Generation Of Noisy Vectors - Normally Distributed.

Aim: To Generate Noisy D-dimensional Vectors whose Mean is M and Covariance Matrix Σ.

Theory: Suppose we had a vector X whose mean was M and Covariance Matrix was Σ.

Let Φ be the eigenvector Matrix of Σ and Λ the eigenvalue Matrix of Σ.

$$\Phi = \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_D \end{bmatrix} \quad \Lambda = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_D \end{bmatrix},$$

ϕ_i is eigenvector of eigenvalue λ_i.

We showed you that to "Diagonalize" X to have independent vector components we use transformation:

$$Z = \Lambda^{-1/2} \Phi^T X$$

Z now has a covariance Matrix $\begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & \ddots \; 1 \end{bmatrix}$.

Thus if we generate samples of type Z

$\Phi \Lambda^{1/2} Z$ will be of type X

i.e. have <u>Covariance Matrix Σ</u>.

This is because $\Phi^{-1} = \Phi^T$.

Algorithm. : Generate Z.

$Z \sim$ Normal - Mean $\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$

Covariance $\begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$

i.e. Z has D independent Gaussian Numbers

all $N(0,1)$.

Compute $X_1 = \Phi \Lambda^{1/2} Z$

Φ - eigen Vector Matrix of Given Σ'

Λ - eigen value Matrix of Given Σ'.

Thus X_1 has $\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$ Mean and Covariance Σ'.

Finally $X = X_1 + M$ $\quad$ M- Required Mean.

Thus X has Mean M and Covariance Σ'.