Performance Analysis of First-Fit Wavelength Assignment Algorithm in Optical Networks

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***Abstract*— This paper proposes a new analytical technique for the performance analysis of all optical networks which use the first-fit algorithm for wavelength assignment. We analyze the wavelength usage on the links to calculate the blocking probability of a source destination pair, taking into account wavelength correlation and load correlation between links. Our model is accurate even in a system with large number of wavelengths.**

I. Introduction

Transmission systems using copper line are limited in bandwidth and constrained by electronic processing speed. With the advance of optical technology, copper lines are gradually replaced by optical fiber. Optical transmission systems provide high bandwidth. Sonet, a system using optical fiber, can transmit data in several gigabytes per second per fiber. Since it uses only one channel for each strand of fiber to transfer data, such optical systems are inefficient in using the bandwidth. Wavelength Division Multiplexing (WDM) technology [1] is proposed as an alternative method for carrying data. With WDM, one strand of fiber can accommodate about 100 channels nowadays (2003) and more in the future. WDM systems are also transparent for data format and data rate (i.e. they can transfer data in any format with any rate), therefore, they promise to integrate data and voice into one telecommunication system. Sitting in the heart of WDM technology is the Routing and Wavelength Assignment (RWA) problem [2]. Different from the routing problem in traditional circuit switched systems, RWA needs to find a path and assign the same wavelength to every link along the path. This is known as the wavelength continuity constraint. This means there are cases in which connections can not be established even if every link along the path has wavelengths available. Thus, the shortest path obtained by executing Dijkstra or Bellman-Ford algorithm [3] is not necessary the best solution to the RWA problem. In the literature, there are two methodologies proposed to tackle the RWA problem. One is to consider the Routing problem and Wavelength Assignment problem as coupled RWA problem and model the RWA as mixed integer linear programming problem [4]. This approach tries to obtain the exact optimal solution at the expense of complexity. The other approach is to consider the Routing problem and Wavelength Assignment problem separately. The solution obtained is suboptimal but practical to use. Several routing algorithms are proposed in the literature of which some representatives are listed below:

- Fixed routing [5]: The path for each source-destination ($s-d$) pair is calculated off-line using an algorithm, say, Dijkstra algorithm.
- Fixed alternate routing [6], [5]: Instead of calculating one path for each pair, fixed alternate routing calculates off-line several alternate paths for each pair.
- Adaptive routing [7]: The paths are calculated on-line, depending on the network state which reflects the resource usage.

Wavelength Assignment algorithms, among possible others, include

- Random wavelength assignment [8]: A wavelength is selected randomly from the available wavelengths.
- First-fit assignment [9], [7]: All wavelengths are numbered. The wavelength with the lowest number is selected from the available wavelengths.
- Most-used assignment [7]: The most used wavelength is the wavelength that has the highest number of links in the network that use the wavelength. The most-used algorithm selects the most used wavelength from the available wavelengths on the path.
- Distributed relative capacity loss (DRCL) [10].

First-fit does not require global knowledge about the network. No storage is needed to keep the network states and no communication overhead is needed. The computational overhead is small and the complexity is low. Moreover, the performance in terms of blocking probability and fairness is among the best. Therefore, first-fit is preferred in practice [10], [11]. Although the performance analysis of optical WDM networks is studied extensively in the literature [8], [12], [13], [14]and [15], there is less work for analyzing the blocking performance of first-fit scheme. The overflow model is used

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to study the first-fit scheme [7], [16]. Reference [7] simply uses Erlang-B formula to approximate the overflow traffic from individual wavelength. However, the overflow traffic is bursty, therefore, modeling it as Poisson process underestimates the link blocking probability. Reference [16] applies the equivalent random method [5] to approximately calculate the blocking probability of the overflow traffic, but the accuracy of the model decreases as the number of wavelength increases [16]. In this paper, we provide a new method to analyze the first-fit scheme which remains reasonably accurate with a large number of wavelengths.

The paper is organized as follows. Section 2 describes the framework for the performance analysis. Numerical and simulation results are provided in section 3. We conclude in section 4.

II. ANALYTICALMODEL FOR FIRST-FIT PERFORMANCE

A. *Network and Traffic Assumptions*

We use the fixed routing algorithm to calculate the path between a source-destination ($s - d$) pair. Therefore, we assume that the path between the $s - d$ pair is calculated off-line and remains fixed, therefore, the path uniquely identifies the $s - d$ pair. When a call comes, a connection will be established on the path if the same wavelength is free on all the links of the path. If there is no such a wavelength free, the call is blocked and lost. The ratio of the calls lost to the total calls is the blocking probability for calls transmitted on the path of the $s - d$ pair. The goal of our analysis is to calculate this blocking probability. In order to do the analysis, we further make the following general assumptions:

- The network is connected in an arbitrary topology. Each link of the network has the same number of C wavelengths.
- For a $s - d$ pair with path P in the network, the call requests arrive according to Poisson Process with rate A^P. The duration (holding time) of each call spending on the path is exponentially distributed with unit mean. Call requests are called the *offered traffic* to the path P.
- The first-fit algorithm [9], [7] is used to assign the wavelengths on the path.
- We also assume the signaling time for establishing the connection between a $s-d$ pair using first-fit algorithm is negligible compared to the traffic interarrival and service times.

B. *A Simple Case*

We will first discuss a simple case to show the traffic interference for the first-fit algorithm and then give a description of general approach to calculate the blocking probability of a $s - d$ pair in the network. Detailed results will be presented later.

The readers should refer to Fig. 1. We have five nodes and four $s - d$ pairs. The number of wavelength used by each $s - d$ pair is also shown. We want to calculate the blocking probability between nodes 2 and 4. If we assume each link has two wavelengths, the traffic between 2 and 4 will be blocked,

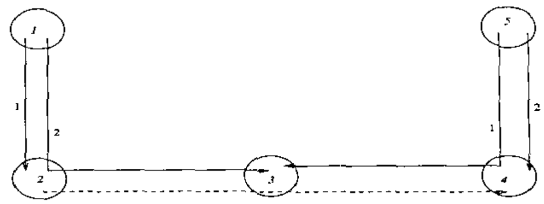

Fig. 1. A simple case.

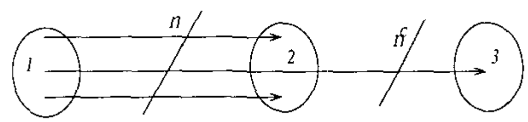

Fig. 2. Link with interference.

although only one wavelength is used in link 23 and link 34. If there were no paths carrying traffic to $(1 - 2)$ and $(5 - 4)$ pairs, the traffic between nodes 2 and 4 would not have been blocked. With respect to path between nodes 2 and 4, we call the $(1 - 3)$ and $(5 - 3)$ pairs new coming paths and $(1 - 2)$ and $(5 - 4)$ pairs interference paths to $(1 - 3)$ pair and $(5 - 3)$ pair respectively. From this example, we can see that not only the new coming paths have influence on blocking probability, but also that the interference paths to new coming paths have a direct influence too.

In order to calculate the blocking probability on the path, we first calculate the probabilities that wavelengths are used in the new coming paths. This is done in the following section.

C. *Wavelengths Usage Probabilities*

When paths share a link, they contend for use of the resources (the wavelengths) on that link. The wavelengths on the link are assigned according to first-fit algorithm. Under the first-fit algorithm, different wavelengths will have different probabilities of being occupied. The probability that a wavelength is used is defined as the **usage probability** of that wavelength. In the following, we will calculate the wavelength usage probabilities in certain cases.

1) Wavelength Usage Probabilities on Single Link: Let us consider a link l. We assume the paths that share l are P_1, P_2, ..., P_n. The call requests rates to these paths are A^{P_1}, A^{P_2}, ..., A^{P_n} respectively. We assume these call requests

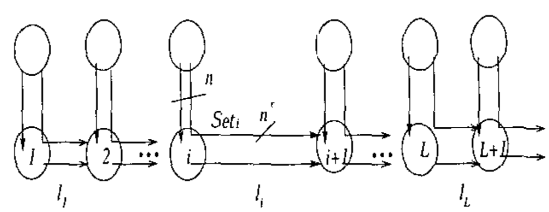

Fig. 3. Path P.

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undergo no interference before and after link l. This implies that the call requests to these paths are totally contributed to the offered traffic of link l without being thinned in other links. If there is interference on these paths before and after link l, the reduced traffic model [17], [18] is used to calculate the offered traffic to a link. However, this interference is small under light traffic, and we do not use it in order to reduce the mathematical complexity. Since the call requests follow the Poisson Process, the offered traffic to link l is also Poisson Process with rate $A^l = \sum_{i=1}^{n} A^{P_i}$. With A^l as input, we can calculate the wavelength usage probabilities on link l. Let $U_k^l (k = 1, 2, \ldots, C)$ be the usage probability of wavelength k on link l. We define that l is in state S $(S = 0, 1, \ldots, C)$ if S wavelengths are used on link l. Let E_S^l be the event that l is in state S. According to Erlang-B Formula, we have

$$P(E_S^l) = \frac{(A^l)^S}{S!} P(E_0^l) \qquad (S = 1, 2, \ldots, C) \qquad (1)$$

$$P(E_0^l) = \frac{1}{1 + \sum_{i=1}^{C} \frac{(A^l)^i}{i!}} \qquad (2)$$

where $P(E_S^l)$ is the probability that E_S^l occurs. Since we assumed $P_1, P_2, \ldots, P_n$ undergo no interference before and after link l and first-fit algorithm is used, we can argue that when l is in state S, wavelengths 1 to S are used. This is an approximation for the first-fit wavelength assignment and implies that when l is in state S, wavelengths with indices bigger than S can be used. Therefore,

$$U_k^l = \sum_{S=k}^{C} P(E_S^l) \qquad (k = 1, 2, \ldots, C) \qquad (3)$$

The formula tends to overestimate the usage probabilities of wavelengths with small indices and underestimate the usage probabilities of wavelengths with big indices.

2) Wavelength Usage Probabilities on Link with Interference: In reality, the interference between paths are complicated such that it is impossible to account all of them. A plausible approach is to make assumption to simplify the problem and yet catch the essence of the problem. For example, In Fig. 1, path $(1-2)$ and $(1-3)$ interfere with each other directly. Path $(2-3)$ and $(2-4)$ also interfere with each other directly. Although path $(1-2)$ and $(2-4)$ do not directly interfere with each other, they do interfere with each other indirectly by both affecting path $(1-3)$. Even path $(1-2)$ and $(5-4)$ have interference with each other. So it is not possible for any analysis to faithfully model all interference. Therefore, we only consider the direct interference between two paths. We first discuss the scenario in Fig. 2. The result will be used to calculate the blocking probability for a large network which is decomposed into parts as in Fig. 2.

In Fig. 2, we have three nodes and link 12 and link 23. We assume there are n paths on link 12 and n^c paths continuing from link 12 to link 23. We will calculate the wavelength usage probabilities on link 23.

We assume a path on link 12 has probability of $\frac{n^c}{n}$ to continue to link 23. Let U_k^{12} be the usage probability of wavelength k on link 12 which can be calculated by (3). Let U_k^{23} be the usage probability of wavelength k on link 23. Then we approximate U_k^{23} by:

$$U_k^{23} = \frac{n^c}{n} U_k^{12} \qquad (k = 1, 2, \ldots, C) \qquad (4)$$

where U_k^{12} can be calculated by (3).

Experiment study shows the approximation is accurate.

D. Wavelengths Correlation Model and Calculation of Path Blocking

Given a $s-d$ pair and its path P, which consists of links $l_1, l_2, \ldots, l_L (L > 1)$ (Fig. 3), we will establish the model to calculate the blocking probability on path P.

Let Set_i $(i = 1, 2, \ldots, L)$ be the set of paths on link l_i that are new (not continuing from link l_{i-1}). Let $Set = \bigcup_{i=1}^{L} Set_i$, i.e. Set is the set of paths that are new along the link $l_1, l_2, \ldots, l_L$. We assume wavelengths used in Set_i $(i = 1, 2, \ldots, L)$ are independent. Let U_k^{Set} be the wavelength k usage probability in Set. Let $U_k^{l_i}$ be the wavelength k usage probability in Set_i $(i = 1, 2, \ldots, L)$. Then we have

$$U_k^{Set} = 1 - \prod_{i=1}^{L} (1 - U_k^{l_i}). \qquad (5)$$

where $U_k^{l_i}$ can be approximated by (4). n and n^c are defined accordingly (the reader may refer to Fig.3).

Let Set_1^c be the set of paths on link l_1 that are not in Set_1. To calculate the blocking probability on path P, we can argue as follows. If C wavelengths are used in Set_1^c, the call will be blocked. If $C-1$ wavelengths are used in Set_1^c and one wavelength is used in Set, the call will also be blocked. Exhausting all the cases, we obtain the following formula for the path blocking probability.

$$B = P(E_C^{l_1^c}) + \sum_{S=0}^{C-1} P(E_S^{l_1^c}) P_{C-S}^{Set} \qquad (6)$$

where $E_S^{l_1^c}$ $(S = 0, 1, \ldots, C)$ is the event that Set_1^c is in state S and P_S^{Set} is the probability that at least S wavelengths are used in Set. $P(E_S^{l_1^c})$ $(S = 0, 1, \ldots, C)$ is calculated by (1) or (2).

In order to calculate P_S^{Set}, we need to analyze the correlation between wavelengths. Since the first-fit algorithm is used to assign wavelengths, a wavelength with low index has higher probability to be used. This leads to the following assumption. We assume the wavelengths used in Set have the following relationship: For any wavelengths with indices ω_1, ω_2 such that $\omega_1 < \omega_2$ $(\omega_1, \omega_2 = 1, 2, \ldots, C)$,

$$E_{\omega_2} \subset E_{\omega_1} \qquad (7)$$

where E_{ω_1} and E_{ω_2} are the events that wavelength ω_1 and ω_2 are used on Set respectively.

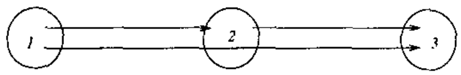

Fig. 4. Case 1: 3 nodes.

This relationship is due to the first-fit algorithm. It indicates if a wavelength is used, then any wavelengths with lower index are also used. Under this assumption, we have

$$P_S^{Set} = P\Big(\bigcup_{\substack{\omega_1\cdots\omega_S\\ 1\le\omega_1<\ldots<\omega_S\le C}} (E_{\omega_1}\bigcap E_{\omega_2}\bigcap\cdots\bigcap E_{\omega_S})\Big) = U_S^{Set} \quad (8)$$

where U_S^{Set} is given by (5).

We first give an example and then the proof of (8). Let $C = 5, S = 3$. We use indices to represent the events and remove $\bigcap$ in the example to simplify the notation.

$$P\Big(\bigcup_{\substack{\omega_1\omega_2\omega_3\\ 1\le\omega_1<\omega_2<\omega_3\le 5}} \omega_1\omega_2\omega_3\Big)$$

$= P(12\underline{3}\bigcup 12\underline{4}\bigcup 12\underline{5}\bigcup 13\underline{4}\bigcup 13\underline{5}\bigcup 14\underline{5}\bigcup 23\underline{4}\bigcup 23\underline{5}\bigcup 24\underline{5}$ $\bigcup 34\underline{5}) = P(3\bigcup 4\bigcup 5) = P(3)$.

For proving (8), we use (7) and get: $E_{\omega_1}\bigcap E_{\omega_2}\bigcap\ldots\bigcap E_{\omega_S}$ $= E_{max\{\omega_1,\omega_2,\ldots,\omega_S\}}$. Therefore,

$$P\Big(\bigcup_{\substack{\omega_1\cdots\omega_S\\ 1\le\omega_1<\ldots<\omega_S\le C}} (E_{\omega_1}\bigcap E_{\omega_2}\bigcap\cdots\bigcap E_{\omega_S})\Big) = P(E_S\bigcup E_{S+1}\bigcup\cdots\bigcup E_C)$$

Also from (7), we have

$$P(E_S\bigcup E_{S+1}\bigcup\cdots\bigcup E_C) = P(E_S).$$

Therefore, formula (8) is true.

We have given the formulas to approximate the blocking probability on path that has length greater than one. For the special case of path comprising a single link, we can simply use Erlang-B formula to approximate the blocking probability. This is because on the single link path the traffic will not be blocked as far as there is an available wavelength on the link. We may have used the reduced traffic model [17], [18] to calculate the blocking probability. However since the blocking probabilities are small, we can get reasonable estimates without using the reduced traffic model.

III. Numerical and Simulation Results

In this section we present the comparison of our numerical results with the simulation results.

In all the simulation cases, we assume the arrival traffic follows a Poisson process and the service time is exponential distributed with mean 1 second. The rates in the simulation of numerical results are expressed in requests/sec.

The first case is for the simple network topology as shown in Fig. 4. We have 3 nodes and 3 $s-d$ pairs: $(1-2)$, $(2-3)$ and $(1-3)$. Each pair has the same arriving rate and the same

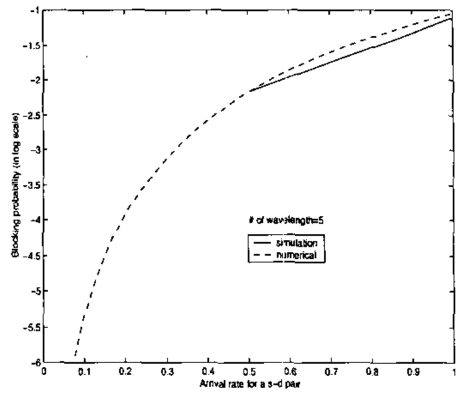

Fig. 5. Blocking prob. of 3 node network for path of length 2.

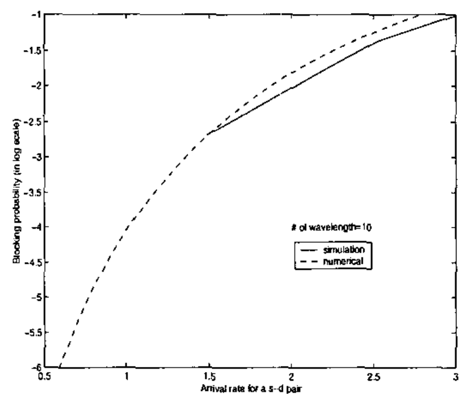

Fig. 6. Blocking prob. of 3 node network for path of length 2.

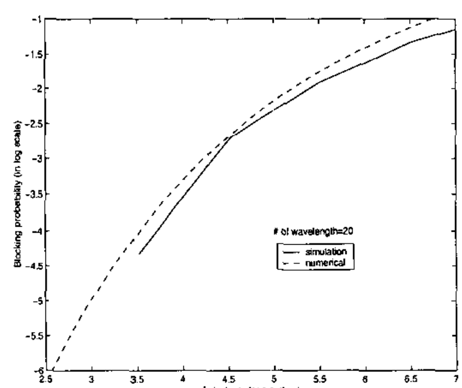

Fig. 7. Blocking prob. of 3 node network for path of length 2.

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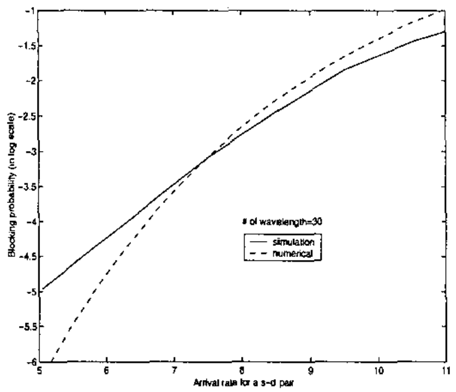

Fig. 8. Blocking prob. of 3 node network for path of length 2.

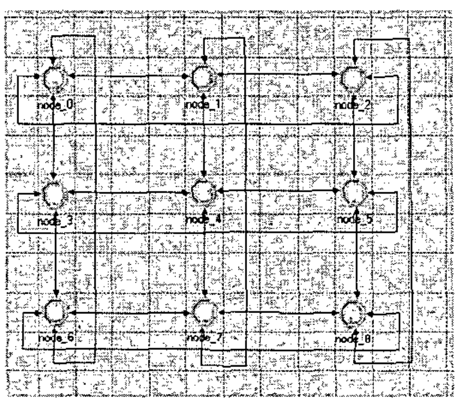

Fig. 9. Case 2: 9 nodes mesh torus.

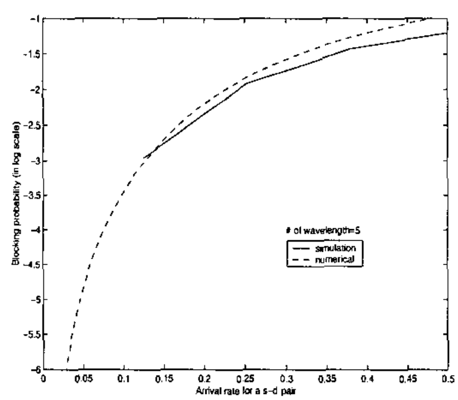

Fig. 10. Blocking prob. of 9 node network for path of length 1.

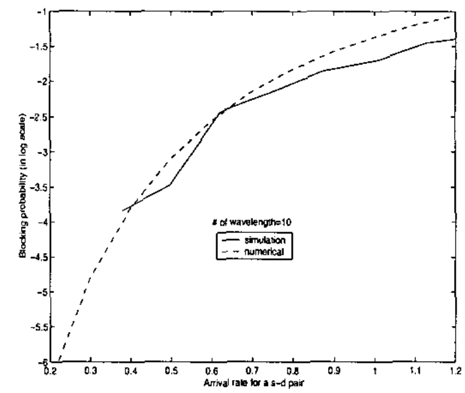

Fig. 11. Blocking prob. of 9 node network for path of length 1.

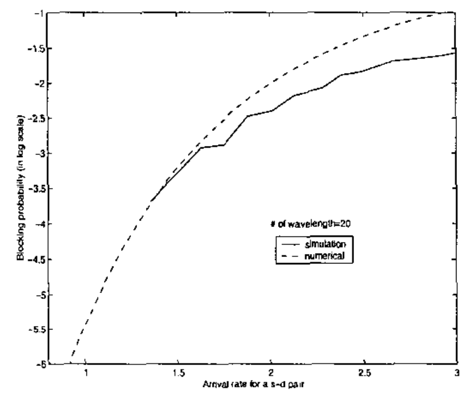

Fig. 12. Blocking prob. of 9 node network for path of length 1.

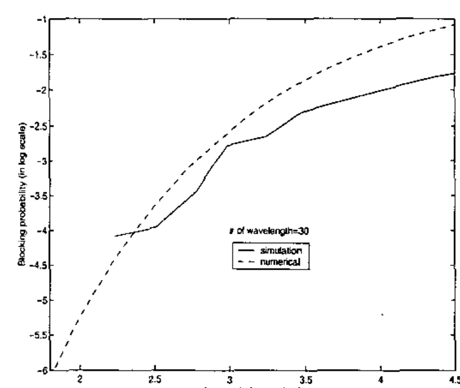

Fig. 13. Blocking prob. of 9 node network for path of length 1.

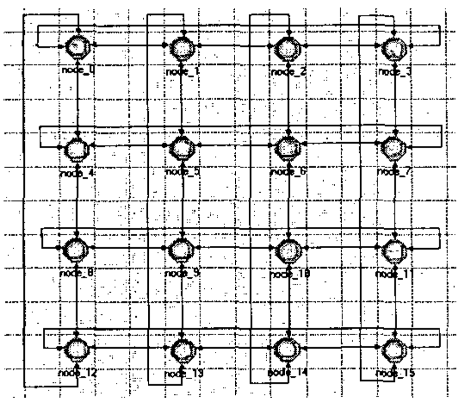

Fig. 14. Case 3: 16 nodes mesh torus.

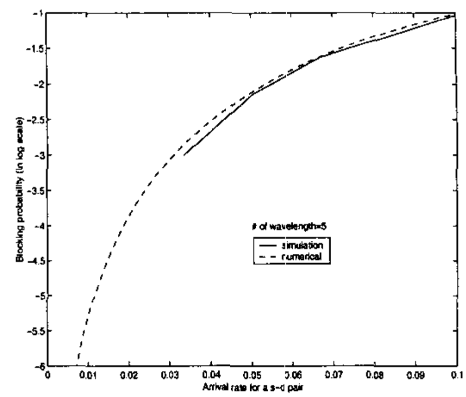

Fig. 15. Blocking prob. of 16 node network for path of length 3.

number of wavelengths. Traffic is one way from source to destination. We calculate the blocking probabilities (1 − 3). We use 4 different numbers of wavelengths on each link i.e. 5, 10, 20 and 30 wavelengths. The results are shown in Fig. 5 - Fig. 8.

The second case is the nine node mesh torus network as shown in Fig. 9. Each node has the same probability to send packet to the rest of nodes. The routing algorithm for the nine node mesh torus network is: 1) shortest path: If one node want to send packet to other node inside the network, it will choose the shortest path. 2) column first: If there are two or more shortest paths, choose the path which goes through the node within the same column of the source node first, e.g., if node 0 wants to send packet to node 4, the packet will be sent to node 3 first, then node 3 will send it to node 4. In this case, we can reckon that each link has 6 $s-d$ pairs to share its resources with. Among which, 4 are new coming paths.

We simulate the blocking probabilities between node 3 and node 4 with 4 different numbers of wavelengths. The results are shown in Fig. 10 - Fig 13.

The third simulation experiment involves the sixteen node mesh torus network as shown in Fig. 14. Each node has the same probability to send packet to the rest of nodes. The routing algorithm is same as for the nine node mesh torus network.

In this case, we can see that each link has 16 $s-d$ pairs to share its resource with. Among which, 12 are new coming pairs. We simulate the blocking probabilities between node 5 and node 7 and between node 4 and node 7 with 3 different numbers of wavelengths. The results are shown in Fig. 15 - Fig 20.

In most cases, the numerical results are a bit higher than the simulation results. A possible reason for this is that we do not use the reduced traffic model [17], [18]. Finally, we see that the results are not sensitive to the number of wavelength and our numerical approach remains accurate even with a relative large number of wavelengths.

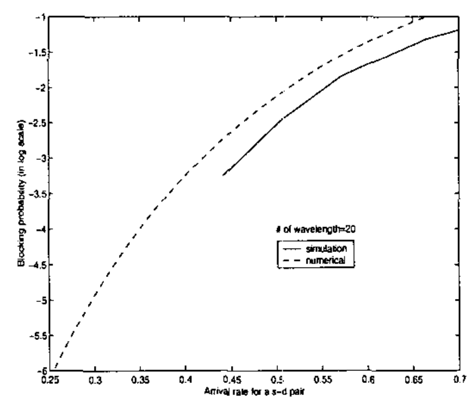

Fig. 16. Blocking prob. of 16 node network for path of length 3.

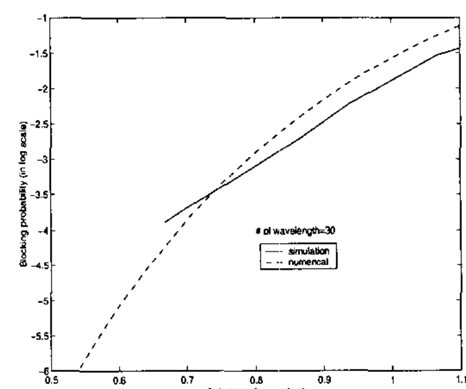

Fig. 17. Blocking prob. of 16 node network for path of length 3.

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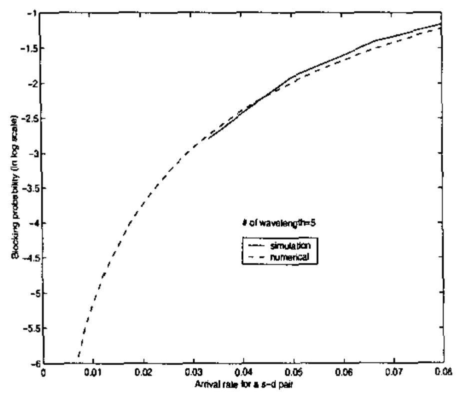

Fig. 18. Blocking prob. of 16 node network for path of length 4.

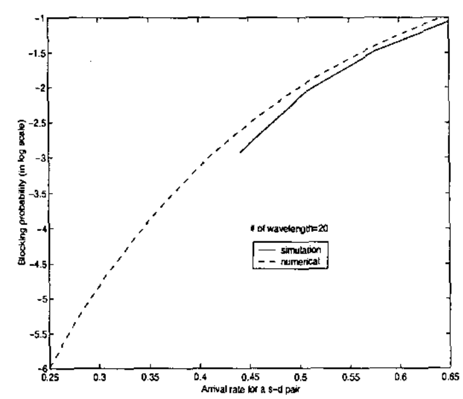

Fig. 19. Blocking prob. of 16 node network for path of length 4.

IV. Conclusion

We have presented a new analytical technique for the performance analysis of optical networks which uses the first-fit algorithm for wavelength assignment. The model is based on state analysis of wavelength usage on the links to calculate the blocking probability of a source destination pair. Through the comparison study of the simulation results and numerical results, we can conclude that that our model can be used to estimate the blocking probabilities even in a network with large number of wavelengths.

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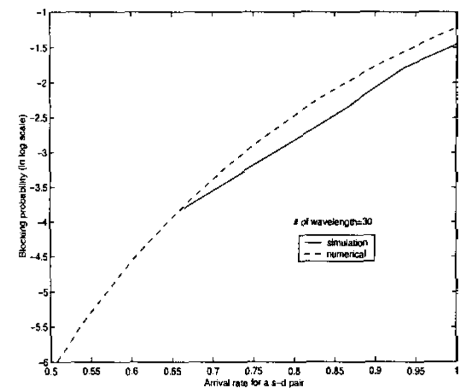

Fig. 20. Blocking prob. of 16 node network for path of length 4.

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