

CARLETON UNIVERSITY
SCHOOL OF COMPUTER SCIENCE
WINTER 2017

Comp 5005

Assignment I

Due Jan. 17, 2017

1. Suppose each of three persons tosses a coin. If the outcome of one of the tosses differs from the other outcomes, then the game ends. If not, then the persons start over and re-toss their coins. Assuming fair coins, what is the probability that the game will end with the first round? If all three coins have a probability $1/5$ of landing heads, what is the probability that the game will end at the first round?
2. Assume that each child born is equally likely to be a boy or a girl. If a family has three children, what is the probability that they will all be boys, given that (a) the eldest is a boy, (b) at least one is a boy?
3. Two fair dice are rolled. What is the probability that at least one is a '4'? If the two faces are different, what is the probability that at least one has a value greater than or equal to '4'?
4. Four fair dice are thrown. What is the probability the same number appears on at least two of the three?
5. Suppose that 4 percent of men and 2 percent of women are color-blind. A color-blind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females.
6. If you have two fair dice, what is the conditional probability that the first die is '5' given that the sum of the dice is seven?
7. In a class there are 'a' first year male students, 'b' second year male students, and 'c' second year female students. How many first year *female* students must be present if gender and class are to be independent when a student is selected at random?
8. Consider two boxes, one containing eight black and two white marbles, the other four black and six white marbles. A box is selected at random and a marble is drawn at random from the selected box. What is the probability that the marble is black? What is the probability that the first box was selected given that the marble is white?
9. Urn 1 contains two white balls and six black balls, while Urn 2 contains six white balls and four black balls. One ball is drawn at random from Urn 1 and placed in Urn 2. A ball is then drawn from Urn 2. It happens to be white. What is the probability that the transferred ball was white?
10. Stores *A*, *B*, and *C* have 40, 60 and 90 employees, and respectively 30, 50, and 70 percent of these are women. Resignations are equally likely among all employees, regardless of gender. One employee resigns and this is a woman. What is the probability that she works in store *C*?
11. (a) A gambler has in his pocket a fair coin and a two-headed coin. He selects one of the coins at random, and when he flips it, it shows heads. What is the probability that it is the fair coin? (b) Suppose that he flips the same coin a second time and again it shows heads. What is now the probability that it is the fair coin? (c) Suppose that he flips the same coin a third time and it shows tails. What is now the probability that it is the fair coin?
12. There are three coins in a box. One is a two-tailed coin, another is a fair coin, and the third is a biased coin which comes up heads 75 percent of the time. When one of the three coins is selected at random and flipped, it shows tails. What is the probability that it was the two-tailed coin?
13. Suppose we have five coins which are such that if the i^{th} one is flipped then heads will appear with probability $i/5$, $i = 1, 2, \dots, 5$. When one of the coins is randomly selected and flipped, it shows heads. What is the conditional probability that it was the fourth coin?
14. Urn 1 has seven white and three black balls. Urn 2 has four white and six black balls. We toss a fair coin. If the outcome is heads a ball from Urn 1 is selected and if the outcome is tails a ball from Urn 2 is selected. Suppose that a white ball is selected. What is the probability that the coin landed tails?
15. A urn contains b black and r red balls. One of the balls is drawn at random, but when it is put back, c additional balls of the same color are put in with it. Suppose that we draw another ball. Show that the probability that the first ball drawn was black given that the second ball drawn was red is $b/(b + r + c)$.