
Arithmetic Flags and Instructions

Chapter 6

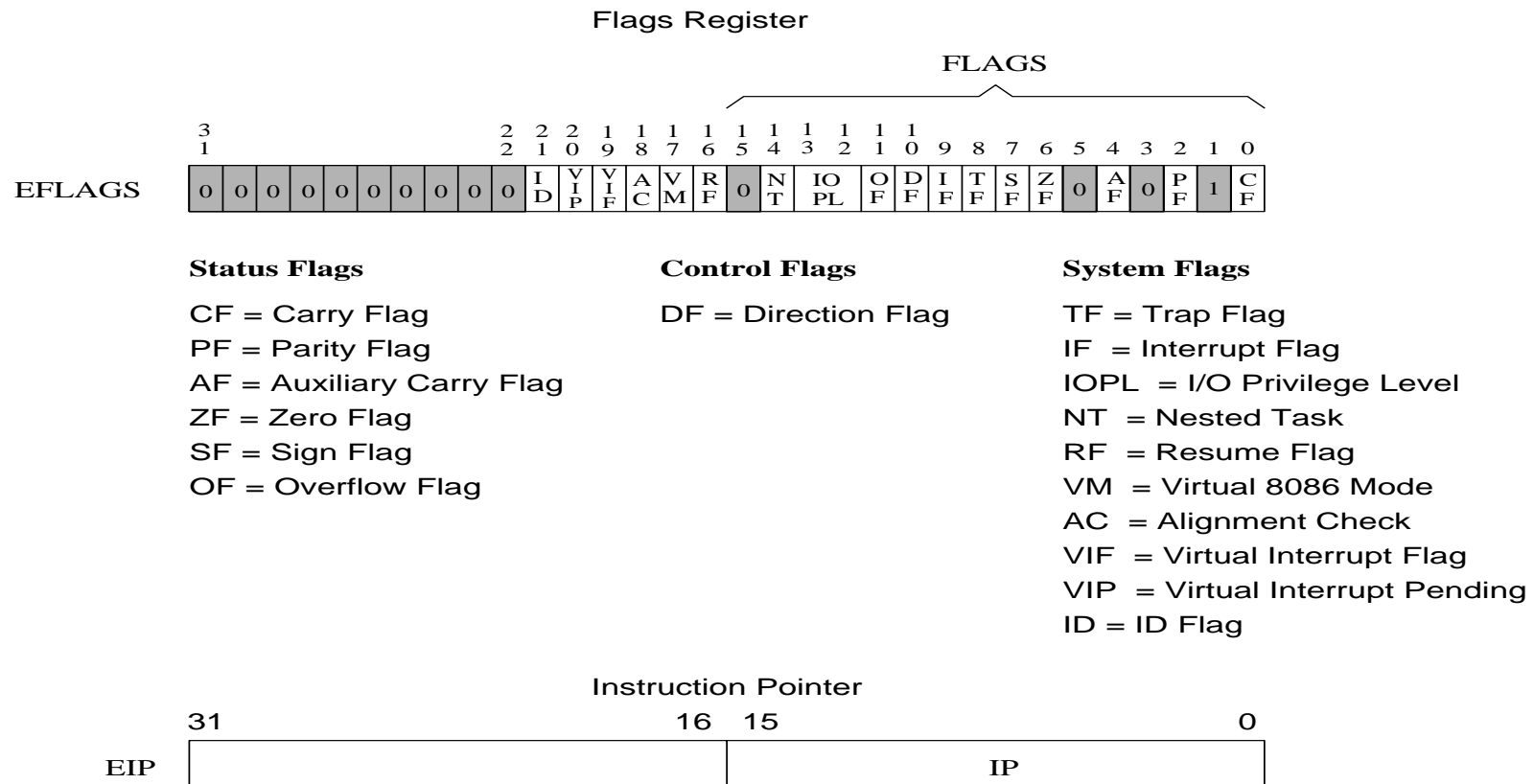
S. Dandamudi

Outline

- Status flags
 - * Zero flag
 - * Carry flag
 - * Overflow flag
 - * Sign flag
 - * Auxiliary flag
 - * Parity flag
- Arithmetic instructions
 - * Addition instructions
 - * Subtraction instructions
 - * Multiplication instructions
 - * Division instructions
- Application examples
 - * PutInt8
 - * GetInt8
- Multiword arithmetic
 - * Addition
 - * Subtraction
 - * Multiplication
 - * Division
- Performance: Multiword multiplication

Status Flags

- Six status flags monitor the outcome of arithmetic, logical, and related operations



Status Flags (cont'd)

- Status flags are updated to indicate certain properties of the result
 - * Example: If the result is zero, zero flag is set
- Once a flag is set, it remains in that state until another instruction that affects the flags is executed
- Not all instructions affect all status flags
 - * **add** and **sub** affect all six flags
 - * **inc** and **dec** affect all but the carry flag
 - * **mov**, **push**, and **pop** do not affect any flags

Status Flags (cont'd)

- Example

```
; initially, assume ZF = 0
mov     AL,55H    ; ZF is still zero
sub     AL,55H    ; result is 0
                        ; ZF is set (ZF = 1)
push    BX       ; ZF remains 1
mov     BX,AX    ; ZF remains 1
pop     DX       ; ZF remains 1
mov     CX,0     ; ZF remains 1
inc     CX       ; result is 1
                        ; ZF is cleared (ZF = 0)
```

Status Flags (cont'd)

- Zero Flag

- * Indicates zero result

- If the result is zero, $ZF = 1$
- Otherwise, $ZF = 0$

- * Zero can result in several ways (e.g. overflow)

<code>mov</code>	<code>AL, 0FH</code>		<code>mov</code>	<code>AX, 0FFFFH</code>		<code>mov</code>	<code>AX, 1</code>
<code>add</code>	<code>AL, 0F1H</code>		<code>inc</code>	<code>AX</code>		<code>dec</code>	<code>AX</code>

» All three examples result in zero result and set ZF

- * Related instructions

`jz` jump if zero (jump if $ZF = 1$)

`jnz` jump if not zero (jump if $ZF = 0$)

Status Flags (cont'd)

- Uses of zero flag
 - * Two main uses of zero flag
 - » Testing equality
 - Often used with **cmp** instruction
 - cmp char, '\$' ; ZF = 1 if char is \$**
 - cmp AX, BX**
 - » Counting to a preset value
 - Initialize a register with the count value
 - Decrement it using **dec** instruction
 - Use **jz/jnz** to transfer control

Status Flags (cont'd)

- Consider the following code

```
sum := 0
for (I = 1 to M)
  for (j = 1 to N)
    sum := sum + 1
  end for
end for
```

- Assembly code

```
sub    AX,AX ; AX := 0
mov    DX,M
outer_loop:
  mov    CX,N
inner_loop:
  inc    AX
  loop  inner_loop
  dec    DX
  jnz   outer_loop
exit_loops:
  mov    sum,AX
```


Status Flags (cont'd)

- Two observations

- * **loop** instruction is equivalent to

```
dec    CX
jnz    inner_loop
```

- » This two instruction sequence is more efficient than the **loop** instruction (takes less time to execute)
 - » **loop** instruction *does not affect* any flags!

- * This two instruction sequence is better than initializing CX to zero and executing

```
inc    CX
cmp    CX,N
jle    inner_loop
```

Status Flags (cont'd)

- Carry Flag

- * Records the fact that the result of an arithmetic operation on *unsigned* numbers is out of range
- * The carry flag is set in the following examples

<code>mov AL, 0FH</code>		<code>mov AX, 12AEH</code>
<code>add AL, 0F1H</code>		<code>sub AX, 12AFH</code>

- * Range of 8-, 16-, and 32-bit unsigned numbers

size	range
8 bits	0 to 255 ($2^8 - 1$)
16 bits	0 to 65,535 ($2^{16} - 1$)
32 bits	0 to 4,294,967,295 ($2^{32} - 1$)

Status Flags (cont'd)

- * Carry flag is not set by **inc** and **dec** instructions

» The carry flag is *not set* in the following examples

mov	AL, 0FFH		mov	AX, 0
inc	AL		dec	AX

- * Related instructions

jc jump if carry (jump if CF = 1)

jnc jump if no carry (jump if CF = 0)

- * Carry flag can be manipulated directly using

stc set carry flag (set CF to 1)

clc clear carry flag (clears CF to 0)

cmc complement carry flag (inverts CF value)

Status Flags (cont'd)

- Uses of carry flag
 - * To propagate carry/borrow in multiword addition/subtraction

1 ← carry from lower 32 bits

x	=	3710	26A8	1257	9AE7H
y	=	489B	A321	FE60	4213H
<hr/>					
		7FAB	C9CA	10B7	DCFAH

- * To detect overflow/underflow condition
 - » In the last example, carry out of leftmost bit indicates overflow
- * To test a bit using the shift/rotate instructions
 - » Bit shifted/rotated out is captured in the carry flag
 - » We can use **jc/jnc** to test whether this bit is 1 or 0

Status Flags (cont'd)

- Overflow flag
 - * Indicates out-of-range result on *signed* numbers
 - Signed number counterpart of the carry flag
 - * The following code sets the overflow flag but not the carry flag

```
mov    AL,72H    ; 72H = 114D
add    AL,0EH    ; 0EH = 14D
```

- * Range of 8-, 16-, and 32-bit signed numbers

size	range	
8 bits	– 128 to +127	2^7 to $(2^7 - 1)$
16 bits	– 32,768 to +32,767	2^{15} to $(2^{15} - 1)$
32 bits	–2,147,483,648 to +2,147,483,647	2^{31} to $(2^{31} - 1)$

Status Flags (cont'd)

- Signed or unsigned: How does the system know?
 - * The processor does not know the interpretation
 - * It sets carry and overflow under each interpretation

Unsigned interpretation

```
mov    AL, 72H
add    AL, 0EH
jc     overflow
no_overflow:
    (no overflow code here)
    ....
overflow:
    (overflow code here)
    ....
```

Signed interpretation

```
mov    AL, 72H
add    AL, 0EH
jo     overflow
no_overflow:
    (no overflow code here)
    ....
overflow:
    (overflow code here)
    ....
```

Status Flags (cont'd)

- * Related instructions

 - jo** jump if overflow (jump if OF = 1)

 - jno** jump if no overflow (jump if OF = 0)

- * There is a special software interrupt instruction

 - into** interrupt on overflow

Details on this instruction in Chapter 12

- Uses of overflow flag

 - * Main use

 - » To detect out-of-range result on signed numbers

Status Flags (cont'd)

- Sign flag

- * Indicates the sign of the result

- Useful only when dealing with signed numbers
- Simply a copy of the most significant bit of the result

- * Examples

mov AL, 15	mov AL, 15
add AL, 97	sub AL, 97
<i>clears</i> the sign flag as the result is 112 (or 0111000 in binary)	<i>sets</i> the sign flag as the result is -82 (or 10101110 in binary)

- * Related instructions

js jump if sign (jump if SF = 1)
jns jump if no sign (jump if SF = 0)

Status Flags (cont'd)

- Usage of sign flag
 - * To test the sign of the result
 - * Also useful to efficiently implement countdown loops

- Consider the count down loop:

```
for (i = M downto 0)
    <loop body>
end for
```

- If we don't use the **jns**, we need **cmp** as shown below:

```
cmp    CX, 0
j1    for_loop ]
```

The count down loop can be implemented as

```
        mov    CX, M
for_loop:
    <loop body>
    dec    CX
    [ jns    for_loop ]
```



Status Flags (cont'd)

- Auxiliary flag
 - * Indicates whether an operation produced a carry or borrow in the low-order 4 bits (nibble) of 8-, 16-, or 32-bit operands (i.e. operand size doesn't matter)
 - * Example

1 ← carry from lower 4 bits

<code>mov</code>	<code>AL, 43</code>	<code>43D = 0010 1011B</code>
<code>add</code>	<code>AL, 94</code>	<code>94D = 0101 1110B</code>
		<hr/>
		<code>137D = 1000 1001B</code>

» As there is a carry from the lower nibble, auxiliary flag is set

Status Flags (cont'd)

* Related instructions

- » No conditional jump instructions with this flag
- » Arithmetic operations on BCD numbers use this flag

aaa	ASCII adjust for addition
aas	ASCII adjust for subtraction
aam	ASCII adjust for multiplication
aad	ASCII adjust for division
daa	Decimal adjust for addition
das	Decimal adjust for subtraction

– Chapter 11 has more details on these instructions

* Usage

- » Main use is in performing arithmetic operations on BCD numbers

Status Flags (cont'd)

- Parity flag

- * Indicates even parity of the low 8 bits of the result
 - PF is set if the lower 8 bits contain even number 1 bits
 - For 16- and 32-bit values, only the least significant 8 bits are considered for computing parity value

- * Example

```
mov  AL, 53      53D = 0011 0101B
add  AL, 89      89D = 0101 1001B
                              
                    142D = 1000 1110B
```

» As the result has even number of 1 bits, parity flag is set

- * Related instructions

```
jp      jump on even parity (jump if PF = 1)
jnp     jump on odd parity (jump if PF = 0)
```

Status Flags (cont'd)

- * Usage of parity flag
 - » Useful in writing data encoding programs
 - » Example: Encodes the byte in AL (MSB is the parity bit)

```
parity_encode PROC
    shl AL
    jp parity_zero
    stc
    jmp move_parity_bit
parity_zero:
    clc
move_parity_bit:
    rcr AL
parity_encode ENDP
```

Arithmetic Instructions

- Pentium provides several arithmetic instructions that operate on 8-, 16- and 32-bit operands
 - » Addition: **add**, **adc**, **inc**
 - » Subtraction: **sub**, **sbb**, **dec**, **neg**, **cmp**
 - » Multiplication: **mul**, **imul**
 - » Division: **div**, **idiv**
 - » Related instructions: **cbw**, **cwd**, **cdq**, **cwde**, **movsx**, **movzx**
- * There are few other instructions such as **aaa**, **aas**, etc. that operate on decimal numbers
 - » See Chapter 11 for details

Arithmetic Instructions (cont'd)

- Addition instructions

- * Basic format

add destination, source

» Performs simple integer addition

destination := destination + source

- * Five operand combinations are possible

add register, register

add register, immediate

add memory, immediate

add register, memory

add memory, register

Arithmetic Instructions (cont'd)

- * Basic format

adc destination, source

» Performs integer addition with carry

destination := destination + source + CF

- * Useful in performing addition of long word arithmetic

- * The three carry flag manipulating instructions are useful

stc set carry flag (set CF to 1)

clc clear carry flag (clears CF to 0)

cmc complement carry flag (inverts CF value)

Arithmetic Instructions (cont'd)

- * The final instruction **inc** requires a single operand

inc destination

- » Performs increment operation

destination := destination + 1

- » The operand is treated as an unsigned number

- * Does not affect the carry flag

- » Other five status flags are updated

- * In general

inc BX

is better than

add BX,1

- » Both take same time but **inc** version takes less space

Arithmetic Instructions (cont'd)

- Subtraction instructions

sub destination, source

» Performs simple integer subtraction

destination := destination - source

sbb destination, source

» Performs integer subtraction with borrow

destination := destination - source - CF

dec destination

» Performs decrement operation

destination := destination - 1

Arithmetic Instructions (cont'd)

- Subtraction instructions (cont'd)

neg destination

» Performs sign reversal

destination := 0 - destination

» Useful in signed number manipulation

cmp destination, source

» Performs subtraction without updating **destination**

destination - source

» Updates all six status flags to record the attributes of the result

» The **cmp** instruction is typically followed by a conditional jump instruction

Arithmetic Instructions (cont'd)

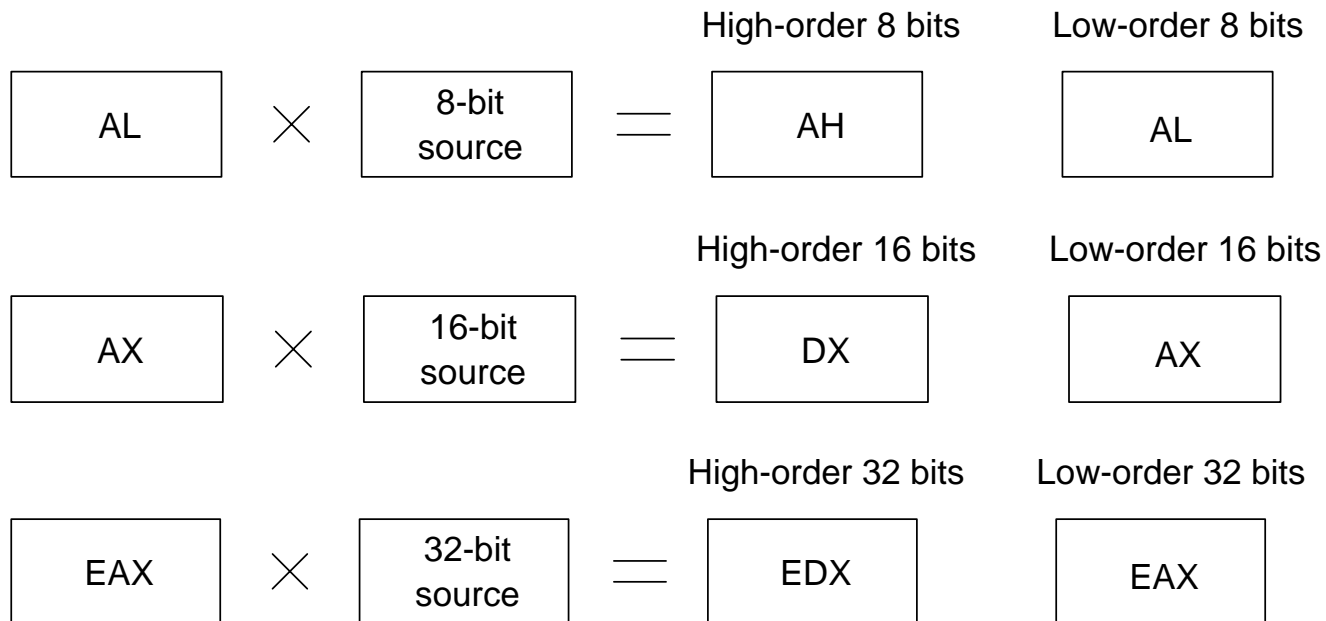
- Multiplication
 - * More complicated than **add/sub**
 - » Produces double-length results
 - E.g. Multiplying two 8 bit numbers produces a result that requires 16 bits
 - » Cannot use a single multiply instruction for signed and unsigned numbers
 - **add** and **sub** instructions work both on signed and unsigned numbers
 - For multiplication, we need separate instructions
 - mul** for unsigned numbers
 - imul** for signed numbers

Arithmetic Instructions (cont'd)

- Unsigned multiplication

`mul` **source**

» Depending on the **source** operand size, the location of the other source operand and destination are selected



Arithmetic Instructions (cont'd)

* Example

```
mov    AL, 10
mov    DL, 25
mul    DL
```

produces 250D in AX register (result fits in AL)

- The **imul** instruction can use the same syntax

» Also supports other formats

* Example

```
mov    DL, 0FFH    ; DL := -1
mov    AL, 0BEH    ; AL := -66
mul    DL
```

produces 66D in AX register (again, result fits in AL)

Arithmetic Instructions (cont'd)

- Division instruction

- * Even more complicated than multiplication

- » Produces two results

- Quotient

- Remainder

- » In multiplication, using a double-length register, there will not be any overflow

- In division, divide overflow is possible

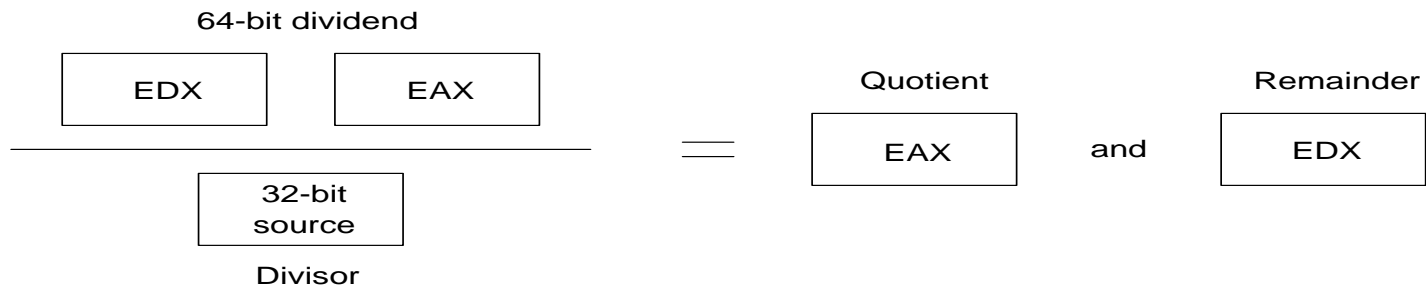
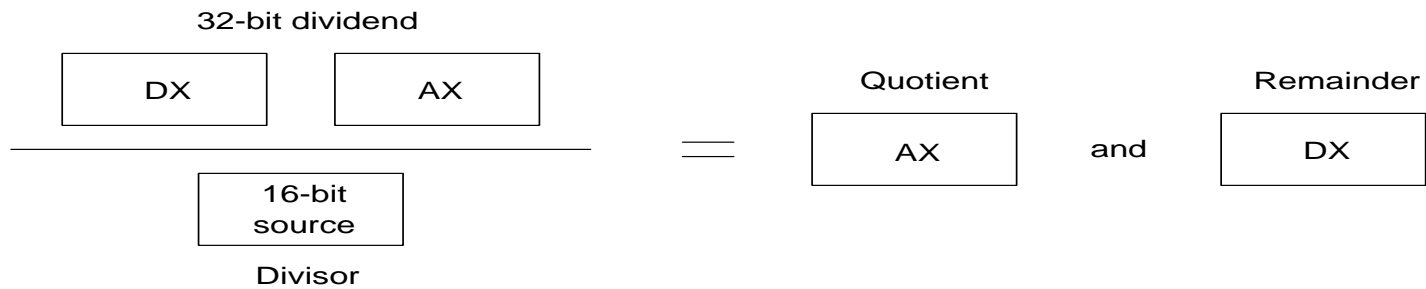
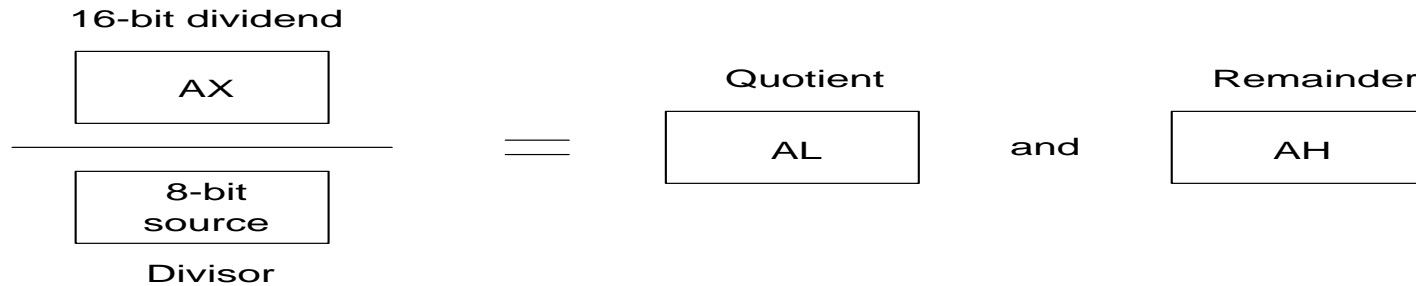
- Pentium provides a special software interrupt when a divide overflow occurs

- * Two instructions as in multiplication

- div** **source** for unsigned numbers

- idiv** **source** for signed numbers

Arithmetic Instructions (cont'd)



Arithmetic Instructions (cont'd)

- Example

```
mov    AX,00FBH    ; AX := 251D
mov    CL,0CH      ; CL := 12D
div    CL
```

produces 20D in AL and 11D as remainder in AH

- Example

```
sub    DX,DX       ; clear DX
mov    AX,141BH    ; AX := 5147D
mov    CX,012CH    ; CX := 300D
div    CX
```

produces 17D in AX and 47D as remainder in DX

Arithmetic Instructions (cont'd)

- Signed division requires some help
 - » We extended an unsigned 16 bit number to 32 bits by placing zeros in the upper 16 bits
 - » This will not work for signed numbers
 - To extend signed numbers, you have to copy the sign bit into those upper bit positions
- * Pentium provides three instructions in aiding sign extension
 - » All three take no operands
 - cbw** converts byte to word (extends AL into AH)
 - cwd** converts word to doubleword (extends AX into DX)
 - cdq** converts doubleword to quadword (extends EAX into EDX)

Arithmetic Instructions (cont'd)

- * Some additional related instructions

- » Sign extension

- cwde** converts word to doubleword
(extends AX into EAX)

- » Two move instructions

- movsx dest,src** (move sign-extended **src** to **dest**)

- movzx dest,src** (move zero-extended **src** to **dest**)

- » For both move instructions, **dest** has to be a register

- » The **src** operand can be in a register or memory

- If **src** is 8-bits, **dest** has to be either a 16 bit or 32 bit register

- If **src** is 16-bits, **dest** has to be a 32 bit register

Arithmetic Instructions (cont'd)

- Example

```
mov     AL,0A1H      ; AL := -95D
cbw                    ; AH = FFH
mov     CL,0CH       ; CL := 12D
idiv   CL
```

produces $-7D$ in AL and $-11D$ as remainder in AH

- Example

```
mov     AX,0EBE5     ; AX := -5147D
cwd                    ; DX := FFFFH
mov     CX,012CH     ; CX := 300D
idiv   CX
```

produces $-17D$ in AX and $-47D$ as remainder in DX

Application Examples

- **PutInt8** procedure

- * To display a number, repeatedly divide it by 10 and display the remainders obtained

	quotient	remainder
108/10	10	8
10/10	1	0
1/10	0	1

- * To display digits, they must be converted to their character form
 - » This means simply adding the ASCII code for zero (see line 24)

```
line 24:          add    AH, '0'
```

Application Examples (cont'd)

- **GetInt8** procedure

- * To read a number, read each digit character
 - » Convert to its numeric equivalent
 - » Multiply the running total by 10 and add this digit

Input digit	Numeric value (N)	Number := Number*10 + N
Initial value	--	0
'1'	1	$0 * 10 + 1 = 1$
'5'	5	$1 * 10 + 5 = 15$
'8'	8	$15 * 10 + 8 = 158$

Multiword Arithmetic

- Arithmetic operations (**add**, **sub**, **mul**, and **div**) work on 8-, 16-, or 32-bit operands
- Arithmetic on larger operands require multiword arithmetic software routines
- Addition/subtraction
 - * These two operations are straightforward to extend to larger operand sizes
 - * Need to use **adc/sbb** versions to include the carry generated by the previous group of bits
 - * Example addition on the next slide

Multiword Arithmetic (cont'd)

```
;-----  
;Adds two 64-bit numbers in EBX:EAX and EDX:ECX.  
;The result is returned in EBX:EAX.  
;Overflow/underflow conditions are indicated  
;by setting the carry flag.  
;Other registers are not disturbed.  
;-----
```

```
add64    PROC  
        add     EAX,ECX  
        adc     EBX,EDX  
        ret  
add64    ENDP
```


Multiword Arithmetic (cont'd)

- Multiplication
 - * We consider two algorithms
 - » Longhand multiplication
 - Uses the method that we are familiar with
 - Needs addition operations only
 - Examines each bit in the multiplier and adds the multiplicand if the multiplier bit is 1
 - Appropriate shifting is required
 - » Using the **mul** instruction
 - Chops the operand into 32-bit chunks and applies **mul** instruction
 - Similar to the addition example seen before

Multiword Arithmetic (cont'd)

* Longhand multiplication

→ Final 128-bit result in P:A

```
P := 0; count := 64
A := multiplier; B := multiplicand
while (count > 0)
    if (LSB of A = 1)
        then P := P+B
            CF := carry generated by P+B
        else CF := 0
    end if
    shift right CF:P:A by one bit position
    count := count-1
end while
```

Multiword Arithmetic (cont'd)

- * Using the **mul** instruction
 - » A 64-bit number is treated as two 32-bit numbers
 - A is considered as consisting of A1A0 (similarly B)
 - Left shift operation replaces zeros on the right

```
temp := A0 × B0
result := temp
temp := A1 × B0
temp := left shift temp
        by 32 bits
result := result + temp
```

```
temp := A0 × B1
temp := left shift temp
        by 32 bits
result := result + temp
temp := A1 × B1
temp := left shift temp
        by 32 bits
result := result + temp
```

Multiword Arithmetic (cont'd)

- Division
 - * To implement n-bit division (A by B), we need an additional n+1 bit register P
 - * Core part of the algorithm
 - » Test the sign of P
 - » if P is negative
 - left shift P:A by one bit position
 - $P := P+B$
 - » else
 - left shift P:A by one bit position
 - $P := P-B$

Multiword Arithmetic (cont'd)

Division Algorithm

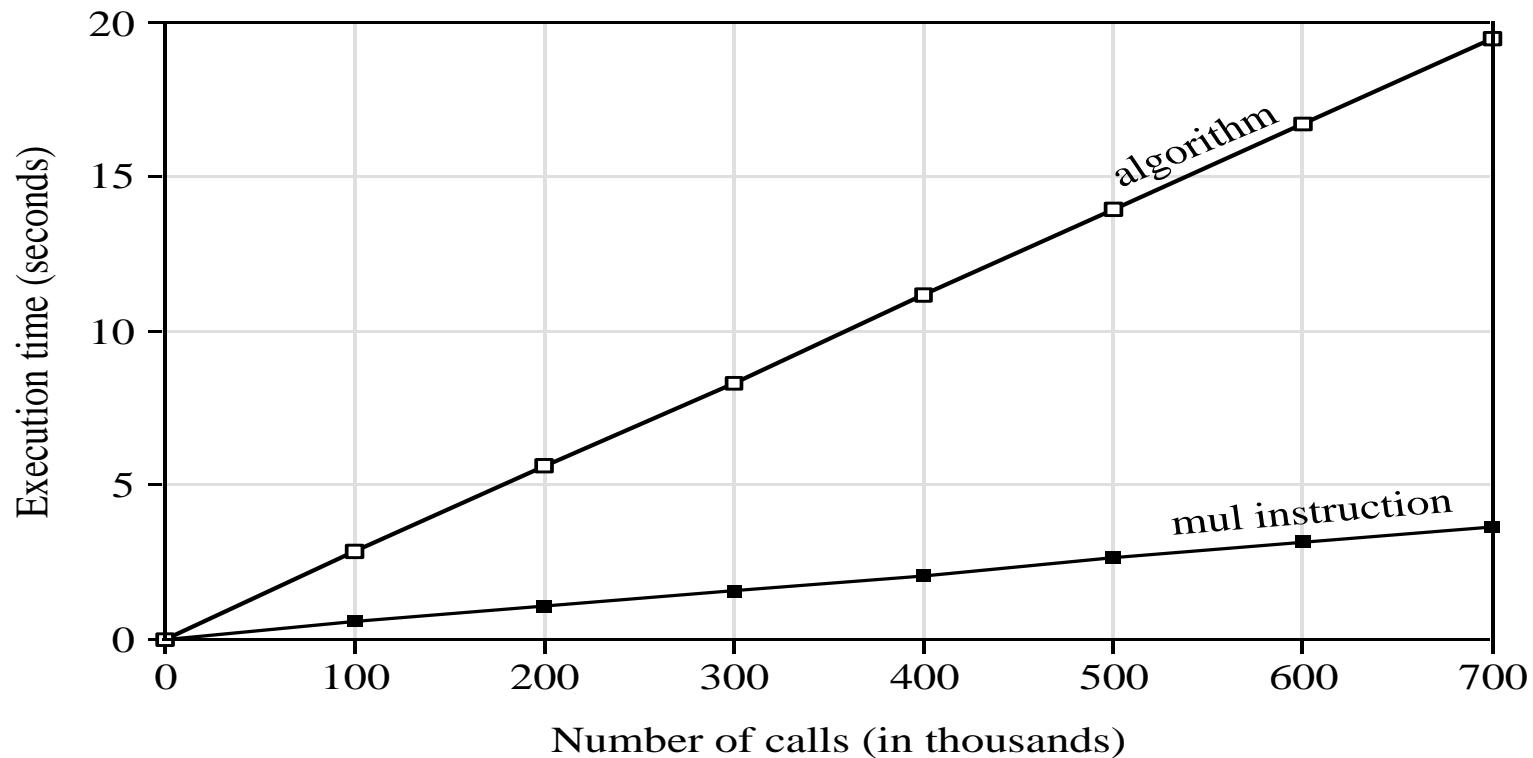
```
P := 0; count := 64
A := dividend
B := divisor
while (count > 0)
  if (P is negative)
  then shift left P:A
      by 1 bit position
      P := P+B
  else shift left P:A
      by 1 bit position
      P := P-B
  end if
```

A = quotient, P = remainder

```
if (P is negative)
then set low-order
      bit of A to 0
else set low-order
      bit of A to 1
end if
count := count-1
end while
if (P is negative)
  P := P+B
end if
```

Performance: Multiword Multiplication

- Longhand version versus **mul** version
 - * To multiply $2^{64}-1 \times 2^{64}-1$



Performance: Multiword Multiply (cont'd)

- Using **add** versus **mul** instruction
 - * Certain special cases of multiplication can be done by a series additions (e.g. power of 2 by shift operations)
 - * Example: Multiplication by 10
 - * Instead of using **mul** instruction, we can multiply by 10 using **add** instructions as follows (performs $AL \times 10$):

```
sub    AH, AH    ; AH := 0
mov    BX, AX    ; BX := x
add    AX, AX    ; AX := 2x
add    AX, AX    ; AX := 4x
add    AX, BX    ; AX := 5x
add    AX, AX    ; AX := 10x
```

Performance: Multiword Multiply (cont'd)

- Multiplication of $2^{32}-1$ by 10

