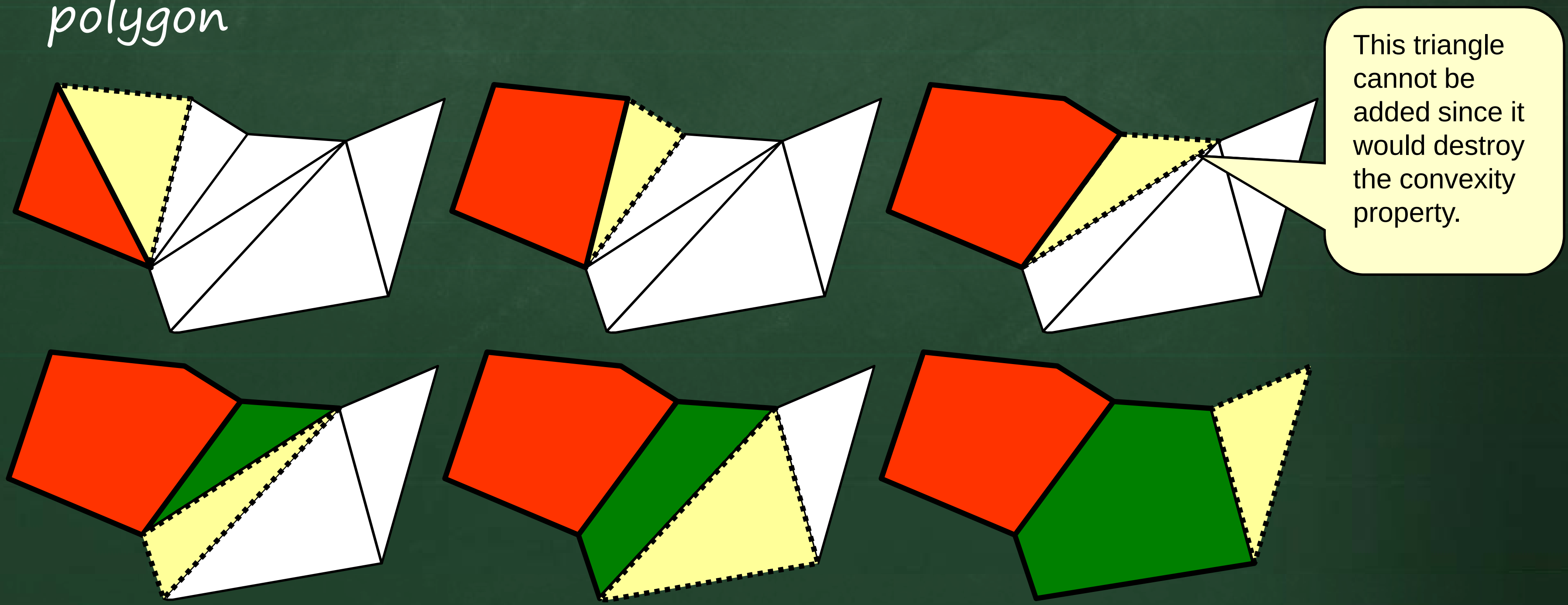
Convex Pieces

- Of course, we can always merge triangles to form convex areas *before* we search the graph
  - reduces need to trim off many edges later



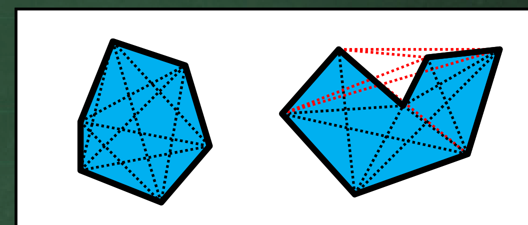
Convex Pieces

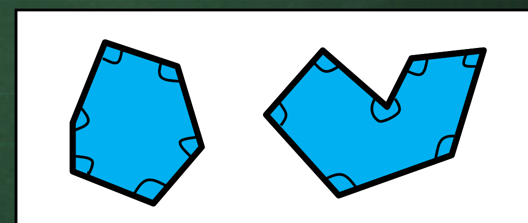
- How do we merge triangles into convex pieces ?
  - traverse dual graph using DFS.
  - build up convex polygon by adding new triangles one at a time … if a new triangle "ruins" convexity, start a new polygon

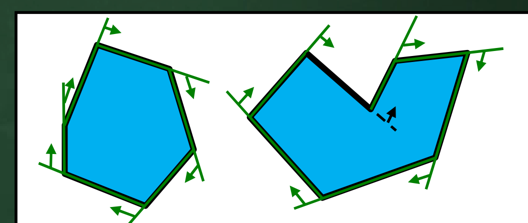


Convex Pieces

- How do we determine if a polygon is convex ?
- There are a variety of ways:
  - check that the line segment between each pair of non-adjacent vertices does not intersect any polygon edge.
  - check that each pair of consecutive edges forms an interior angle ≤180°.
  - traverse the polygon CW and make sure that each consecutive edge makes a right turn.

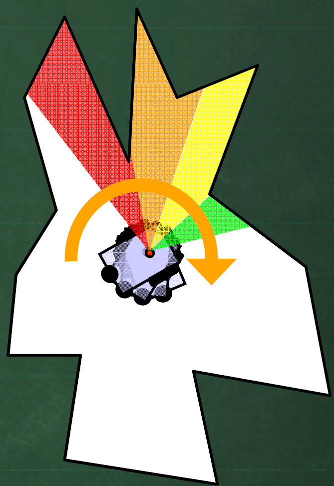




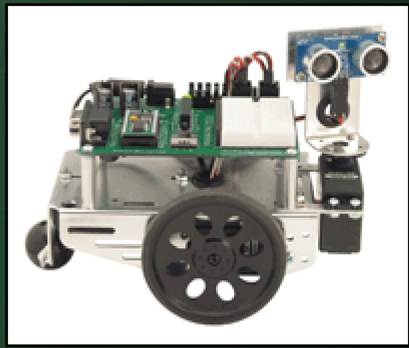


Limited Direction Visibility

- What if the robot cannot sense omni-directionally ?
- Recall that robot can turn at each search point:
  - can be time consuming
  - try to minimize search locations



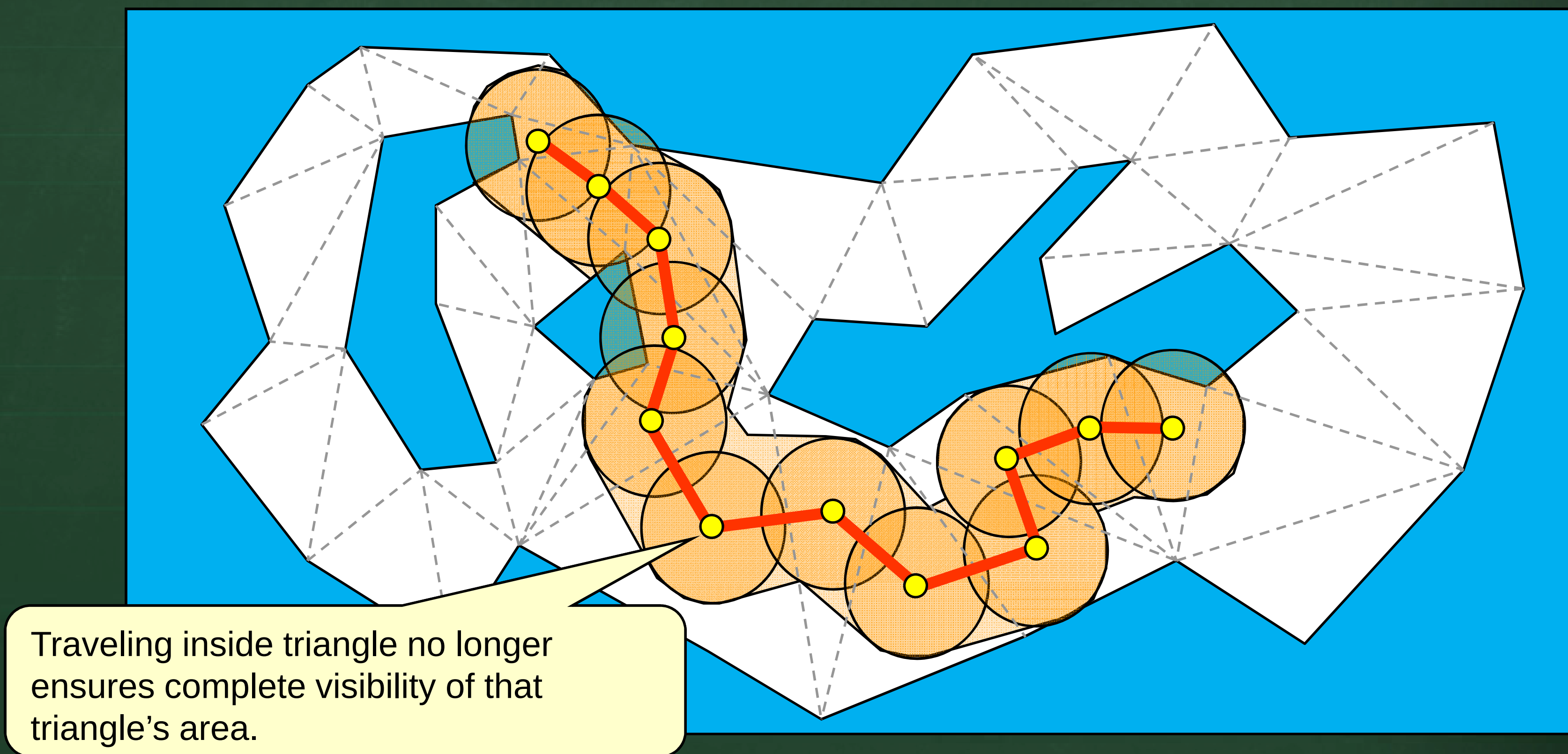
- Alternatively, some robots are equipped with ***head turrets*** that can turn 360°.



Searching With Limited Visibility

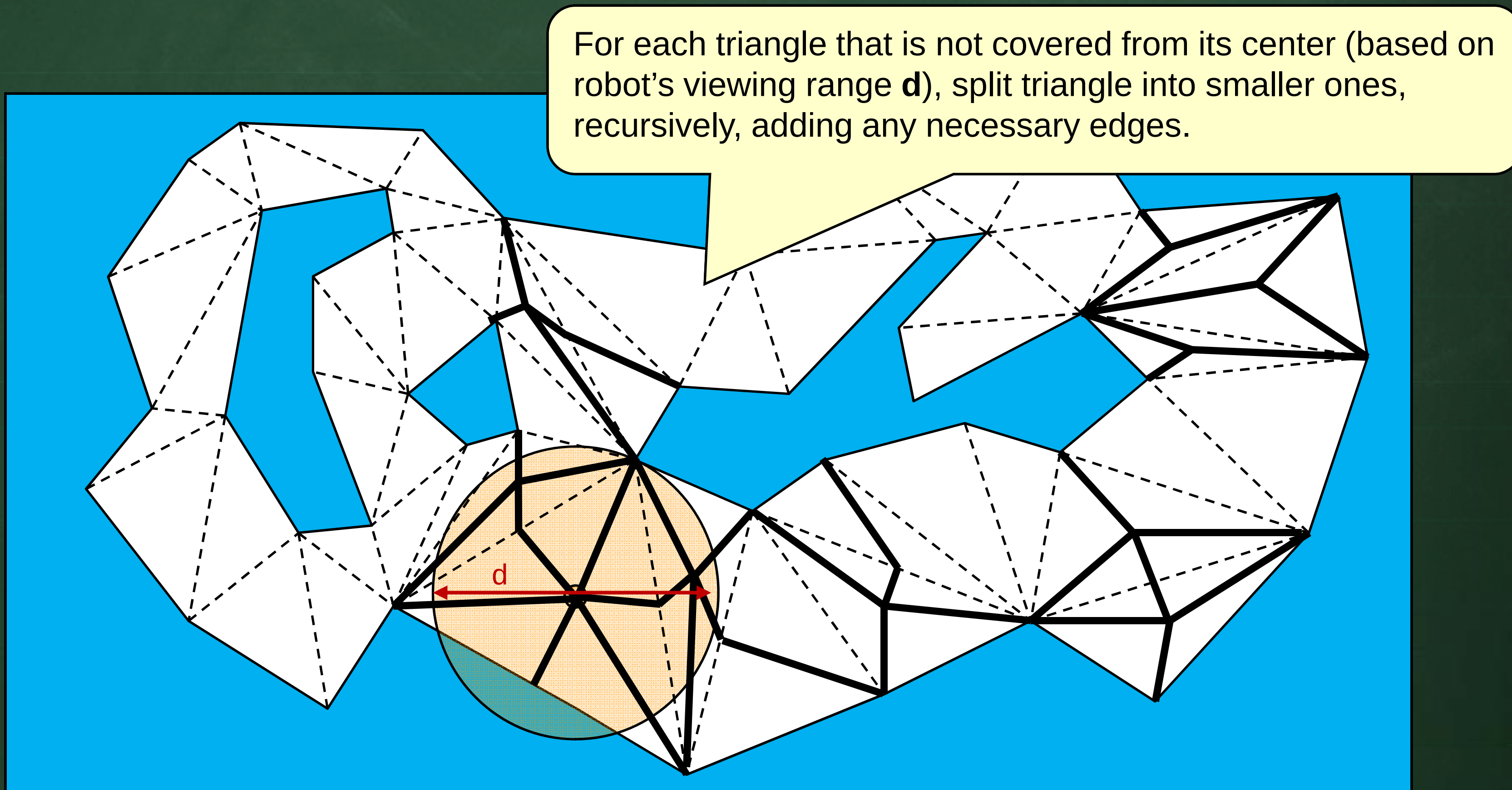
Limited Range Visibility

- The problem changes when the robot has limited sensing range:



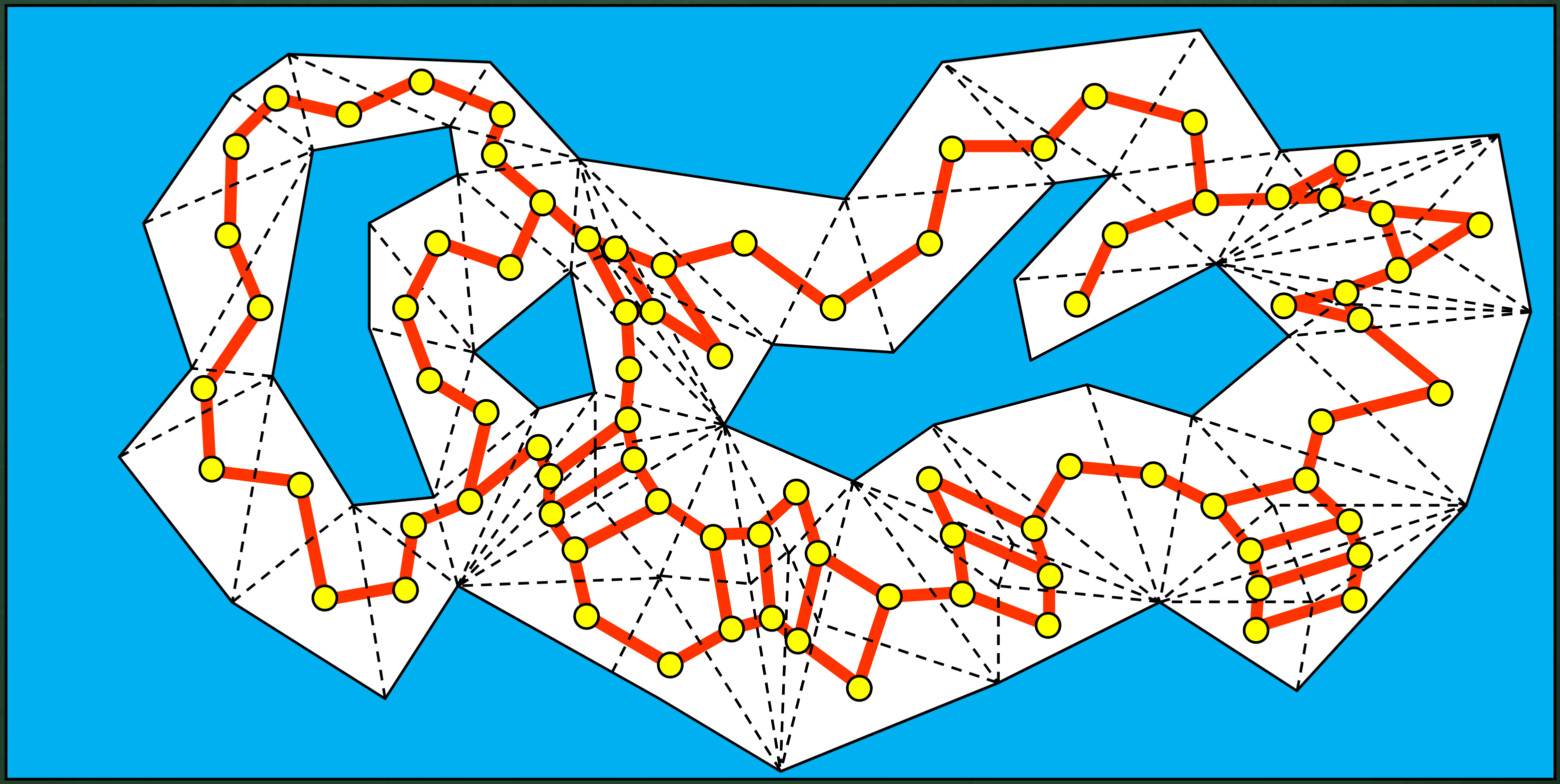
Recursively Decompose

- *One option is to ensure that each **triangle is small enough to be covered** by the robot's range:*



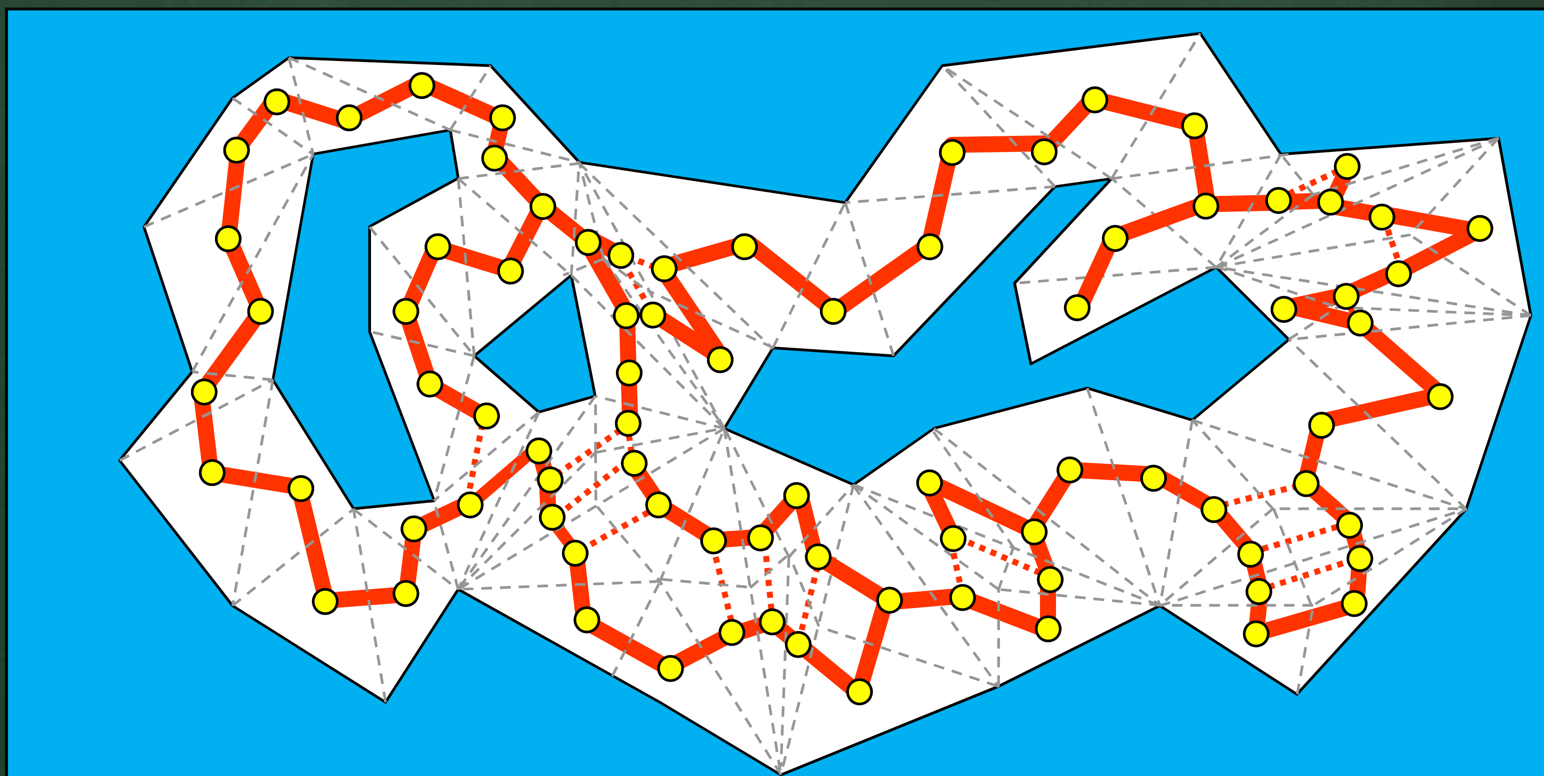
Limited Range Paths

- Again, form path from dual graph:
  - more loops now
  - cannot trim vertices now, only edges.



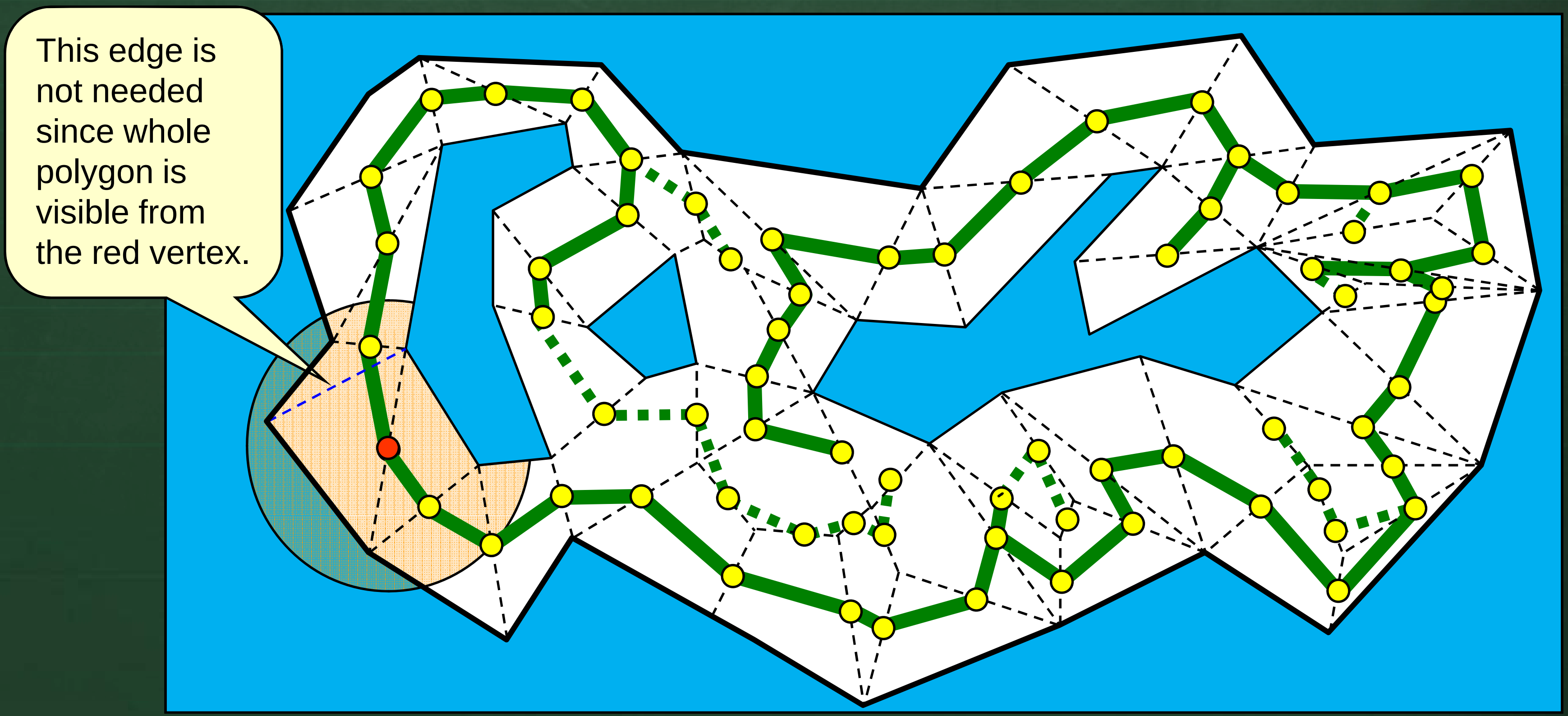
Refining Limited Range Paths

- May trim as many edges as possible, provided that the removal of the edge does not disconnect the graph.



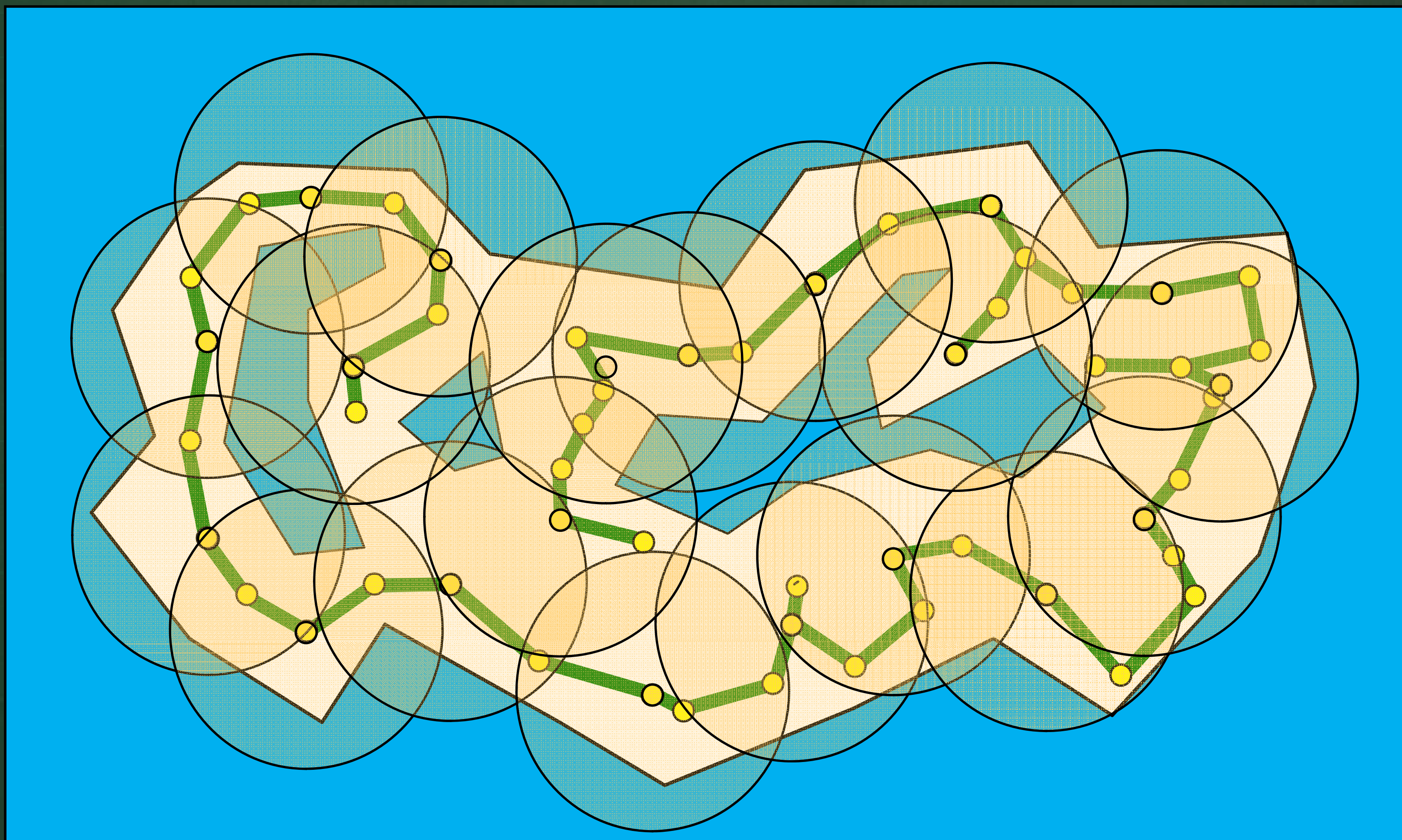
Convex Pieces

- Again, we can merge into convex polygons first, provided that the convex polygons are fully visible from each edge:



Limited Range Visibility Coverage

- Result is that entire area is covered:



Summary

- You should now understand:
  - How to compute paths that cover an environment
  - Different ways of covering an environment
  - How to compute a set of robot locations that see the entire environment
  - A simple way to search an environment with robots that have sensors with unlimited or limited range as well as omni-directional or limited direction.