

Generation Of Noisy Vectors - Normally Distributed.

Aim: To generate Noisy D-dimensional Vectors whose Mean is μ and Covariance Matrix Σ .

Theory: Suppose we had a vector X whose mean was μ and Covariance Matrix was Σ .

Let Φ be the eigenvector Matrix of Σ and Λ the eigenvalue Matrix of Σ .

$$\Phi = [\phi_1 \ \phi_2 \ \dots \ \phi_D] \quad \Lambda = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \\ & & & \lambda_D \end{bmatrix}, \quad \begin{array}{l} \phi_i \text{ is eigenvector of} \\ \text{eigenvalue } \lambda_i. \end{array}$$

We showed you that to "Diagonalize" X to have independent vector components we use transformation:

$$Z = \Lambda^{-1/2} \Phi^T X$$

Z now has a covariance Matrix $\begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$.

Thus if we generate samples of type Z

$$\Phi \Lambda^{1/2} Z \text{ will be of type } X$$

i.e. have Covariance Matrix Σ .

This is because $\Phi^{-1} = \Phi^T$.