CARLETON UNIVERSITY
SCHOOL OF COMPUTER SCIENCE
WINTER 2015
COMP. 3803
INTRODUCTION TO THEORY OF COMPUTATION
ASSIGNMENT I
DUE: FRIDAY JAN. 30, 2015 (11:30 AM)

Assignment Policy: Late assignments will **not** be accepted. You are expected to work on the assignments on your own. Past experience has shown conclusively that those who do not put adequate effort into the assignments do not learn the material and have a probability near 1 of doing poorly on the exams.

Important note: When writing your solutions, you must follow the guidelines below.

- Please write your name and student number clearly. Your last name must be in Upper Case.
- The answers should be concise, clear and neat. Make sure that your TA can read your solutions.
- Please submit the solutions *in the order of the problems*, the solution to Problem 1, then to Problem 2 and so on.
- When presenting proofs, every step should be justified so as to get partial credit.
- Assignments should be stapled (or in an unsealed envelope) *with your name and student number*.

Substantial departures from the above guidelines will not be graded.

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1. Prove that the *sum* of n real numbers is rational if all of them are rational. Is the converse true? Prove or disprove that the product of n real numbers is rational (resp. irrational) if they are all rational (resp. irrational).

2. Prove that if n is a positive integer, then n is odd *if and only if* 5n+6 is odd.

3. Show by induction that $n^5 – n$ is divisible by 5 for all $n \geq 0$.

4. Show by induction that $n^3 – n$ is divisible by 3 for all $n \geq 0$.

5. We had shown in class that the set of real numbers in the interval [0, 1] is uncountable. What can you then say about the cardinality of the set of real numbers in the interval [0.5, 0.6]? If it is countable, why is it? If it is uncountable, present the arguments in the same way we did the proof for the interval [0, 1]. Given what was taught in class, could you have come up with an easier proof?

6. Let ***A*** be the set of all even natural numbers, and ***B*** be the set of natural numbers divisible by 3. Prove that the set of fractions a/b where $a \in \boldsymbol{A}$ and $b \in \boldsymbol{B}$ is countable.

7. For arbitrary strings X and Y, show that $(X.Y)^R = Y^R.X^R$, where, by notation, V^R is the string obtained by reversing the string V. In this question, and in the questions below, '.' is used to represent concatenation.

8. For any language ***A***, let $\boldsymbol{A}^R$ be $\{ X^R \mid X \in \boldsymbol{A} \}$. Then, for arbitrary *languages* ***A*** and ***B***, show that $(\boldsymbol{A}.\boldsymbol{B})^R = \boldsymbol{B}^R.\boldsymbol{A}^R$, and that $(\boldsymbol{A} \cup \boldsymbol{B})^R = \boldsymbol{A}^R \cup \boldsymbol{B}^R$. Your arguments must be brief but accurate.

9. If the languages ***A*** and ***B*** are countably infinite and we use the notation of Question 8, what can you say about the size of the language obtained by concatenating $(\boldsymbol{A}.\boldsymbol{B})^R$ and $(\boldsymbol{A} \cup \boldsymbol{B})^R$? Is the cardinality of the set $(\boldsymbol{A}.\boldsymbol{B})^R.(\boldsymbol{A} \cup \boldsymbol{B})^R$ any larger or smaller than the size of the language obtained by concatenating $\boldsymbol{B}^R$ and $\boldsymbol{A}^R$?