CARLETON UNIVERSITY
SCHOOL OF COMPUTER SCIENCE
WINTER 2022

COMP 5107   **Assignment I**   **Due: January 21, 2022**

1.   In roulette there are 16 slots into which a ball may land. They are numbered 0, 00, 000, 0000 and 1 through 12. The odd numbers are red, even numbers are black, and the zeroes are green. A ball is thrown randomly into a slot.
   - (a)   What is the probability that it is green?
   - (b)   What is the probability that it is number 8?
   - (c)   What is the probability that it is either red, or between 4 and 8, but not both?
   - (d)   What is the probability that it is not red?

2.   A coin with the probability of heads being 0.4 and a fair six-sided die are used for a random experiment. The coin is first tossed. If the outcome is heads, the die is thrown twice. If the outcome is tails, the die is thrown only once. Find the probability of:
   - (a)   Throwing a sum less than 8.
   - (b)   Throwing a sum of which is an odd number greater than or equal to 7.
   - (c)   Throwing a sum of 7, 9 *or* 11.

3.   A box contains 600 silicon transistors, of which 20% are bad, and 300 germanium transistors, of which 30% are bad. If a transistor is selected at random and found to be good, what is the probability that it is germanium? If a transistor is selected at random and found to be bad, what is the probability that it is silicon?

4.   A candy machine has 20 buttons to make selections. One button never works, two buttons each work with a probability of 0.6, and the others work with a probability of 0.9. If the buttons are pushed at random with equal probability:
   - (a) Find the probability that the machine will not work on any one trial.
   - (b) If the machine did not work, find the probability that one of the buttons that never work was the one pushed.

5.   A coin with the probability of heads being 0.8 is tossed 4 times.
   - (a) What is the probability that the coin shows "tails" all the four times?
   - (b) What is the probability that 3 "heads" are obtained?
   - (c)   What is the probability that at least 2 consecutive "heads" are obtained?

6.   Assume that an old used car fails to start 10% of the time. The other 90% of the time it drives for a time period which is uniformly distributed in the interval 20 to 40 minutes. In this interval all subintervals of time of equal width between 20 to 40 minutes are equally likely.
   - (a) What is the probability that the car drives for at least 30 minutes **given** that it starts?
   - (b) What is the probability that the car drives for at least 30 minutes?

7.   Consider two urns with the following composition: Urn 1 contains 7 red and 3 black balls, Urn 2 contains 4 red and 6 black balls. Our random experiment consists of choosing one of the two urns with equal probability and then drawing a sample (without replacement) of 3 balls. If 3 red balls are drawn, what is the probability they came from Urn 1? How would the result change if the initial choice of the urn was as per the rule: Choose Urn 1 with probability 0.7 and Urn 2 with probability 0.3?

8.   Assume that a class contains 20 students. If the probability of a student obtaining an "A" is 0.7, a "B" is 0.2 and a "C" is 0.1, answer the following:
   - (a)   What is the probability that 7 students will obtain A's, 3 obtain B's, and 10 obtain C's?
   - (b)   What is *this* probability if we are given that at least two students receive C's?
   - (c)   If A has a grade value of 4, B has a grade value of 3 and C has a grade value of 1, what is the <u>expected</u> value of the grade of the class? Which law of probability will you use to compute this?