## Chapter 3

## User Authentication—Passwords, Biometrics and Alternatives

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## Chapter 3

## User Authentication-Passwords, Biometrics and Alternatives

Computer users regularly enter a username and password to access a local device or remote account. Authentication is the process of using supporting evidence to corroborate an asserted identity. In contrast, identification (recognition) establishes an identity from available information without an explicit identity having been asserted-such as picking out known criminals in a crowd, or finding who matches a given fingerprint; each crowd face is checked against a list of database faces for a potential match, or a given fingerprint is tested against a database of fingerprints. For identification, since the test is one-to-many, problem complexity grows with the number of potential candidates. Authentication involves a simpler one-to-one test; for an asserted username and fingerprint pair, a single test determines whether the pair matches a corresponding stored template.

Corroborating an asserted identity may be an end-goal (authentication), or a sub-goal towards the end-goal of authorization-determining whether a requested privilege or resource access should be granted to the requesting entity. For example, users may be asked to enter a password (for the account currently in use) to authorize installation or upgrading of operating system or application software.

This chapter is on user authentication-humans being authenticated by a computer system. Chapter 4 addresses machine-to-machine authentication and related cryptographic protocols. The main topics of focus herein are passwords, hardware-based tokens, and biometric authentication. We also discuss password managers, CAPTCHAs, graphical passwords, and background on entropy relevant to the security of user-chosen passwords.

### 3.1 Password authentication

Passwords provide basic user authentication. Each user authorized to use a system is assigned an account identified by a character string username (or numeric userid). To gain access ("log in") to their account, the user enters the username and a password. This pair is transmitted to the system. The system has stored sufficient information to test
whether the password matches the one expected for that userid. If so, access is granted.
A correct password does not ensure that whoever entered it is the authorized user. That would require a guarantee that no one other than the authorized user could ever possibly know, obtain, or guess the password-which is unrealistic. A correct match indicates knowledge of a fixed character string-or possibly a "lucky guess". But passwords remain useful as a (weak) means of authentication. We summarize their pros and cons later.

Storing hashes vs. Cleartext. To verify entered userid-password pairs, the system stores sufficient information in a password file $F$ with one row for each userid. Storing cleartext passwords $p_{i}$ in $F$ would risk directly exposing all $p_{i}$ if $F$ were stolen; system administrators and other insiders, including those able to access filesystem backups, would also directly have all passwords. Instead, each row of $F$ stores a pair (userid, $h_{i}$ ), where $h_{i}=H\left(p_{i}\right)$ is a password hash; $H$ is a publicly known one-way hash function (Chapter 2). The system then computes $h_{i}$ from the user-entered $p_{i}$ to test for a match.

PRE-COMPUTED DICTIONARY ATTACK. If password hashing alone is used as described above, an attacker may carry out the following pre-computed dictionary attack.

1. Construct a long list of candidate passwords, $w_{1}, \ldots, w_{t}$.
2. For each $w_{j}$, compute $h_{j}=H\left(w_{j}\right)$ and store a table $T$ of pairs $\left(h_{j}, w_{j}\right)$ sorted by $h_{j}$.
3. Steal the password file $F$ containing stored values $h_{i}=H\left(p_{i}\right)$.
4. "Look up" the password $p_{i}$ corresponding to a specifically targeted userid $u_{i}$ with password hash $h_{i}$ by checking whether $h_{i}$ appears in table $T$ as any value $h_{j}$; if so, the accompanying $w_{j}$ works as $p_{i}$. If instead the goal is to trawl (find passwords for arbitrary userids), sort $F$ 's rows by values $h_{i}$, then compare sorted tables $F$ and $T$ for matching hashes $h_{j}$ and $h_{i}$ representing $H\left(w_{j}\right)$ and $H\left(p_{i}\right)$; this may yield many matching pairs, and each accompanying $w_{j}$ will work as $u_{i}$ 's password $p_{i}$.

Exercise (Pre-computed dictionary). Using diagrams, illustrate the above attack.
$\ddagger$ Exercise (Morris worm dictionary). Describe the dictionary that the Internet worm of 1988 used. (Hint: [22], [53], [56], [54, pages 19-23]. This incident, mentioned again in Chapter 7, contributed to the rise of defensive password composition policies.)

TARGETED VS. TRAWLING SCOPE. The pre-computed attack above considered:

- a targeted attack specifically aimed at pre-identified users (often one); and
- a password trawling attack aiming to break into any account by trying many or all accounts. (Section 3.8 discusses related breadth-first attacks.)
Approaches to defeat password authentication. Password authentication can be defeated by several technical approaches, each targeted or trawling.

1. Online password guessing. Guesses are sent to the legitimate server (Section 3.2).
2. Offline password guessing. No per-guess online interaction is needed (Section 3.2).
3. Password capture attacks. An attacker intercepts or directly observes passwords by means such as: observing sticky-notes, shoulder-surfing or video-recording of entry, hardware or software keyloggers or other client-side malware, server-side interception, proxy or middle-person attacks, phishing and other social engineering, and pharming. Details of these methods are discussed in other chapters.


Figure 3.1: Password attacks. Attack labels match attacks in Figure 1.9 (Chapter 1).
4. Password interface bypass. The above three attacks are direct attacks on password authentication. In contrast, bypass attacks aim to defeat authentication mechanisms by avoiding their interfaces entirely, instead gaining unauthorized access by exploiting software vulnerabilities or design flaws (e.g., as discussed in Chapter 6).
5. Defeating recovery mechanisms. This is discussed in Section 3.3.
$\ddagger$ Exercise (Locating attacks on a network diagram). a) Locate the above password attack approaches on a network architecture diagram. b) Expand this to include the additional attacks noted in Figure 3.1 (the labels save space and simplify the end diagram).

PASSWORD COMPOSITION POLICIES AND "STRENGTH". To ease the burden of remembering passwords, many users choose (rather than strings of random characters) words found in common-language dictionaries. Since guessing attacks exploit this, many systems impose ${ }^{1}$ password composition policies with rules specifying minimum lengths (e.g., 8 or 10), and requiring password characters from, e.g., three (or perhaps all) LUDS categories: lowercase (L), uppercase (U), digits (D), special characters (S). Such passwords are said to be "stronger", but this term misleads in that such increased "complexity" provides no more protection against capture attacks, and improves outcomes (whether an attack succeeds or not) against only some guessing attacks. Users also predictably modify dictionary words, e.g., to begin with a capital, and end with a digit. More accurately, such passwords have higher resilience to (only) simple password-guessing attacks.

DISADVANTAGES OF PASSWORDS. Usability challenges multiply as the numbers of passwords that users must manage grows from just a few to tens or hundreds. Usability disadvantages include users being told, for example:

1. not to write their passwords down ("just memorize them");
2. to follow complex composition policies (with apparently arbitrary rules, some excluding commas, spaces and semi-colons while others insist on special characters);
3. not to reuse passwords across accounts;
4. to choose each password to be easy to remember but difficult for others to guess (this is meaningless for users not understanding how password-guessing attacks work);
5. to change passwords every $30-90$ days if password expiration policies are in use.
[^0]A further cognitive challenge often overlooked is password interference: the requirement to match passwords with accounts. Beyond these usability issues, security issues noted earlier include vulnerability to offline guessing, online guessing attacks (for predictable user-chosen passwords—Section 3.2), and password capture (cf. Figure 3.1).

Advantages of Passwords. Among offsetting advantages, passwords:

1. are simple, easy to learn, and already understood by all current computer users;
2. are "free" (requiring no extra hardware at the client or system/server);
3. require no extra physical device to carry;
4. allow relatively quick login, and password managers may help further (for smallkeyboard mobile devices, apps commonly store passwords);
5. are easy to change or recover if lost-electronic recovery is typically immediate with no physical travel needed or delay awaiting physical shipment (cf. Section 3.3);
6. have well-understood failure modes (forgetful users learn to write passwords down somewhere safe);
7. require no trust in a new third party (in contrast, public-key certificates, per Chapter 8 , require trust in organizations beyond either the client or server organization);
8. are easily delegated (e.g., to a spouse or secretary while on vacation), an underrated benefit despite the security drawback that retracting delegation is rarely done.
Passwords remain the dominant means of Internet user authentication. No alternative has displaced them to date. One might conclude that the advantages outweigh the disadvantages, or that their historical position as a default authentication method, provides strong inertia. To displace an incumbent, often a new technology must be not just marginally but substantially better due to inertia, universal support enjoyed by an incumbent, and interoperability with existing systems. About cost, passwords are "free" only if no costs are charged for usability or user memory. That cost is small for one password, but much larger for remembering hundreds of passwords and which one goes with each account.

### 3.2 Password-guessing strategies and defenses

Password-guessing attacks fall into two categories so distinct that using the term "password guessing" for both can be more confusing than helpful. This will be clear shortly.

ONLINE PASSWORD GUESSING AND RATE-LIMITING. An online guessing attack can be mounted against any publicly reachable password-protected server. A human attacker or automated program is assumed to know one or more valid userids; these are typically easily obtained. Userid-password pairs, with password guesses, are submitted sequentially to the legitimate server, which conveniently indicates whether the attempt is correct or not-access is granted or denied. An obvious defensive tactic, for sites that care at all about security, is to rate-limit or throttle guesses across fixed time windowsenforcing a maximum number of incorrect login attempts per account. This may "lock
out" legitimate users whose accounts are attacked, ${ }^{2}$ a drawback that can be ameliorated by account recovery methods (Section 3.3). A variation is to increase delays, e.g., doubling system response time after successive incorrect login: $1 \mathrm{~s}, 2 \mathrm{~s}, 4 \mathrm{~s}$, and so on.

OfFLINE PASSWORD GUESSING. In offline guessing attacks (e.g., by pre-computed dictionary per page 57, or on-the-fly below), it is assumed that an attacker has somehow acquired the system password hash file. (Chapter 5 discusses how Unix-based systems store password hashes in /etc/passwd and related files. Password hashing is more common than reversible encryption, as that also requires a means to protect the encryption key itself.) This indeed occurs often in practice, but is a significant requirement absent for online guessing attacks. The hash file provides verifiable text (Chapter 4), i.e., data allowing a test of correctness of password guesses without contacting the legitimate server. Consequently, the number of offline guesses that can be made over a fixed time period is limited only by the computational resources that an attacker can harness; in contrast for online guessing, even without rate-limiting, the number of guesses is limited by the online server's computing and bandwidth capacity.

Iterated hashing (password stretching). Offline password guessing attacks can be slowed down using a tactic called iterated hashing (or password stretching). Ideally this defense is combined with salting (below). The idea is that after hashing a password once with hash function $H$, rather than storing $H\left(p_{i}\right)$, the result is itself hashed again, continuing likewise $d$ times, finally storing the $d$-fold hash $H\left(\ldots H\left(H\left(p_{i}\right)\right) \ldots\right)$, denoted $H^{d}\left(p_{i}\right)$. This increases the hashing time by a factor of $d$, for both the legitimate server (typically once per login) and each attacker guess. Practical values of $d$ are limited by the constraint that the legitimate server must also compute the iterated hash. A value $d=1000$ slows attacks by a factor of 1000 , and $d$ can be adjusted upward as computing power increases, e.g., due to advances in hardware speeds. ${ }^{3}$

Password salting. To combat dictionary attacks (above), common practice is to salt passwords before hashing. For userid $u_{i}$, on registration of each password $p_{i}$, rather than storing $h_{i}=H\left(p_{i}\right)$, the system selects, e.g., for $t \geq 64$, a random $t$-bit value $s_{i}$ as salt, and stores ( $u_{i}, s_{i}, H\left(p_{i}, s_{i}\right)$ ) with $p_{i}, s_{i}$ concatenated before hashing. Thus the password is altered by the salt in a deterministic way before hashing, with $s_{i}$ stored cleartext in the record to enable verification. For trawling attacks, the above dictionary attack using a precomputed table is now harder by a factor of $2^{t}$ in both computation (work) and storage-a table entry is needed for each possible value $s_{i}$. For attacks on a targeted userid, if the salt value $s_{i}$ is available to an insider or read from a stolen file $F$, the salt does not increase the time-cost of on-the-fly attacks where candidate passwords are hashed in real time vs. using pre-computed tables of hashes (above); confusingly, both are called dictionary attacks.

A bonus of salting is that two users who happen to choose the same password, will almost certainly have different password hashes in the system hash file. A salt value $s_{i}$ may also combine a global system salt, and a user-specific salt (including, e.g., the userid).

Pepper (Secret salt). A secret salt (sometimes called pepper) is like a regular

[^1]salt, but not stored. The motivation is to slow down attacks, by a method different than iterated hashing but with similar effect. When user $u_{i}$ selects a new password $p_{i}$, the system chooses a random value $r_{i}, 1 \leq r_{i} \leq R$; stores the secret-salted hash $H\left(p_{i}, r_{i}\right)$; and then erases $r_{i}$. To later verify a password for account $u_{i}$, the system sequentially tries all values $r^{*}=r_{i}$ in a deterministic order (e.g., sequentially, starting at a random value in $[1, R]$, wrapping around from $R$ to 1 ). For each $r^{*}$ it computes $H\left(p_{i}, r^{*}\right)$ and tests for a match with the stored value $H\left(p_{i}, r_{i}\right)$. For a correct $p_{i}$, one expects a successful match on average (i.e., with $50 \%$ probability) after testing half the values $r^{*}$, so if $R$ is 20 bits, one expects on average a slow-down by a factor $2^{19}$. Pepper can be combined with regular salt as $H\left(p_{i}, s_{i}, r_{i}\right)$, and with iterated hashing. (Aside: if the values $r^{*}$ are tested beginning at a fixed point such as 0 , timing data might leak information about the value of $r_{i}$.)

Specialized password-hashing functions. General crypto hash functions $H$ from the 1990s like MD5 and SHA-1 were designed to run as fast as possible. This also helps offline guessing attacks, wherein hash function computation is the main work; relatively modest custom processors can exceed billions of MD5 hashes per second. As attackers improved offline guessing attacks by leveraging tools such as Graphics Processing Units (GPUs), parallel computation, and integrated circuit technology called FPGAs (field-programmable gate arrays), the idea of specialized password-hashing functions to slow down such attacks arose. This led to the international Password Hashing Competition (PHC, 2013-2015), with winner Argon2 now preferred; prior algorithms were bcrypt and scrypt. Specialized hash functions called key derivation functions (KDFs) are also used to derive encryption keys from passwords. As an older example, PBKDF2 (password-based KDF number 2) takes as inputs ( $p_{i}, s_{i}, d, L$ ) -a password, salt, iteration count, and desired bitlength for the resulting output to be used as a crypto key.

Example (GPU hashing). GPUs are particularly well-suited to hash functions such as MD5 and SHA-1, with cost-efficient performance from many inexpensive custom cores. For example, the circa-2012 Tesla C2070 GPU has 14 streaming multiprocessors (SMs), each with 32 computing cores, for 448 cores in one GPU. Machines may have, e.g., four GPUs. As a result, password-hashing functions are now designed to be "GPU-unfriendly".

SyStem-assigned passwords and brute-force guessing. Some systems use system-assigned passwords. ${ }^{4}$ The difficulty of guessing passwords is maximized by selecting each password character randomly and independently. An $n$-character password chosen from an alphabet of $b$ characters then results in $b^{n}$ possible passwords, i.e., a password space of size $b^{n}$. On such systems, there is no guessing strategy better than bruteforce guessing: simply guessing sequentially using any enumeration (complete listing in any order) of the password space. The probability of success is $100 \%$ after $b^{n}$ guesses, with success expected on average (i.e., with $50 \%$ probability) after $b^{n} / 2$ guesses. If the passwords need not be a full $n$ characters, a common attack strategy would first try all one-character passwords, then all two-character passwords, and so on. System-assigned passwords are little used today. Their higher security comes with poor usability-humans

[^2]are unable to manually remember large numbers of random strings for different accounts, even without password expiration policies (below). Random passwords are more plausible (usable) when password manager tools are used (Section 3.6).

Probability of guessing success. Simple formulas giving the probability that system-assigned passwords are guessed can help inform us how to measure vulnerability. (For user-chosen passwords, these simple formulas fail, and partial-guessing metrics per Section 3.8 are used.) The baseline idea is to consider the probability that a password is guessed over a fixed period (e.g., 3 months or one year). A standard might allow maximum guessing probabilities of 1 in $2^{10}$ and 1 in $2^{20}$, respectively, for Level 1 (low security) and Level 2 (higher security) applications. The parameters of interest are:

- $G$, the number of guesses the attacker can make per unit time;
- $T$, the number of units of time per guessing period under consideration;
- $R=b^{n}$, the size of the password space (naive case of equiprobable passwords).

Here $b$ is the number of characters in the password alphabet, and $n$ is the password length. Assume that password guesses can be verified by online or offline attack. Then the probability $q$ that the password is guessed by the end of the period equals the proportion of the password space that an attacker can cover. If $G T>R$ then $q=1.0$, and otherwise

$$
\begin{equation*}
q=G T / R \text { for } G T \leq R \tag{3.1}
\end{equation*}
$$

Passwords might be changed at the end of a period, e.g., due to password expiration policies. If new passwords are independent of the old, the guessing probability per period is independent, but cumulatively increases with the number of guessing periods.

Example (Offline guessing). For concreteness, consider $T=1$ year ( $3.154 \times 10^{7} \mathrm{~s}$ ); truly random system-assigned passwords of length $n=10$ from an alphabet of $b=95$ printable characters yielding $R=b^{n}=95^{10}=6 \times 10^{19}$; and $G=100$ billion guesses per second (this would model an attack with a relatively modest number of modern GPUs, but otherwise favorable offline attack conditions assuming a password hash file obtained, a fast hash function like MD5, and neither iterated hashing nor secret salts). A modeling condition favoring the defender is the assumption of system-assigned passwords, which have immensely better guess-resistance than user-chosen passwords (below). Given these model strengths and weaknesses, what do the numbers reveal?

$$
\begin{equation*}
q=G T / R=\left(10^{11}\right)\left(3.154 \times 10^{7}\right) / 6\left(10^{19}\right)=0.05257 \tag{3.2}
\end{equation*}
$$

Oops! A success probability over $5 \%$ far exceeds both $2^{-10}$ and $2^{-20}$ from above. These conditions are too favorable for an attacker; a better defensive stance is needed.
$\ddagger$ Exercise (Password expiration/aging). Password expiration policies require users to change passwords regularly, e.g., every 30 or 90 days. Do such policies improve security? List what types of attacks they stop, and fail to stop. (Hint: [14], [62].)

LOWER BOUND ON LENGTH. Equation (3.1) can be rearranged to dictate a lower bound on password length, if other parameters are fixed. For example, if security policy specifies an upper bound on probability $q$, for a fixed guessing period $T$ and password alphabet of $b$ characters, we can determine the value $n$ required (from $R=b^{n}$ ) if we have
a reliable upper bound estimate for $G$, since from $R=b^{n}$ and (3.1) we have:

$$
\begin{equation*}
n=\lg (R) / \lg (b) \text { where } R=G T / q \text {. } \tag{3.3}
\end{equation*}
$$

Alternatively, to model an online attack, (3.1) can determine what degree of rate-limiting suffices for a desired $q$, from $G=q R / T$.

USER-CHOSEN PASSWORDS AND SKEWED DISTRIBUTIONS. Many systems today allow user-chosen passwords, constrained by password composition policies and (as discussed below) password denylists and other heuristics. Studies show that the distribution of user-chosen passwords is highly skewed: some passwords are much more popular than others. Figure 3.2 illustrates this situation. Attackers tailor their guessing strategies by trying more popular (higher estimated probability) passwords first. While originally the term dictionary attack loosely implied leveraging words in common-language dictionaries, it now often refers to using ordered lists of password candidates established by heuristic means (e.g., based on empirical password databases including huge lists published after compromises), often with on-the-fly computation of hashes (cf. p.60).

PASSWORD DENYLISTS AND PROACTIVE PASSWORD CRACKING. Due to the phenomenon of skewed password distributions, another simple defense against (especially online) password-guessing attacks is denylisting of passwords. This involves composing lists of the most-popular passwords, e.g., observed from available password distributions publicly available or within an enterprise organization. These denylists, originally based on a modified regular-language dictionary, need not be huge-e.g., as short as $10^{4}$ to $10^{6}$ entries. The idea then, sometimes also called proactive password checking, is to disallow any password, at the time a user tries to select it, if it appears on the denylist. A related idea
(a) What we want: randomly distributed passwords

(b) What we get: predictable clustering, highly skewed distribution



Figure 3.2: Password distributions (illustrative). Ideally, chosen passwords are unique $(y=1)$ with most unchosen $(y=0)$. Diameter represents frequency a password is chosen.
is for a system to try to "crack" its own users' passwords using only the password hashes and background computing cycles, in variations of dictionary attacks described earlier; account owners of cracked passwords are sent email notices to change their passwords because they were too easily found. ${ }^{5}$
$\ddagger$ Exercise (Heuristic password-cracking tools). (a) Look up and experiment with common password-cracking tools such as JohnTheRipper and oclHashcat. (b) Do they use pre-computed tables, or hash password guesses on-the-fly? Discuss. (c) Explain what mangling rules are. (Hint: [57].)

Login passwords vs. passkeys. Recalling KDFs (above), passwords may be used to derive cryptographic keys, e.g., for file encryption. Such password-derived encryption keys (passkeys) are subject to offline guessing attacks and require high guessingresistance. For site login passwords, complex composition policies are now generally recognized as a poor choice, imposing usability burdens without necessarily improving security outcomes; alternatives such as rate-limiting, denylisting, salting and iterated hashing appear preferable (Table 3.1). In contrast, for passkeys, complex passwords are prudent; memory aids include use of passphrases, the first letters of words in relatively long sentences, and storing written passwords in a safe place. Note that in contrast to site passwords, where "password recovery" is a misnomer (Section 3.3), recovering a forgotten password itself is a requirement for passkeys-consider a passkey used to encrypt a lifetime of family photographs. Users themselves are often responsible for managing their own password recovery mechanism for passkeys (some realize this only too late).

| Defensive measure | Primary attack addressed | Notes |
| :--- | :--- | :--- |
| rate-limiting | online guessing | some methods result in user lockout |
| denylisting | online guessing | disallows worst passwords |
| salt | pre-computed dictionary | increases cost of generic attacks |
| iterated hashing | offline guessing | combine with salting |
| pepper | offline guessing | alternative to iterated hashing |
| MAC on password | offline guessing | stolen hash file no longer useful |

Table 3.1: Defenses against web-login password attacks. For offline attacks, a file of password hashes is also needed and thus one may assume that salts are also known.

Example (Password management: NIST SP 800-63B). U.S. government password guidelines were substantially revised in 2017. They include: mandating use of password denylists to rule out common, highly predictable, or previously compromised passwords; mandating rate-limiting to throttle online guessing; recommending against composition rules, e.g., required combinations of lowercase, uppercase, digits and special characters; recommending against password expiration, but mandating password change upon evidence of compromise; mandating secure password storage methods (salt of at least 32 bits, hashing, suitable hash iteration counts, e.g., cost-equivalent to 10,000 iterations for PBKDF2); recommending a further secret key hash (MAC) and if so mandating that the key be stored separately (e.g., in a hardware security module/HSM).

[^3]$\ddagger$ Exercise (Password guidelines: others). Compare the revised U.S. guidelines above [29, Part B] to those of governments of: (i) U.K., (ii) Germany, (iii) Canada, (iv) Australia.
$\ddagger$ Exercise (Password-guessing defenses). An expanded version of Table 3.1 includes password composition rules (length and character-set requirements), password expiration, and password meters. (a) Discuss the pros and cons of these additional measures, including usability costs. (b) Discuss differences in guessing-resistance needed for passwords to resist online guessing vs. offline guessing attacks. (Hint: [24].)

### 3.3 Account recovery and secret questions

Password-based authentication inevitably leads to forgotten passwords. Since not all users write them down in a safe place for retrieval, some means of password recovery is essential. Site (account) authentication passwords are rarely literally "recovered", as best practice avoids storing cleartext passwords at servers. Rather, what is typically recovered is access to password-protected accounts, by some password reset method.

RECOVERY PASSWORDS AND RECOVERY LINKS. A common reset method is to send to users, using a recovery email address set up during registration, a temporary password or web page link that serves as an authenticator. On following the link or entering the temporary code, the user is prompted to immediately create a new password. Here for obvious reasons, registering the new password does not require authorization by entering an existing password (as typically required for changing passwords); the temporary capability plays that role.

Loss of primary email password (CODES SENT TO TELECOM DEVICE). A complication arises if the forgotten password is for a primary email account itself. One solution is to register a secondary email address and proceed as above. An alternative is to pre-register an independent device or channel, most commonly by a phone number (mobile, or wireline with text-to-voice conversion) to which a one-time recovery code is sent by text message. Note: a compromised primary email account may be used to compromise all other accounts that use that email address for password recovery.

Question-based recovery. Another account-recovery method to address forgotten passwords is secret questions, also called challenge questions. (The term personal knowledge questions is also used, but risks confusion with a type of targeted attack relying on personal knowledge.) Typically, secret questions are not literally questions that are secret, but rather secrets cued by user-selectable questions. On account registration, a user provides answers to a selected subset of questions. On later forgetting a password, correctly answering questions allows the user to re-establish a new account password.

Usability aspects. The idea is that using questions to cue information already in a user's long-term memory is an easier memory task than remembering text passwords. (Cued recall is discussed further in Section 3.6.) However, recovery by such challenge questions fails surprisingly often in practice, including because:

1. recovery may be long removed in time from when answers are set;
2. answers may be non-unique or change over time (e.g., favorite movie);
3. some users register false answers but forget this, or the false answers themselves. ${ }^{6}$

Security aspects. Challenge questions are at best "weak" secrets-the answer spaces are often small (pro baseball team, spouse's first name) or highly skewed by popularity (favorite food or city), making statistical guessing attacks easy. For targeted attacks, some answers are easily looked up online (high school attended). User-created questions, as allowed in some systems, are also notoriously bad (e.g., favorite color). Trying to salvage security by requiring answers to more questions reduces efficiency, and increases rejection of legitimate users due to incorrect answers. The problem remains: the answers are often not secret or too easily guessed. If more easily guessed than the primary password, this introduces a weak link as a new attack vector. Recovery questions then violate principle P13 (DEFENSE-IN-DEPTH); a minor mitigation is to limit the validity period of recovery codes or links.

Summary. Secret questions are poor both in security (easier to guess than userchosen passwords) and reliability (recovery fails more often than for alternatives). In addition, secret answers are commonly stored plaintext (not hashed per best practice for passwords) to allow case-insensitive matching and variations in spacing; this leaves the answers vulnerable to server break-in, which a large company then apologizes for as an "entirely unforeseeable" event. They then ask you to change your mother's maiden name-in their own system, and every other system where that question is used. (This may however be easier than changing your fingerprints when biometric data is similarly compromised. All of a sudden, regular passwords start to look not so bad, despite many flaws!) Overall, the general consensus is that secret questions are best abandoned; any use should be accompanied by additional authenticators, e.g., a link sent to an email account on record, or a one-time password texted to a registered mobile phone.
$\ddagger$ Example (Password Reset Attack). Password reset processes that rely on secret questions or SMS codes may be vulnerable to an interleaving attack. The attacker requires control of a web site (site-A), and a way to get user victims to visit it, but need not intercept Internet or phone messages. Site-A offers download of free resources/files, requiring users to register first. The registration solicits information such as email address (if the goal is to take over that email address), or SMS/text phone contact details (see below); the latter may be solicited ("for confirmation codes") as the user attempts to download a resource. The user visits site-A and creates a new account as noted. In parallel, the attacker requests password reset on an account of the same victim at a service provider site-B (often the userid is the same email address)-an email provider or non-email account. Site-B, as part of its reset process, asks secret questions before allowing password reset; these are forwarded by the attack program to the victim on site-A, positioned as part of site-A's registration. Answers are forwarded to site-B and authorize the attacker to set a new account password on site-B. If site-B's reset process involves sending one-time codes to the contact number on record, the attacker solicits such codes from the victim, positioned as part of registration or resource download from site-A. If site-B sends CAPTCHA challenges (Section 3.7) to counter automated attacks, they are similarly relayed to the victim on

[^4]site-A. The attack requires synchronization and seems complicated, but has been demonstrated on a number of major real-world services. The attack fails for sites sending reset messages to email addresses on record, but resets relying on secret questions or SMS codes are common, e.g., to address users having lost access to recovery email accounts, or when the account host is itself an email provider and users lack alternate email.

### 3.4 One-time password generators and hardware tokens

A major security issue with ordinary passwords is their static nature. If observed and captured by a passive attacker (eavesdropper), simple replay of the password defeats security. A step forward is one-time passwords (OTPs)_passwords valid for one use only. A challenge is how to pre-share lists of one-time passwords between the party to be authenticated (claimant) and the verifier. For electronic account access, some banks give customers paper lists of passwords to be used once each (then crossed off); the server keeps corresponding records for verification. Another method is to use one-way hash functions to generate sequences of one-time passwords from a seed (Lamport, below).

OTPS RECEIVED BY MOBILE. As Section 3.3 noted, mobile phones may be used as an independent channel for one-time codes via "text" or SMS (Short Message Service) messages. These OTPs are generated system-side and sent to the user's number on record. Beyond use for account recovery, they can serve as a (stand-alone) password alternative; or as a "what you have" second factor (Fig. 3.5, p.70). In all cases, it should be verified that the phone number is bound to a physical phone (versus, e.g., a voice-over-IP number).

SIM SWAP ATTACK. Sending OTPs to a mobile phone opens a new attack vector: by social engineering, an attacker, asserting a lost or stolen phone, tricks a mobile phone provider into transferring a victim's mobile number to a new subscriber identity module (SIM), the in-phone chip-card by which phone numbers are mapped to mobile phones. Thereafter, SMS OTPs intended for the legitimate user (victim) go to the attacker's phone.

OTPS FROM LAMPORT HASH CHAINS. Starting with a random secret (seed) w, user $A$ can authenticate to server $B$ using a sequence of one-time passwords as follows (Fig. 3.3). $H$ is a one-way hash function (Chapter 2) and $t$ is an integer (e.g., $t=100$ ). Let a hash chain of order $t$ be the sequence: $w, H(w), H(H(w)), H^{3}(w), \ldots, H^{t}(w) . H^{t}$ means


Figure 3.3: Lamport hash chain. Illustrated with $t=100$ for session $i=76(t-i=24)$. Setup value $h_{0}$ must initially be transferred over a secure channel and associated with $A$.
$t$ nested iterations. The elements in the sequence are used once each in the order:

$$
\begin{equation*}
h_{1}=H^{99}(w), h_{2}=H^{98}(w), \ldots, h_{98}=H(H(w)), h_{99}=H(w), h_{100}=w \tag{3.4}
\end{equation*}
$$

For $1 \leq i \leq 100$ the password for session $i$ will be $h_{i}=H^{t-i}(w)$. As set-up, $A$ sends as a shared secret to $B$, the value $h_{0}=H^{100}(w)$, over a channel ensuring also data origin authenticity (so $B$ is sure it is from $A$, unaltered); $B$ stores $h_{0}$ as the next-verification value $v$. Both parties set $i=1$. Now to authenticate in session $i$ (for $t$ sessions, $1 \leq i \leq t$ ), $A$ computes the next value $h_{i}$ in the chain and sends to $B$ : $\left(\mathrm{id}_{A}, i, h_{i}\right)$. (The notation is such that the values are used in reverse order, most-iterated first.) $B$ takes the received value $h_{i}$, hashes it once to $H\left(h_{i}\right)$, checks for equality to the stored value $v$ (which is $h_{i-1}$ ), and that the $i$ received is as expected. Login is allowed only if both checks succeed; $B$ replaces $v$ by the received $h_{i}$ (for the next authentication), and $i$ is incremented. $A$ can thus authenticate to $B$ on $t$ occasions with distinct passwords, and then the set-up is renewed with $A$ choosing a new $w$ (one-time passwords must not be reused). Note that for each session, $A$ provides evidence she knows $w$, by demonstrating knowledge of some number of iterated hashes of $w$; and even if an attacker intercepts the transferred authentication value, that value is not useful for another login (due to the one-way property of $H$ ).
$\ddagger$ Example (Pre-play attack on OTPs). OTP schemes can be attacked by capturing one-time passwords and using them before the system receives the value from the legitimate user. Such attacks have been reported-e.g., an attacker socially engineers a user into revealing its next five banking passwords from their one-time password sheet, "to help with system testing". Thus even with OTPs, care should be exercised; ideally OTPs would be sent only to trusted (authenticated) parties.
$\ddagger$ Example (Alternative OTP scheme). The following might be called a "poor man's" OTP scheme. Using a pre-shared secret $P$, A sends to $\mathrm{B}:(r, H(r, P))$. B verifies this using its local copy of $P$. Since a replay attack is possible if $r$ is reused, $r$ should be a time-varying parameter (TVP) such as a constantly increasing sequence number or time counter, or a random number from a suitably large space with negligible probability of duplication. As a drawback, a cleartext copy of the long-term secret $P$ is needed at B.
$\ddagger$ Exercise (Forward guessing attack). Explain how the method of the example above can be attacked if $P$ is a "poorly chosen" secret, i.e., can be guessed within a feasible number of guesses (here "feasible" means a number the attacker is capable of executing).

Passcode generators. A commercial form of one-time passwords involves inexpensive, calculator-like passcode generators (Fig. 3.4). These were originally specialized devices, but similar functionality is now available using smartphone apps. The device holds a user-specific secret, and computes a passcode output with properties similar to OTPs. The passcode is a function of this secret and a TVP challenge. The TVP might be an explicit (say eight-digit) string sent by the system to the user for entry into the device (in this case the device requires a keypad). Alternatively, the TVP can be an implicit challenge, such as a time value with, say, one-minute resolution, so that the output value remains constant for one-minute windows; this requires a (loosely) synchronized clock. The OTP is typically used as a "second factor" (below) alongside a static password. The user-specific secret is stored in cleartext-recoverable form system-side, to allow the sys-


Figure 3.4: Passcode generator using a keyed one-way function $f$. User-specific secret $s_{A}$ is shared with the system. Response $r$ is like a one-time password.
tem to compute a verification value for comparison, from its local copy of the secret and the TVP. Generating OTPs locally via passcode generators, using a synchronized clock as an implicit challenge, can replace system-generated OTPs transmitted as SMS codes to users-and without the risk of an SMS message being intercepted.

Hardware tokens. Passcode generators and mobile phones used for user authentication are instances of "what you have" authentication methods. This class of methods includes hardware tokens such as USB keys and chip-cards (smart cards), and other physical objects intended to securely store secrets and generate digital tokens (strings) from them in challenge-response authentication protocols (Chapter 4). As a typical example, suppose a USB token holds user A's RSA (public, private) signature key pair. The token receives a random number $r_{B}$ as a challenge. It sends in response a new random number $r_{A}$, and signature $S_{A}\left(r_{A}, r_{B}\right)$ over the concatenated numbers. This demonstrates knowledge of $A$ 's private key in a way that can be verified by any holder of a valid (e.g., pre-registered) copy of A's public key. The term authenticator is a generic descriptor for a hardware- or software-based means that produces secret-based strings for authentication.

USER AUTHENTICATION CATEGORIES. User authentication involves three main categories of methods (Fig. 3.5). Knowledge-based means ("what you know") include things remembered mentally, e.g., passwords, PINs, passphrases. The "what you have" category uses a computer or hardware token physically possessed (ideally, difficult to replicate), often holding a cryptographic secret; or a device having hard-to-mimic physical properties. The "what you are" category includes physical biometrics (Section 3.5), e.g., fingerprints; related methods involve behavioral biometrics or distinguishing behavioral patterns. A fourth category, "where you are", requires a means to determine user location.

Multiple factors. This chapter discusses several user authentication alternatives to passwords. These can either replace, or be used alongside passwords to augment them. In the simplest form, two methods used in parallel both must succeed for user authentication. Two-factor authentication (2FA) does exactly this, typically requiring that the methods be from two different categories (Fig. 3.5); a motivation is that different categories are more likely to deliver independent protection, in that a single attack (compromise) should not defeat both methods. Multi-factor authentication is defined similarly. Most such fac-

| User authentication categories based on type of verification evidence | 1 what you know | 5 | $\begin{gathered} \text { what }{ }^{2} \\ \text { you have } \end{gathered}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | multi- | 6 | 7 |  |
|  | where you <br> 4 are | 8 |  |  |  |

Figure 3.5: User authentication categories 1-3 are best known. Here (3) includes physical and behavioral biometrics; behavioral patterns could be considered a separate category, e.g., observed user location patterns (8). Location-based methods (4) may use geolocation of a user-associated device. A secret written on paper (because it is critical yet might be forgotten, or rarely used) may be viewed as something you have (5). Devices may receive one-time passwords (6). Device fingerprinting is shown as a sub-category of (7).
tors have traditionally required some form of explicit user involvement or action; in this case the additional factors impose usability costs. If authentication is user-to-device and then device-to-web, we call it two-stage authentication.

Exercise (Two-factor principles). Explain how 2FA is related to: P12 SUfFicient-WORK-FACTOR, P13 DEFENSE-IN-DEPTH, P18 INDEPENDENT-CONFIRMATION.

Example (Selecting factors). As a typical default, static passwords offer an inexpensive first factor from "what you know". Common two-factor schemes are (password, biometric) and (password, OTP from passcode generator). Automated banking machines commonly require (PIN, chip-card)—something you know plus something you have. If, against advice, you write your PIN on the card itself, the two factors are no longer independent and a single theft allows defeat.
$\ddagger$ COMPLEMENTARY FACTORS AND PROPERTIES. Multiple factors should be combined with an eye to the complementary nature of the resulting combined properties. Using two "what you know" factors (two passwords) increases memory burden; a hardware token avoids the cognitive burden of a second password. However, hardware authenticators must be carried-imagine having as many as passwords! When multiple schemes are used in parallel, if they are independent (above), their combined security is at least that of the weaker, and ideally stronger than each individually-otherwise there is little benefit to combine them. Regarding usability, however, inconveniences of individual factors are typically also additive, and the same is true for deployability barriers/costs.
\#Signals vs. factors. Some systems use "invisible" or "silent" authentication checks behind the scenes, which do not require explicit user involvement. Beyond earlierdiscussed authentication factors, which require explicit user actions, the broader class of authentication signals includes also implicit means such as:

- IP-address checks of devices previously associated with successful logins;
- browser cookies stored on devices after previously successful authentication;
- device fingerprinting, i.e., means to identify devices associated with legitimate users (previous successful logins), by recognizing hardware or software characteristics.

Individual signals may use secrets assumed to be known or possessed only by legitimate users, or devices or locations previously associated with legitimate users. Silent signals offer usability advantages, as long as they do not trigger false rejection of legitimate users.
$\ddagger$ FACTORS, PRIMARY AUTHENTICATION, AND RECOVERY. Are second factors suitable for stand-alone authentication? That depends on their security properties-but for a fixed application, if yes, then there would seem little reason to use such a factor in combination with others, except within a thresholding or scoring system. As a related point, any password alternative may be suitable for account recovery if it offers sufficient security-but in reverse, recovery means are often less convenient/efficient (which is tolerable for infrequent use) and therefore often unsuitable for primary authentication.

### 3.5 Biometric authentication

As discussed, passwords have well-known disadvantages in both usability and security. Authentication approaches based on hardware tokens often do well on security but suffer due to token cost, being forgotten, lost or stolen, and inconvenience. This leads to promotion of biometric-based authentication, with usability a primary motivation: nothing for users to carry, no cognitive burden, a general appearance of ease of use (a fingerprint is quicker than typing a password on a mobile phone), and scalability in terms of burden on users (unlike passwords, the burden does not increase with the number of accounts). These are powerful advantages, varying somewhat based on the biometric used. We now discuss biometrics in more detail, and find their security generally less than expected, while other disadvantages render them inappropriate for remote authentication.

Certain human characteristics are unique to individuals even across large populations. Biometric authentication methods leverage this. Physical biometrics (based on static physiological characteristics) provide the "what you are" category of authentication; behavioral biometrics (based on behavioral features related to physiology) are part of a "what you do" category. Behavioral characteristics independent of human physiology, such as geolocation patterns and call-patterns (phone numbers called), can also be used in non-biometric behavioral authentication approaches. A set of biometric features that can be used for authentication is called a biometric modality. Table 3.2 gives examples.

Biometrics are non-SECRETS. Authentication approaches that rely on demonstrating knowledge of a secret, such as passwords, rely on an assumption: the secret is known only to authorized parties. (That this assumption is commonly violated for passwords is a security weakness.) Biometric characteristics are not secrets-fingerprints are left on many surfaces, and faces are readily visible. Thus biometric authentication relies on a different assumption: that samples are input by a means providing some assurance of being "tightly bound" to the user present. Consider a fingerprint sampled at a supervised entrance, e.g., at an airport or corporate facility. The supervision provides some assurance that the individual is not presenting an appendage other than their own for physical measurement, nor injecting a data string captured earlier. Biometrics thus implicitly rely on some form of trusted input channel; this generally makes them unsuitable for remote

| Modality | Type | Notes |
| :--- | :---: | :--- |
| fingerprints | P | common on laptops and smartphones |
| facial recognition | P | used by some smartphones |
| iris recognition | P | the part of the eye that a contact lens covers |
| hand geometry | P | hand length and size, also shape of fingers and palm |
| retinal scan | P | based on patterns of retinal blood vessels |
| voice authentication | M | physical-behavioral mix |
| gait | B | characteristics related to walking |
| typing rhythm | B | keystroke patterns and timing |
| mouse patterns | B | also scrolling, swipe patterns on touchscreen devices |

Table 3.2: Biometric modalities: examples. P (physical), B (behavioral), M (mixed). Fingerprint (four digits) and iris biometrics are used at U.S.-Canadian airport borders.
authentication over the Internet. Note that your iPhone fingerprint is not used directly for authentication to remote payment sites; a two-stage authentication process involves user-to-phone authentication (biometric verification by the phone), then phone-to-site authentication (using a protocol leveraging cryptographic keys).

Failure to enroll/failure to capture. Failure to enroll (FTE) refers to how often users are unsuccessful in registering a template. For example, a non-negligible fraction of people have fingerprints that commercial devices have trouble reading. The FTE-rate is a percentage of users, or percentage of enrollment attempts. Failure to capture (FTC), also called failure to acquire, refers to how often a system is unable to acquire a sample of adequate quality to proceed. FTE-rate and FTC-rate should be examined jointly with FAR/FRR rates (below), due to dependencies.

Disadvantages (biometrics). Many modalities require custom client-side hardware. Adding to system complexity, fallback mechanisms are needed to accommodate rejection of legitimate users (sometimes surprisingly frequent), and FTE and FTC issues. Scalability also has a downside: using fingerprints across many systems, each sampling and storing templates, results in a scenario analogous to password reuse: a compromise of any system puts others at risk. From experience, believing no such compromise will occur is naive; but here, because a foundational requirement for biometrics is that they cannot be changed, the consequences are severe. This inherent "feature" of biometrics being unrevokable is a daunting show-stopper. Moreover, as biometrics are non-secrets, their "theft" does not require breaking into digital systems-thus the criticality of ensuring fresh samples bound to individuals, and trusted input channels. Aside from this, the security of biometrics is often over-stated-even uniqueness of biometric characteristics across individuals (which measurement limitations reduce) would not preclude circumvention; security often depends more on system implementation details than modality.

Example (iPhone fallback authentication). Biometric authentication is generally considered stronger protection than short numeric passwords (PINs). In 2013, iPhone fingerprint authentication replaced (four-digit) login PINs. Face recognition on 2017 iPhones replaced this using 3D face models. If a PIN is the fallback for such systems (if the biometric fails to recognize the user after a few tries), then the overall system is no stronger
than this fallback. ${ }^{7}$ Fraudulently entering a PIN does, however, require phone possession.
Summary. Biometrics offer usability advantages, have some deployability disadvantages, are generally less secure than believed, and have failure modes with severe negative externalities (i.e., consequences for unrelated parties or systems). Thus, biometrics are by no means a "silver bullet" solution. Their suitability depends, as usual, on the target environment of use; they suit supervised environments better than remote authentication.

BIOMETRIC PROCESS: ENROLLMENT AND VERIFICATION. A biometric modality is implemented by selecting a suitable set of measurable features-e.g., for fingerprints, the arches, loops and whorl patterns formed by skin ridges, their length, relative locations and distances between them. For each user (account), several sample biometric measurements are taken in an enrollment phase. Features are extracted to build a reference template. For subsequent user authentication, a freshly taken sample is compared to the template for the corresponding implied or asserted account, and a matching score $s$ is computed; higher scores indicate higher similarity, e.g., $s=0$ could denote no similarity, with $s=100$ denoting $100 \%$ agreement. A threshold $t$ is set (discussed below). Then if $s \geq t$, the system declares the sample to be from the same individual as the template.

Exercise (Biometric system flow chart). Illustrate the process of biometric enrollment and verification in a flow-chart relating architectural components (hint: [36, Figure 1]).

False rejects, false accepts. Two types of errors occur in biometric systems. In a false reject, a legitimate user's new sample is declared to not match their own template. In a false accept, an imposter's sample is (wrongly) declared to match the legitimate user's template. The frequency of these errors depends on both the threshold $t$ and system limitations (inaccuracies in sampling, measurement and feature representation). Measurement accuracy is affected by how user features present to sensors; environmental factors also come into play, e.g., dry skin from cold weather impacts fingerprint readings.

A stricter threshold (larger $t$, requiring stronger matches) results in more false rejects, but fewer false accepts; this negatively affects usability and availability, but improves security. A looser tolerance (smaller $t$, accepting weaker matches) results in fewer false rejects, but more false accepts; this improves usability and availability for legitimate users, but obviously decreases security. What is acceptable as a tradeoff between false accepts and false rejects depends on the application; $t$ is adjusted to suit application scenarios. High-security (security-sensitive) applications demand stricter matching, tolerating more false rejects in order to preserve security; low-security applications prioritize usability over security, setting looser tolerances in order to reduce false rejects.

False accept/reject rates. Fixing a threshold $t$ and legitimate user $L$ with reference template $X_{L}$, let $X_{V}$ denote the biometric samples to be verified. The false accept rate (FAR) is the probability the system declares $X_{V}$ matches $X_{L}$ when in fact $X_{V}$ is not from $L$; this assumes sampling over the user population. Theoretically, to determine a system FAR, the above could be computed over all users $L$ and reported in composite. Aside: FAR reflects random sampling, but we expect serious attacks do better than using random samples in impersonation attempts. This may be viewed as reflecting naive at-

[^5]

Figure 3.6: Biometric system tradeoffs. Curves model probability distributions for an intruder and legitimate user's matching scores; higher scores match the user's biometric template better. The $y$ axis reflects how many biometric samples get matching score $x=s$.
tacks, or benign errors. Security researchers thus view FAR as misleadingly optimistic, giving more weight to resilience under malicious scenarios ("circumvention", below).

The false reject rate (FRR) is the probability of a false reject, i.e., prob(system declares $X_{V}$ does not match $X_{L}$, when sample $X_{V}$ is actually from $L$ ); sampling is over repeated trials from user $L$. The equal error rate (EER) is the point at which FAR $=\mathrm{FRR}$ (Fig. 3.7). Although unlikely to be the preferred operational point in practice, EER is used for simplified single-point comparisons-the system with lower EER is preferred.

TWO-DISTRIBUTION OVERLAP: USER/INTRUDER MATCH SCORES. For a fixed user $L$ with template $X_{L}$, two graphs illustrate how altering the threshold $t$ trades off false accepts and false rejects. The first (Fig. 3.6) has $x$-axis giving matching score of samples (against $X_{L}$ ), and $y$-axis for probability of such scores across a large set of samples. Two collections of samples are shown, giving two probability distributions: $D_{I}$ (on left) for intruder samples, $D_{L}$ (on right) for samples from user $L$. Each value $t$ defines two shaded areas (shown), respectively corresponding to false rejects and false accepts; moving $t$ left or right changes the relative sizes of these areas (trading false rejects for false accepts). Noting each shaded area as a percentage of its corresponding distribution and interpreting this as a rate allows a second graph to be drawn (Fig. 3.7, left). Its $x$-axis denotes FAR, $y$-axis FRR, and curve point $(x, y)$ indicates $\mathrm{FRR}=y$ when $\mathrm{FAR}=x$. Each point implicitly corresponds to a value $t$ in the system of the first graph. The DET graph (detection error tradeoff) thus shows a system's operating characteristics (tuning options) across implicit values $t$. Such analysis is common in binary classifier systems. DET graphs are closely related to relative/receiver operating characteristic curves or ROC curves (Fig. 3.7, right). Both arise in analyzing four diagnostic outcomes of a binary classifier (true/false positive,


Figure 3.7: DET graph and ROC curve. These depict a system's characteristics for different values of a decision threshold $t$, and allow comparisons between systems. If EER is used as a single comparison point, System 2 is preferred. System 3's FRR decreases slowly as parameters are adjusted to admit a higher FAR; in contrast, System 1's FRR decreases more rapidly in return for an increased FAR. An upper-left ROC curve is better (A). In binary classification of events in the intrusion detection scenario (Chapter 11), the analogous terminology used is True/False Positive Rate, and True/False Negative Rate.
true/false negative). DET graphs plot the two error outcomes.
Evaluating biometrics using standard criteria. To evaluate and compare biometric authentication systems, a suite of properties and standardized criteria are used. Basic requirements must first be met for a modality to be given serious consideration. The following aspects of a modality's characteristics are considered.

- universality: do all users have the characteristic? This relates to failure-to-enroll.
- distinguishability: do the characteristics differ sufficiently across pairs of users to make benign matches unlikely? This requires sufficient variability in measurable features, and impacts false accept rates.
- invariance: are characteristics stable over time (even for behavioral biometrics)?
- ease-of-sampling: how easily are samples obtained and measured? For example, consider DNA vs. fingerprints. Retinal scans typically involve contact with an eyepiece. Physical biometrics may be obscured (e.g., by hair, glasses).
Beyond basic requirements, other criteria important for use in practice are as follows.
- accuracy: metrics discussed earlier include FAR, FRR, EER, FTE-rate, FTC-rate (all for selected thresholds $t$ ). These reflect operation in anticipated or target operating environments and conditions, albeit with benign participants.
- cost: this includes time (sampling; processing), storage, hardware/software costs.
- user acceptance: do users willingly use the system? Some users worry about privacy of biometric data in general, or its use for tracking. Examples of modality-specific concerns are invasiveness (e.g., discomfort about light or objects near the eyes-for retinal scans, users must peer into eyepieces that send visible light into the eye), and negative cultural associations (some cultures associate fingerprinting with implied criminal activity). This relates to principle P11 (USER-BUY-IN).
- attack-resistance: can the system avoid adversarial false accepts, i.e., resist user impersonation (spoofing), substitution, injection, or other attempted circumvention?
Circumvention: attacks on biometric authentication. The basic security question for a biometric system is: How easily can it be fooled into accepting an imposter? This asks about malicious false accepts, whereas FAR measures benign false accepts. Related questions are: What attacks work best, are easiest to mount, or most likely to succeed? How many authentication trials are needed by a skilled attacker? These questions are harder to address than those about performance measures noted above.
$\ddagger$ Exercise (Circumventing biometrics). (i) Outline generic system-level approaches to defeating a biometric system, independent of the modality (hint: [35, Fig.9]). (ii) For each of five selected modalities from Table 3.2, summarize known modality-specific attacks. (iii) For which modalities are liveness detectors important, or possible?

Biometrics: authentication vs. identification. This section has considered biometrics mainly for user authentication, e.g., to replace passwords or augment them as a second factor. In this usage, a username (account) is first asserted, then the biometric sample is matched against the (single) corresponding user template. An alternate usage is user identification (i.e., without asserting specific identity); the system then must do a one-to-many test-as explained in this chapter's first paragraph. For local identification to a laptop or smartphone, the number of accounts registered on that device is typically small; access is granted if any match is found across registered accounts, and the one-to-many matching has relatively negligible impact on computation (time) or security. Relieving the user of the task of entering a username improves convenience.

However, for systems with large user bases, one-to-many matching is a poor fit with access control applications. The probability of a benign match between a single attackerentered sample and any one of the many legitimate user templates is too high. The natural application of "user identification" mode is, unsurprisingly, identification-e.g., to match a target fingerprint against a criminal database, or use video surveillance face recognition to match crowd faces against a targeted list (the shorter the better, since the latter case is many-to-many). The issue of false accepts is handled here by using biometric identification as the first-stage filter, with human processing for second-stage confirmation.
$\ddagger$ Exercise (Comparing modalities). Select six biometric modalities from Table 3.2, plus two not listed. (i) For each, identify primary advantages and limitations. (ii) Using these modalities as row labels, and bulleted criteria above as column labels, carry out a qualitative comparison in a table, assigning one of (low, medium, high) to each cell; word the criteria uniformly, such that "high" is the best rating. (iii) For each cell rated "low", briefly justify its rating in one sentence. (Hint: [36], [10].)

## $3.6 \ddagger$ Password managers and graphical passwords

Password managers, including those that auto-fill web site username-password pairs, store and retrieve passwords as a means to cope with overwhelming numbers of passwords. ${ }^{8}$

[^6]Instead of remembering many passwords, a user remembers one master password. It provides access to the others. Efficiency and usability improve, with reduced memory burden. Resilience to phishing improves if the tool records the domain associated with outgoing passwords, and disallows (or warns) on subsequent attempts to send a password to a domain not previously associated. Ideally, security improves by allowing users to put more cognitive energy into choosing and remembering a single master password with high guessing-resistance, while random (unguessable) individual site passwords can be used as they need no longer be remembered. In practice, master passwords may be weaker than hoped, and the individual site passwords managed remain not only static (thus replayable) but often remain user-chosen (thus guessable) for reasons explained below. Overall, password managers thus deliver fewer security advantages than expected, while introducing new risks (below); their main advantage is improved usability.

Software packaging. A password manager may be integrated as an operating system utility (macOS Keychain uses the OS login password as master password), or be a stand-alone client application, a browser built-in feature or plug-in/add-on, or a cloudbased service. Some managers synchronize passwords across devices, i.e., make them accessible from (stored encrypted on) multiple designated user devices; otherwise, the passwords managed are available only on a primary device.

PASSWORD MANAGER APPROACHES. The two main approaches are as follows:

- password wallet (or vault): here the tool manages an existing collection of passwords, automatically selecting the password needed based on prior association with the domain of use. Passwords are stored in the wallet individually encrypted under a password derived from the master password; poorly designed tools leave them plaintext. The master password is entered at the start of each manager session (e.g., start of day); caching it allows later use without re-entry by the user. Since the tool now remembers site passwords rather than the user, and can generate long, random passwords, the main barriers to making all site passwords unguessable should be gone; but in practice, wallets typically manage pre-existing passwords, because "migrating" existing passwords site-by-site to new random passwords consumes user time. Thus many password wallets manage existing (guessable) passwords. The wallet is stored on the local device in client-side tools, or at a server in cloud-based tools.
- derived passwords: here application-specific or site-specific passwords are derived from a master password plus other information such as the target domain. This provides some protection against phishing attacks; a rogue site attracting a target user receives the hash value $H$ (master, rogue.domain.com) rather than the authorizing string $H$ (master, true.domain.com) that allows account access on the true domain. Derived site passwords (vs. user-chosen) can increase guessing-resistance through (client-side) iterated hashing, and protect against pre-computed dictionary attacks by using userspecific salts (Section 3.2).
Exercise (Offline attack on master password). The phishing protection just noted comes with a risk: a rogue site may mount an offline guessing attack to recover a master password. Explain how this is done, and what factors affect the likelihood of attacker success.

SECURITY AND RISKs. Password managers are password "concentrators", thus also concentrating risk by creating a single point of failure and attractive target (recall principle P13, DEFENSE-IN-DEPTH). Threats to the master password include capture (e.g., by client-side malware, phishing, and network interception), offline guessing (user-chosen master passwords), and online guessing in the case of cloud-based services. Individual site passwords managed, unless migrated to random passwords, remain subject to guessing attacks. Managers thus expose new attack surface (cf. P1 SIMPLICITY-AND-NECESSITY), but reduced user entry of site passwords may lower susceptibility to keylogger attacks.

RISK If PASSWORD MANAGER FAils. Once a password generator is used to generate or remember passwords, users rely on it (rather than memory); if the tool becomes unavailable or malfunctions, any password recovery mechanisms in place through a web site may allow recovery of (some of) the managed passwords. However, typically no such recovery service is available for the master password itself, nor any managed passwords used for password-derived keys for stand-alone applications, e.g., local file/disk encryption. If access to such a password is lost, you should expect the locally encrypted files to be (catastrophically) unrecoverable.

Compatibility with existing password servers. An advantage of password wallets for managing existing passwords is that they introduce no server incompatibilities and thus can be deployed without any server-side changes or cooperation. In the case of generating new (random) passwords, both password wallet managers and derivedpassword managers must satisfy server-defined password composition policies-and automatically generated passwords will not always satisfy policies on the first try. Thus derived-password managers cannot regenerate site passwords on the fly from master passwords alone; they may still need to store, for each user, site-specific information beyond standard salts, as a type of additional salt to satisfy site-specific policies. As another compatibility issue, some sites disallow auto-filled passwords.

Exercise (Analysis and user study of password managers). For each of (i) PwdHash, and (ii) Password Multiplier, answer the following: (a) Explain the technical design of this manager tool, and which manager approach it uses. (b) Summarize the tool's strengths and weaknesses related to each of: security, usability, deployability. In particular for usability, describe how users invoke the tool to protect individual site passwords, and for any automated actions, how users are signaled that the tool is operating. (c) Describe how the tool performs on these standard password management tasks: day-to-day account login, password update, login from a new device, and migration of existing passwords to those compatible with the manager tool. (Hint: [15].)

Graphical passwords: overview. Like password managers, graphical password schemes aim to ease the burden of too many passwords, here by schemes that depend in some way on pictures or patterns. Like regular passwords, a graphical password is encoded to a string that the system can verify. The idea is that because human memory is better for pictures, graphical passwords might impose a lighter memory burden than text passwords; and security might also be increased, if this allows users to choose harder-toguess passwords. Another motivation is to improve input usability on mobile phones and touchscreen devices, where typing is less convenient than on desktop machines.

Classes of graphical password schemes. There are three basic classes.

1. Pure recall. The user essentially reconstructs a pattern starting from a blank sheet. As a simple example, Android touchscreen devices commonly use a swipe pattern over a nine-dot background; this replaces use of a login PIN or password.
2. Cued recall. The user is aided by a graphical cue. For example, the user is presented with a picture and asked to choose five click-points as their password. The user must later re-enter those points (within reasonable tolerance) to gain account access.
3. Recognition schemes. The users must recognize a previously seen image (or set of images). For example, a user is presented with four panels sequentially, each with nine faces: eight distractors and one face familiar to the user (a set of familiars is selected during registration). The user must click on a familiar face in each of the four panels. Other sets of common objects can be used instead of faces, e.g., house fronts. Cognitive psychology research indicates that people are better at recognizing previously encountered items (recognition memory) than in tasks involving (pure) recall.
Analysis. Study of graphical password schemes has found the following:
a) The best among graphical password proposals offer minor usability (memorability) improvements. In some cases this is due to strategies that could similarly be used with text passwords (e.g., cued recall). Many proposals require significant training, longer password registration phases, and moderately longer password entry times.
b) Promised security improvements are often elusive due to the tendency of users to choose predictable graphical passwords-some are typically much more popular than others-just as users choose predictable text passwords. In contrast, the ideal is equiprobable passwords from a suitably large password space-e.g., for text passwords, consisting of characters chosen truly at random; for cryptographic keys, the ideal is truly random secret bit strings.
c) Overall, any advantages of graphical passwords over text passwords have, to date, been insufficient to displace text passwords in general, an exception being on smartphones.
From a security viewpoint, the simplest graphical password schemes boil down to fixed authentication strings, which, if captured, are replayable like captured text passwords. More compelling benefits appear necessary to displace text passwords and alternatives.

## $3.7 \ddagger$ CAPTCHAS (humans-in-the-loop) vs. automated attacks

Free web services are easy targets for automated programs, which might, e.g., try to acquire in bulk free e-mail accounts (e.g., to send spam email), or make bulk postings of spam or malware to online discussion boards. A countermeasure is to present a task relatively easily done by humans, but difficult for computer programs-to distinguish humans from malicious programs ("robots" or bots). Thus many sites began to ask users to type in text corresponding to distorted character strings. This is an example of a CAPTCHA ${ }^{9}$

[^7]or Automated Turing Test (ATT). These are often based on character recognition (CR), audio recognition (AUD), image recognition (IR), or cognitive challenges involving puzzles/games (COG). Below we see how CAPTCHAs can stop automated online guessing.

As noted earlier, to mitigate online guessing attacks, a server may rate-limit the number of online login attempts. However, if account "lock-out" results, this inconveniences the legitimate users of attacked accounts. A specific attacker goal might even be, e.g., to lock out a user they are competing with precisely at the deadline of an online auction. A defensive alternative is to make each login guess "more expensive", by requiring that a correct ATT response accompany each submitted password-but this inconveniences legitimate users. The cleverly designed protocol outlined next does better.

Pinkas-Sander login protocol. The protocol of Fig. 3.8 imposes an att on only a fraction $p$ of login attempts (and always when the correct password is entered but the device used is unrecognized). It assumes legitimate users typically log in from a small set of devices recognizable by the server (e.g., by setting browser cookies or device fingerprinting), and that any online dictionary attack is mounted from other devices. Device recognition is initialized once a user logs in successfully. Thereafter the legitimate user faces an ATT only when either logging in from a new device, or on a fraction $p$ of occurrences upon entering an incorrect password.

Two TECHNICAL DETAILS. In Fig. 3.8, note that requiring an ATT only upon entry of the correct password would directly signal correct guesses. Following an adjunct to principle P3 (OPEN-DESIGN), the protocol refrains from disclosing such free information to an attacker's benefit. Also, whether to require an ATT for a given password candidate must be a deterministic function of the submitted data, otherwise an attack program could quit any login attempt triggering an ATT, and retry the same userid-password pair to see whether an ATT is again required or if a "login fails" indication results.

To an attacker-expected to make many incorrect guesses-imposing an ATT on even

```
fix a value for system parameter \(p, 0<p \leq 1\) (e.g., \(p=0.05\) or 0.10 )
user enters username-password pair
if (user device has cookie) then server retrieves it endif
if (entered username-password correct) then
    if (cookie present \& validates \& unexpired \& matches username) then
        "login passes"
    else \% i.e., cookie failure
        ask an ATT; "login passes" if answer correct (otherwise "login fails")
    endif
else \% i.e., incorrect username-password pair
    set AskAnATT to TRUE with probability \(p\) (otherwise FALSE) \(\dagger\)
    if (AskAnATT) then
        ask an ATT; wait for answer; then, independent of answer, say "login fails"
    else immediately say "login fails" endif
endif
```

Figure 3.8: Protocol to counter online dictionary attacks (simplified Pinkas-Sander [51]). $\dagger$ Setting is a deterministic function of userid-password pair (same each time for that pair)
a small fraction of these (e.g., $5 \%$ ) is still a large cost. The attacker, assumed to be submitting guesses from an unrecognized machine, must always "pay" with an ATT on submitting a correct guess, and must similarly pay a fraction $p$ of the time for incorrect guesses. But since the information available does not reveal (before answering the ATT) whether the guess is correct, abandoning an ATT risks abandoning a correct guess.

Exercise (Pinkas-Sander password protocol analysis). This protocol (Fig. 3.8) can be analyzed under two attack models: (i) an automated program switches over to a human attacker to answer ATTs; (ii) the program makes random guesses as ATT answers, assuming an ATT answer space of $n$ elements (so an ATT guess is correct with probability 1 in $n$ ). To simplify analysis, assume a space of $S$ equiprobable passwords. (a) Under model (i), for an optimal attacker, determine the expected number of ATTs answered before successfully guessing a password; express your answer as a function of $p$ and $S$, and assume an attack on a single account. (b) Under model (ii), determine the expected number of password guesses needed before success, as a function of $p, S$ and $n$. (Hint: [51].)

CAPTCHA FUTURES. For several reasons, the ongoing value of CAPTCHAs in security is unclear. For many types of CAPTCHAs, automated solvers are now so good that CAPTCHA instances sufficiently difficult to resist them are beyond the annoyance and complexity level acceptable for legitimate users-so these CAPTCHAs cease to be useful Turing Tests. The efficacy of CR CAPTCHA solvers in particular has resulted in more IR CAPTCHAS. Another attack on CAPTCHAs is to maliciously outsource them by redirection to unsuspecting users. Similarly, the core idea of distinguishing humans from bots is defeated by redirecting CAPTCHAs to willing human labour pools-"sweat shops" of cheap human solvers, and Amazon Mechanical Turkers.

Example (Google recaptcha). In 2014, the Google recaptcha project replaced CAPTCHAS with checkboxes for users to click on, labeled "I'm not a robot". A human-or-bot decision is then made from analysis of browser-measurable elements (e.g., keyboard and mouse actions, click locations, scrolling, inter-event timings). If such first-level checks are inconclusive, a CR or IR CAPTCHA is then sent. In 2017 even such checkboxes were removed; the apparent trend is to replace actions triggered by requesting clicking of a checkbox by pre-existing measurable human actions or other recognition means not requiring new explicit user actions.

## $3.8 \ddagger$ Entropy, passwords, and partial-guessing metrics

This supplemental section discusses information-theoretic (Shannon) entropy, guessing entropy, and partial-guessing metrics useful to understand guessing attacks on user-chosen passwords. Historically, common misunderstandings about entropy have led to incorrect models, assumptions and conclusions about password security. We aim to avoid this.

Example (Data, information representation, and entropy). A 16-bit word might be used to convey four values (Table 3.3), representing events that are equiprobable over a large number of samples. The same information can be conveyed in 2 bits. For the given probabilities, in information theory we say there are two bits of entropy. Below, we

| Information | Probability | Hex representation | Binary alternative |
| :---: | :---: | :---: | :---: |
| red | 0.25 | 0000 | 00 |
| green | 0.25 | $00 F F$ | 01 |
| blue | 0.25 | FFOO | 10 |
| black | 0.25 | FFFF | 11 |

Table 3.3: Alternative representations for conveying four known values. The same information is conveyed, whether two bytes of data are used to represent it, or two bits.
explain further, using password distributions both for concreteness and relevance.
ShANNON ENTROPY. Let $q_{i}>0$ be the probability of event $x_{i}$ from an event space $X$ of $n$ possible events ( $1 \leq i \leq n$, and $\sum q_{i}=1$ ). In our exposition, $x_{i}=P_{i}$ will be a userchosen password from a space of $n$ allowable passwords, with the set of passwords chosen by a system's $m$ users considered an experimental outcome (e.g., consider the passwords being drawn from a known distribution). In math-speak, a random variable $X$ takes value $x_{i}=P_{i}$ with probability $q_{i}$, according to a probability distribution $D_{X}$. We write $X \stackrel{D_{X}}{\longleftarrow} X$ and $D_{X}: q_{i} \rightarrow x_{i} . D_{X}$ models the probability of users choosing specific passwords, e.g., as might be derived from an unimaginably large number of real-world iterations. Now the Shannon entropy of this discrete distribution is defined as: ${ }^{10}$

$$
\begin{equation*}
H(X)=H\left(q_{1}, q_{2}, \ldots, q_{n}\right)=\sum_{i=1}^{n} q_{i} \cdot \lg \left(1 / q_{i}\right)=-\sum_{i=1}^{n} q_{i} \cdot \lg \left(q_{i}\right) \tag{3.5}
\end{equation*}
$$

(Note that only the probabilities $q_{i}$ are important, not the events themselves.) Here the units are bits of entropy, $l g$ denotes a base- 2 logarithm, and by convention $0 \cdot \lg (0)=0$ to address $\lg (0)$ being undefined. $H(X)$ measures the average uncertainty of $X . H(X)$ turns out to be the minimum number of bits needed (on average, across the probability distribution) to convey values $X=x_{i}$, and the average wordlength of a minimum-wordlength code for values of $X$.

Interpretation of entropy. To help understand the definition of $H(X)$, for each outcome $x_{i}$ define $I\left(x_{i}\right)=-\lg \left(q_{i}\right)$ as the amount of information conveyed by the event $\left\{X=x_{i}\right\}$. It follows that the less probable an outcome, the more information its observation conveys; observing a rare event conveys more than a common event, and observing an event of probability 1 conveys no information. The average (expected value) of the random variable $I$ is then $H(X)=E_{X}\left(I_{X}\right)=E_{X}\left(-\lg \left(q_{i}\right)\right)$. Viewing $q_{i}$ as a weight on $\lg \left(q_{i}\right), H(X)$ is now seen to be the expected value of the $\log$ of the probabilities.

Entropy properties. The following hold for $H(X)$ with event space of size $n$ :

1. $H(X) \geq 0$. The minimum 0 occurs only when there is no uncertainty at all in the outcome, i.e., when $q_{i}=1$ for some $i$ (forcing all other $q_{j}$ to 0 ).
2. $H(X) \leq \lg (n)$. The maximum occurs only when all $q_{i}=\frac{1}{n}$ (all events equiprobable). Then $H(X)=\sum_{i=1}^{n} 1 / n \cdot \lg (n)=\lg (n)$. Thus a uniform ("flat") distribution maximizes entropy (gives greatest uncertainty), e.g., randomly chosen cryptographic keys (whereas user-chosen passwords have highly skewed distributions).

[^8]3. Changes towards equalizing the $q_{i}$ increase $H(X)$. For $q_{1}<q_{2}$, if we increase $q_{1}$ and decrease $q_{2}$ by an equal amount (diminishing their difference), $H(X)$ rises.
Example (Entropy, rolling a die). Let $X$ be a random variable taking values from rolling a fair eight-sided die. Outcomes $X=\{1,2,3,4,5,6,7,8\}$ all have $q_{i}=\frac{1}{8}$ and $H(X)=\lg (8)=3$ bits. For a fair six-sided die, $q_{i}=\frac{1}{6}$ and $H(X)=\lg (6)=2.58$ bits. If the six-sided die instead has outcomes $X=\{1,2,3,4,5,6\}$ with resp. probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{32}$, then $H\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{32}\right\}=\frac{1}{2} \cdot 1+\frac{1}{4} \cdot 2+\frac{1}{8} \cdot 3+\frac{1}{16} \cdot 4+2\left(\frac{1}{32} \cdot 5\right)=$ 1.9375 bits, which, as expected, is less than for the fair die with equiprobable outcomes.

Exercise (Entropy, rolling two dice). Let $X$ be a random variable taking values as the sum on rolling two fair six-sided dice. Find the entropy of $X$. (Answer: 3.27 bits.)

Example (binary entropy function). Consider a universe of $n=2$ events and corresponding entropy function $H=-(p \cdot \lg (p)+q \cdot \lg (q))$, where $q=1-p$. A 2D graph (Fig. 3.9) with $p$ along $x$-axis $[0.0,1.0]$ and $H$ in bits along $y$ axis $[0.0,1.0]$ illustrates that $H=0$ if and only if one event has probability 1 , and that $H$ is maximum when $q_{i}=\frac{1}{n}$. This of course agrees with the above-noted properties. (Source: [55] or [42, Fig.1.1].)

Single most-probable event. Which single password has highest probability is a question worth studying. If an attacker is given exactly one guess, the optimal strategy is to guess the most-probable password based on available statistics. A company might analyze its password database to find the percentage of users using this password, to measure maximum expected vulnerability to a single-guess attack by an "optimal attacker" knowing the probability distribution. The expected probability of success is $q_{1}=\max _{i}\left(q_{i}\right)$; this assumes the target account is randomly selected among system accounts. A formal measure of this probability of the most likely single event is given by the min-entropy formula:

$$
\begin{equation*}
H_{\infty}(X)=\lg \left(1 / q_{1}\right)=-\lg \left(q_{1}\right) \tag{3.6}
\end{equation*}
$$

If $q_{i}=\frac{1}{n}$ for all $i$, then $H_{\infty}(X)=-\lg \left(\frac{1}{n}\right)=\lg (n)$, matching Shannon entropy in this case.


Figure 3.9: Binary entropy function for $n=2$ events. The $x$-axis is the probability $p$ of one of the events; $y=H(x)=-x \cdot \log _{2}(x)+(1-x) \log _{2}(1-x)$, in bits.

Example (Most-probable password). Suppose the most popular password in a distribution is "Password1", with $1 \%$ of users choosing it. Knowing this, but not which account(s) use this password, an attacker tries it on a randomly chosen account from this distribution; assume account names are easily obtained. The attacker then has a $1 \%$ probability of success with this one guess.

Example (Optimal breadth-first guessing). If passwords are ordered $P_{1}, P_{2}, \ldots$ in highest to lowest probability order, then for an attacker allowed multiple online guesses, the optimal (breadth-first) strategy to minimize expected time to success is to guess $P_{1}$ on all accounts, then $P_{2}$ on all accounts, and so on. If $1 \%$ of users use $P_{1}$ per the previous example, then success is expected ( $50 \%$ chance) after guessing $P_{1}$ on 50 accounts.

GUESSWORK (GUESSING FUNCTION). With notation as above, let $q_{i} \geq q_{i+1}$, modeling an optimal guessing-attack order. The guessing index $g(X)$ over a finite domain $X$ assigns a unique index $i \geq 1$, the guess number, to each event $X=x_{i}$ under this optimal ordering. Then for $X \stackrel{D_{X}}{\leftrightarrows} \mathcal{X}$ ( $X$ drawn from the event universe according to distribution $D_{X}$, above), the expected (on average) number of trials to get a YES on asking "Is $X=x_{i}$ ?", for sequential candidates $x_{i}$ tested in optimal order, is given by the guessing function

$$
\begin{equation*}
G_{1}(X)=E[g(X)]=\sum_{i=1}^{n} i \cdot q_{i} \quad(\text { units }=\text { number of guesses }) \tag{3.7}
\end{equation*}
$$

Like $H(X), G_{1}$ gives an expectation averaged over all events in $X$. Thus its measure is relevant for an attack executing a full search to find all user passwords in a dataset-but not one, e.g., quitting after finding a few easily guessed passwords. If $q_{i}=1 / n$ for all $i$,

$$
\begin{equation*}
G_{1}(n \text { equally probable events })=\sum_{i=1}^{n} i \cdot 1 / n=(1 / n) \sum_{i=1}^{n} i=(n+1) / 2 \tag{3.8}
\end{equation*}
$$

since $\sum_{i=1}^{n} i=n(n+1) / 2$. Thus in the special case that events are equiprobable, success is expected after guessing about halfway through the event space; note this is not the case for user-chosen passwords since their distributions are known to be heavily skewed.

Example (Guesswork skewed by outliers). As a guesswork example, consider a system with $m=32$ million $\approx 32 \cdot 2^{20}$ users, whose dataset $\mathcal{R} \subset X$ of $m$ non-unique user passwords has a subset $\mathcal{S} \subset \mathcal{R}$ of 32 elements that are 128-bit random strings (on average 1 per $2^{20}$ elements in $\mathcal{R}$ ) . Let $\mathcal{U}_{2}{ }^{128}$ denote the set of all 128 -bit random strings. From (3.8), $G_{1}\left(\mathcal{U}_{2}{ }^{128}\right)>2^{127}$. Per individual password in $\mathcal{S}$ we thus expect to need at least $2^{127}$ guesses. How does this affect $G_{1}$ overall? From (3.7) and averaging estimates, ${ }^{11}$ it follows that $G_{1}(\mathcal{R})>2^{127} \cdot 2^{-20}=2^{107}$ guesses independent of any passwords outside $\mathcal{S}$. Thus the guesswork component from difficult passwords swamps (obscures) any information that $G_{1}$ might convey about easily guessed passwords. (Motivation: [7, Ch.3].)

[^9]Utility of entropy and guesswork. For user-chosen passwords in practice, entropy and guesswork mislead us-because their metrics do not model real attacks. Estimating attacker success involves assuming a guessing strategy, then projecting its success using estimated password probabilities. The historical entropy and guesswork metrics implicitly assume attackers guess through entire password spaces. In contrast, commonly used attack tools try passwords in priority order, i.e., (estimated) highest probability first, over only a subset of the password space. As a secondary issue, computing entropy and guesswork requires probabilities for complete password spaces, yet it is difficult to obtain even plausible estimates for probability distributions of real-world passwords. Most analysis to date has been on large "leaked" (stolen, then published) datasets. This has increased knowledge, especially for building password denylists (Section 3.2). But complete probability distributions $D_{X}$ for password spaces remain beyond reach.

Metrics useful in practice. For equiprobable event spaces, such as randomly chosen cryptographic keys and system-assigned random passwords, Shannon entropy $H(X)$ is a useful metric; but its use for passwords too often results in falsely assuming that user-chosen passwords are random and that attacks proceed by uninformed exhaustive search. Also, $H(X)$ cannot measure the strength of single passwords or subsets-as a sum over all events, it gives an average over the full space. Thus it cannot model real attacks that, as just noted, guess in priority order and "quit early" to avoid expending effort on hard passwords. Guesswork is similarly a full-distribution average subject to distortion by outliers (example above). So both metrics poorly measure real attacks. Min-entropy provides useful single-point information, albeit coarse (modeling only a one-guess attack), but does move us towards partial-guessing metrics (next) that are useful as metrics that compute expectations not over full event spaces, but over partial spaces.

Cumulative probability. The first of two partial-guessing metrics we mention is the cumulative probability of success, with $q_{i}$ as defined above for password space $X$ :

$$
\begin{equation*}
\operatorname{CPS}(b)=\sum_{i=1}^{b} q_{i} \tag{3.9}
\end{equation*}
$$

This is the expected success rate for an attack capable of $b$ guessing trials on an account from this space. It simply sums the probabilities of the first (highest-probability) $b$ passwords, giving the probability fraction $(\leq 1.0)$ these cumulatively cover.

Example (Cumulative probability). Consider the meaning of $C P S(10,000)=0.20$ for a company password database. An administrator finds, from analysis of the database, that the most popular $b=10,000$ distinct passwords are used by $20 \%$ of its users: $q_{1}+q_{2}+$ $\ldots+q_{10,000}=0.20$. Then an attacker able to mount $b$ guesses on one account (randomly chosen from all accounts) might be expected to guess the correct password with probability 0.20 , or break into $20 \%$ of accounts if given $b$ guesses on each account. This might result from an online attack if the system does not rate-limit login attempts.

Guess count. A second partial-guessing metric, for $q_{i}$ as above, is the guess count

$$
\begin{equation*}
G C(p)=\min \left\{b \mid \sum_{i=1}^{b} q_{i} \geq p\right\} \quad(0 \leq p \leq 1) \tag{3.10}
\end{equation*}
$$

This gives the number $b$ of per-account guesses needed to find passwords for a proportion $p$ of accounts, or for one account to correctly guess its password with probability at least $p$; or correspondingly, the number of words $b$ in an optimal (smallest) dictionary to do so.

Example (Guess count). Choosing $p=0.20$, (3.10) tells us how many per-account guesses are expected to be needed to guess $20 \%$ of accounts (or break into one account, drawn randomly from system accounts, with probability 0.20 ). For the previous example's scenario, this metric would return a guess count of $b=10,000$ (to achieve $20 \%$ success).

Example use of metrics. Partial-guessing metrics can be used to reason about choices for password denylist size and rate-limiting. For example, equation (3.9) allows comparison of the protection offered by denylists of $b_{1}=1,000$ entries vs. $b_{2}=10,000$. If a system $S$ rate-limits login attempts to $b$ incorrect guesses (e.g., $b=10$ or 25) over time period $T$, then (3.9)'s probability indicates exposure to online guessing attacks over $T$. If in addition $S$ denylists the 10,000 most popular passwords, then the $q_{i}$ used in (3.9) for this second case should be for passwords beyond the denylist, i.e., starting at $q_{10,001}$.

## $3.9 \ddagger$ End notes and further reading

The 1985 "Green Book" [19] (rainbow series) discusses system-generated random passwords; on password aging it recommends a maximum password lifetime of one year. Many older Unix systems support password aging. NIST 800-63-2 [11] discusses Equation (3.1), setting a limit of 100 login attempts over 30 days, and (to avoid lock-out) still accepting login attempts from allowlisted IP addresses from which successful logins previously occurred. Secret salts are discussed by Manber [40] and Abadi [1]; attacks on secret salts (but not on iterated hashing) can be parallelized. Oechslin's rainbow tables [50] allow an advanced attack on unsalted passwords, using a time-memory tradeoff; see also Narayanan [48]. Provos [52] explains a future-adaptable backwards-compatible hashing scheme whereby a hash iteration count can be increased over time as computing power increases. Florêncio [24] summarizes password research for systems administrators. Hatzivasilis [33] provides an overview of the 2013-2015 Password Hashing Competition candidates and winner Argon2 [5]; see also the Balloon [6] memory-hard hashing function. Dürmuth [20] explores offline guessing attacks leveraging parallel-computing architectures for fast hashing, including GPUs.

From a large-scale empirical study, Bonneau [9] concludes: "it appears next to impossible to find secret questions that are both secure and memorable." The password reset attack is from Gelernter [28]. The description of passcode generators and Lamport's OTP chain is adapted from Menezes [43]; see also Bellcore's S/KEY one-time password system [30, 31]. Bonneau [10] discusses comparative analysis of password alternatives, including hardware tokens, biometrics, password managers, and secret questions. Security Keys [39] are a FIDO-standardized second-factor token originating from Google internal use. NIST SP 800-63B [29, Part B] makes clear the view of OTP via SMS as a weak second-factor authentication (2FA), following concerns about large-scale SMS message redirection and interception; Konoth [38] describes attacks on SMS-based 2FA as leav-
ing it "entirely compromised", including due to cross-platform synchronization features breaking the assumption of a mobile phone as an independent second factor.

For usability of password managers see Chiasson [15]; Florêncio [25] explores managing password portfolios. Biddle [4] surveys graphical passwords. For Captchas, Hidalgo [34] gives an authoritative survey; Motoyama [46] explores underground CAPTCHAsolving economies; Egele [21] discusses CAPTCHA farms that leverage malware. Van Oorschot [58] extends the usability of the Pinkas-Sander protocol [51] by tracking failed login counts. Garfinkel [26] surveys usable security work, especially authentication. For password meters, see de Carné de Carnavalet [17] and zxcvbn [61].

For authoritative surveys on biometrics, see Jain [36, 35]. For effective attacks on fingerprint systems, including using "gummy bears" (gelatin), see Matsumoto [41]. For iris recognition, see Daugman [16, 32]. Ballard [3] notes "the evaluation of biometric technologies is usually conducted under fairly weak adversarial conditions", encourages use of more realistic attack models (e.g., "trained" forgers), and demonstrates successful impersonation attacks on handwriting (a behavioral biometric) using generative attacks that synthesize forged inputs. For attacks on touchscreen implicit authentication schemes (another behavioral biometric), see Khan [37]. For background on keystroke dynamics (latencies), see Monrose [45]. For using biometrics for cryptographic key generation, see Ballard [2]. For an introduction to ROC analysis, see Metz [44]. For a primer on secure Internet geolocation, see Muir [47].

Figure 3.9 follows Shannon [55, p.20], who introduces entropy and succinctly derives basic properties; see also McEliece [42], Garrett [27], Bonneau [7, Ch.3], Cachin [13] and Ferguson [23]. Weak password subspaces [59] are related to partial-guessing metrics and predictable user choices. Heuristics for estimating password strength based on Shannon entropy, popularized by SP 800-63 [12, Appendix A], were widely used to justify password composition policies despite explicit warning therein that they were very rough rules of thumb; the revision 13 years later [29, Part B], mentioned in Section 3.2, recommends against not only password aging (following [62, 14]), but also against composition policies. This followed expositions of the heuristics poorly matching practical attack strategies-by Weir [60], and Bonneau [8] who also gives a mechanism allowing collection of password statistics without human access to cleartext databases.

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[^0]:    ${ }^{1}$ This unpopular imposition on users is viewed as a failure to fully meet principle P11 (USER-BUY-IN).

[^1]:    ${ }^{2}$ The Pinkas-Sander protocol (Section 3.7) avoids this denial of service (DoS) problem.
    ${ }^{3}$ This follows the principle of DESIGN-FOR-EVOLUTION (HP2).

[^2]:    ${ }^{4}$ For example, in 1985, FIPS 112 [49] noted that many user-chosen passwords are easily guessed, and therefore all passwords should be system-generated.

[^3]:    ${ }^{5}$ Because the most-popular passwords are also most easily guessed, failure to use denylists goes against two principles: P12, SUFFICIENT-WORK-FACTOR and P13, DEFENSE-IN-DEPTH (equal-height fences).

[^4]:    ${ }^{6}$ Some users believe false answers improve security; empirical studies have found the opposite.

[^5]:    ${ }^{7}$ Recall we want equal-height fences (P13 DEFENSE-IN-DEPTH); cf. recovery channels (Section 3.3).

[^6]:    ${ }^{8}$ Alternate strategies include regularly relying on password resets, and single sign-on means (Chapter 4).

[^7]:    ${ }^{9}$ The acronym originated from Completely Automated Public Turing test to tell Computers and Humans Apart, but as common implementations are often proprietary, the middle part may better be Program to Tell.

[^8]:    ${ }^{10}$ Here in Section 3.8, following tradition, $H$ denotes the Shannon entropy function (not a hash function).

[^9]:    ${ }^{11} G_{1}$ 's sum in (3.7) assigns, to each event in $X$, a guess-charge $i$ weighted by a probability $q_{i}$, with optimal order dictating $q_{i} \geq q_{i+1}$. For the 32 elements in $S$ alone, we expect to need $2^{5} \cdot 2^{127}=2^{132}$ guesses; but if the average guess-charge for each of the sum's first $m$ terms were $2^{107}$, the guesswork component for these $m<n$ terms would be $m \cdot 2^{107}=2^{132}$. All terms in the sum are non-negative. It follows that $G_{1}(\mathcal{R})>2^{107}$.

